Topic: Basic Elements of a Quantum Computer

Key Questions: Which components (or features) does a generic quantum computer have?

Example: Memory to store information.

How are those different from a classical computer?

Your Background:
- General knowledge of how computers work
- Knowledge of quantum mechanics

COMBINE

Goal:
- Understand basic structure and operation of a quantum computer.
- Make use of this 'hardware independent' knowledge about quantum computers to evaluate different physical realizations

Outline:

Aspect of Classical Information Processing → Structure of a quantum computer → Features of its components

Classical Information Processing

- Carrier of information in binary representation

BIT — possible values: 1, 0 — physical representation:

example:

V
5 V
0 V

voltage level in a circuit

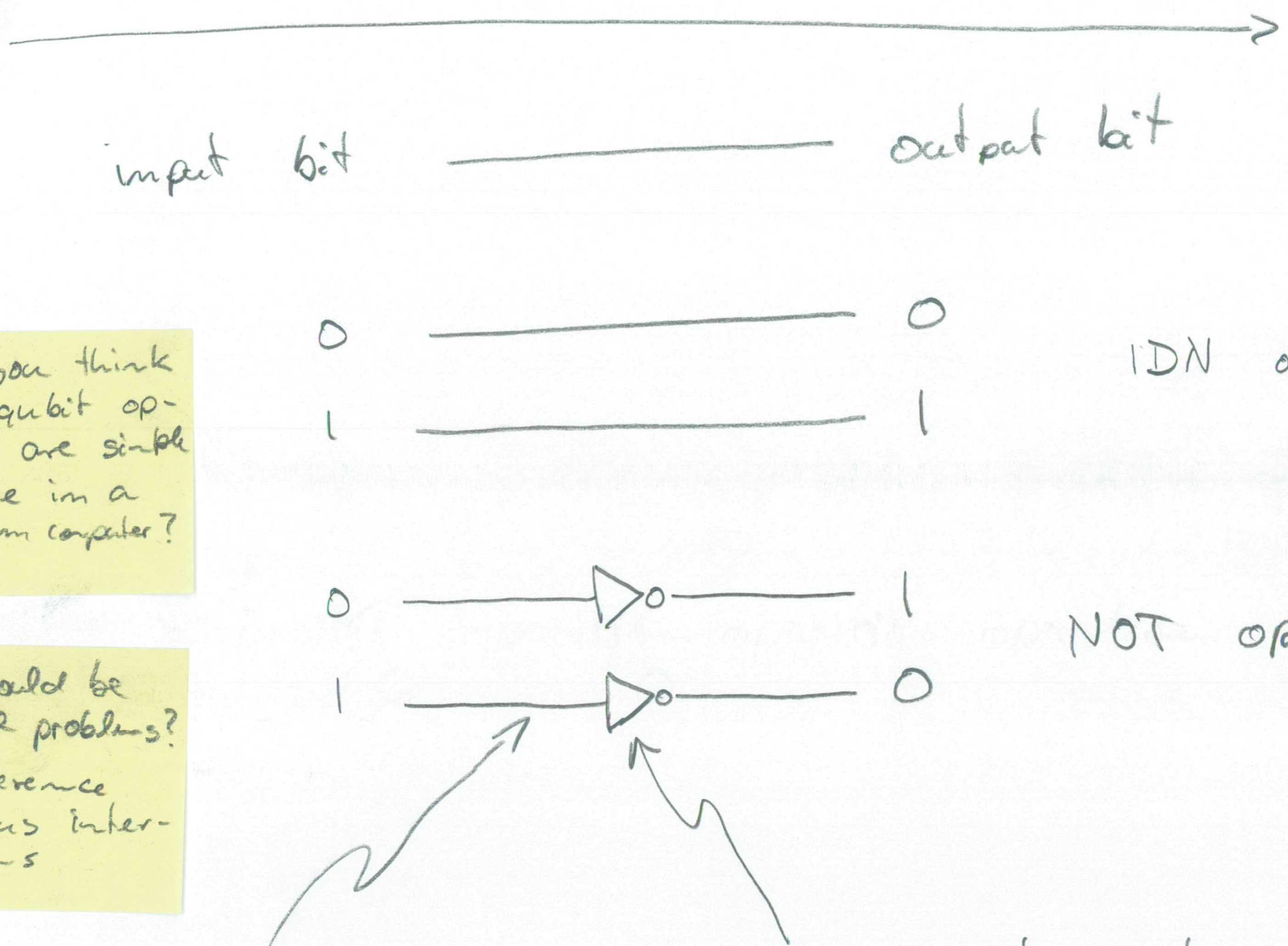
- modification of information in BIT by operating with a physical process on the BIT

> How is information physically modified in physical realizations of bits? Give examples.
>
> - CD
> - hard disk
> - RAM
>
> ⇒ The same question will be important in quantum computers!

- any logical operation on bits can be decomposed into single and two-bit operations

> - Why is this useful?
> - You may want to think about why this is possible at all!
> - Same is true for quantum computers.

Single Bit Operation



IDN operation

NOT operation

Would you think single qubit operations are simple to realize in a quantum computer?

What could be potential problems?
- decoherence
- spurious interactions
- ...

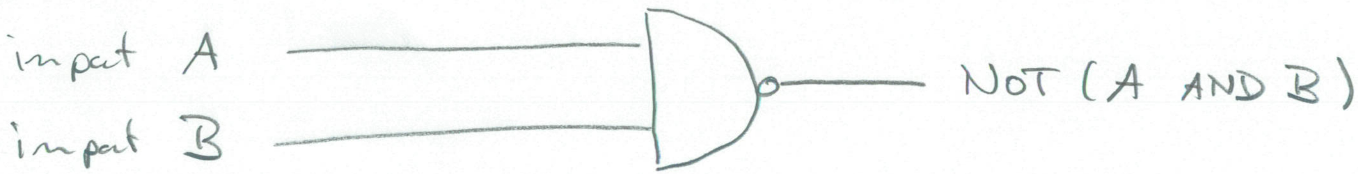
- wire represents bit
- preserves state

- symbol represents operation
- changes state

Same representation of information in circuit model for quantum computation

Two Bit Operations

The NAND gate



truth table

A	B	NOT (A AND B)
0	0	1
0	1	1
1	0	1
1	1	0

properties: – Universal logic gate

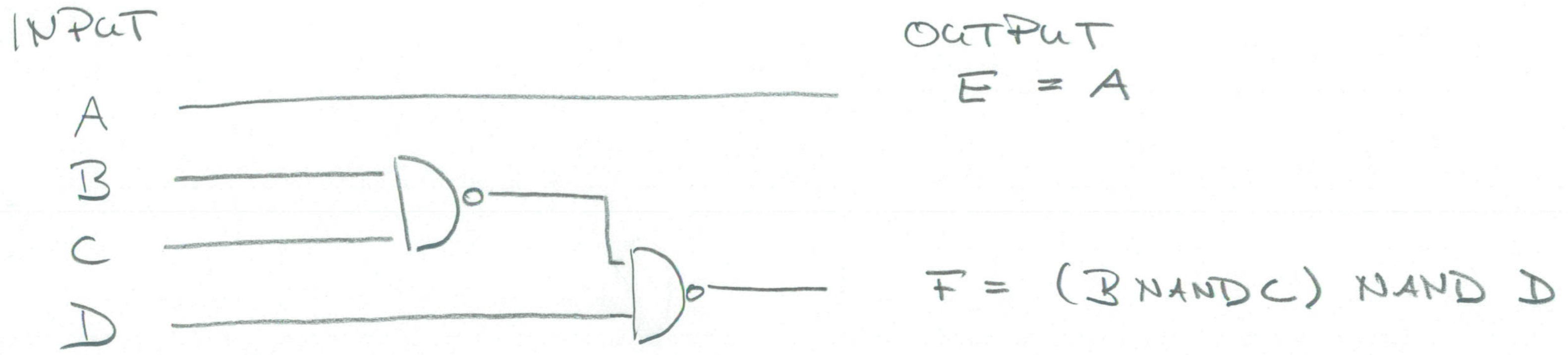
⇝ any function operating on bits can be computed using NAND gates

⇝ examples: AND, OR, XOR, NOR
(ancilla bits & possibility to make copies are required)

Why do you think universality is useful!

NOTE: Universal logic gates also do exist for quantum computers!

Circuit Representation



Any computable function can be represented as a circuit composed of logic universal gates acting on a set of input bits generating a set of output bits

Circuit properties :

- bits can be copied (FAN OUT)
- additional working bits are allowed (ANCILLAS)
- values of bits can be interchanged (CROSSOVER)
- number of output bits may be smaller than number of input bits
- loops are not allowed

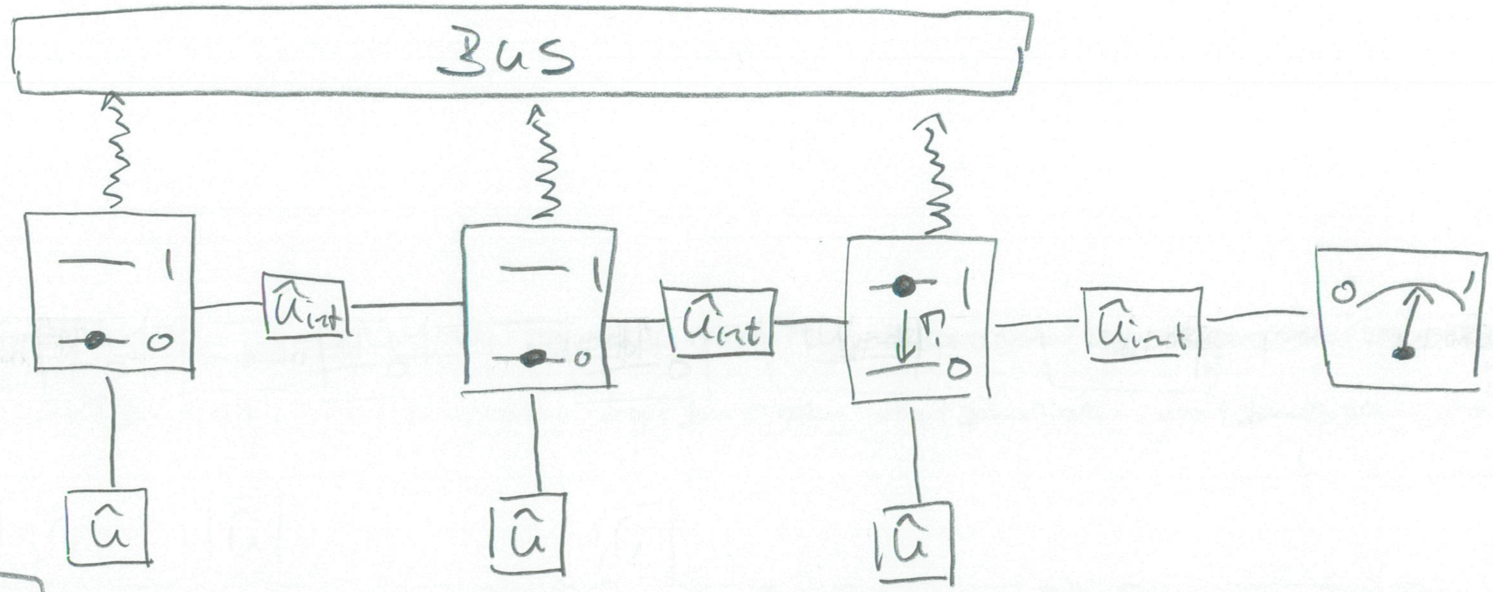
In your opinion, which of these properties might be easy / hard to realize in a quantum computer?

- reversibility ?
- preservation of state ?
- copying of information
- universality ?

A Generic Quantum Processor

features:

1. quantum bits
2. initialization
3. coherence
4. universal gates
5. read-out



DiVincenzo Criteria

6. conversion
7. transfer

In your opinion, what kind of components are absolutely essential to realize any quantum computer?

The Quantum Bit

Give examples of systems with this energy level spectrum.
Under which conditions would you be allowed to approximate a given quantum system as a two-level system?

- quantum mechanical system with two distinct states

- representation of qubit basis states

$$|0\rangle = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$$

$$|1\rangle = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

vectors in 2D Hilbert space
(1st QM postulate)

- general qubit state

$$|\psi\rangle = \alpha\,|0\rangle + \beta\,|1\rangle \qquad \text{with } \alpha, \beta \in \mathbb{C}$$

properties:
- qubit can be in superposition of states $|0\rangle$ and $|1\rangle$
- probability of states $P_0 = |\alpha|^2$

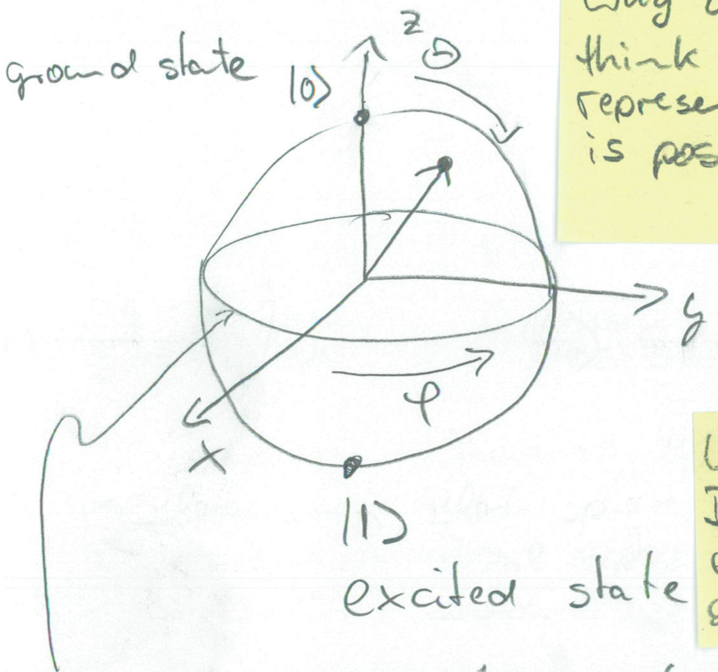
$$P_1 = |\beta|^2$$

with $P_1 + P_2 = 1$

What is the main difference in comp. to a classical bit?

The Bloch Sphere

representation of pure single qubit state as vector to the surface of a sphere



Why do you think this representation is possible?

- general state $\quad |\psi\rangle = \alpha|0\rangle + \beta|1\rangle$
- 4 parameters $\quad \mathrm{Re}\,\alpha,\ \mathrm{Im}\,\alpha,\ \mathrm{Re}\,\beta,\ \mathrm{Im}\,\beta$

  \+ 1 normalization constraint $\quad |\alpha|^2 + |\beta|^2 = 1$

- $|\psi\rangle = e^{i\gamma}\left[\cos\frac{\Theta}{2}|0\rangle + e^{i\varphi}\sin\frac{\Theta}{2}|1\rangle\right]$

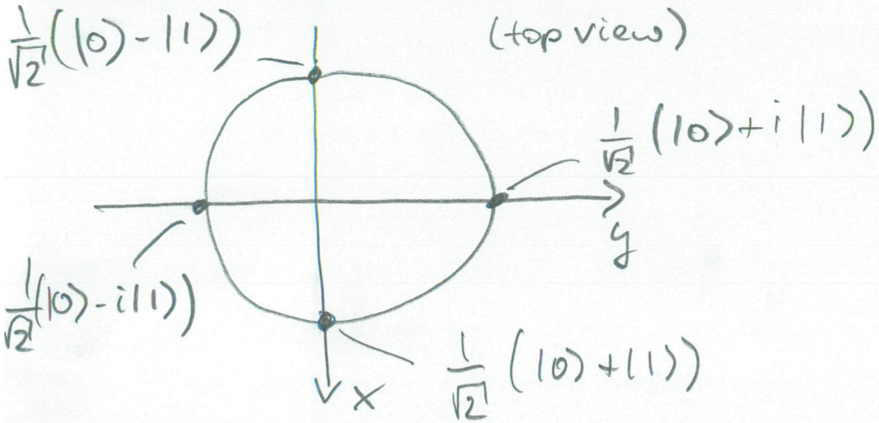
with γ : global phase factor

Θ : polar angle

φ : azimuthal angle

Where on the Bloch sphere are equal superposition states?

equal superposition states:



equivalent representation

- $|\psi\rangle = \frac{1}{\sqrt{2}}\left(|0\rangle + e^{i\varphi}|1\rangle\right)$

$\Rightarrow \Theta = \frac{\pi}{2}$: defines equator

$\varphi = 0, \pi/2, \pi, \frac{3}{2}\pi$ defines phase angle

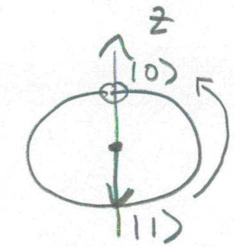
Single Qubit Gates

circuit representation

$$|\psi\rangle_{IN} \;—\!\!—\boxed{\hat{O}}—\!\!—\; |\psi\rangle_{OUT}$$

Work out the effect of the single qubit operations on some simple state on the Bloch sphere!

- $\hat{I} = \hat{\mathbb{1}} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$ Identity
- $\hat{X} = \hat{\sigma}_x = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$ Bit flip

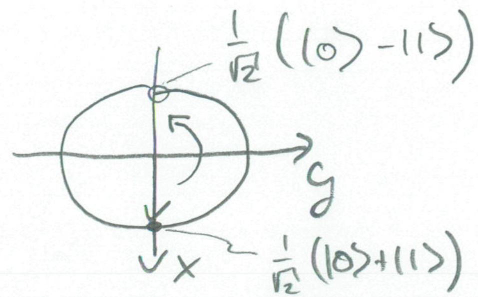


- $\hat{Y} = \hat{\sigma}_y = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}$ Conjugate bit flip

$|0\rangle \to -i\,|1\rangle$

$|1\rangle \to i\,|0\rangle$

- $\hat{Z} = \hat{\sigma}_z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$ Phase flip



Pauli matrices

Basic Properties of Single Qubit Gates

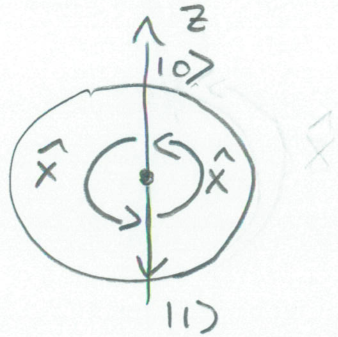
- unitary $\hat{U}^{+}\hat{U} = \hat{I}$ for $\hat{U} = (\hat{X}, \hat{Y}, \hat{Z}, \hat{I})$

- repeated action

$$\hat{U}^{2n} = \underbrace{\hat{U}\hat{U} \ldots \hat{U}}_{2n \text{ times}} = \hat{I} \qquad \text{for } \hat{U} = \hat{X}, \hat{Y}, \hat{Z}$$

$$U^{2n+1} = \ldots = \hat{U}$$

simple to visualize on Bloch sphere



The Hadamard Gate

generation of superposition from basis states

$$|0\rangle \;\text{———}\; \boxed{\hat{H}} \;\text{———}\; \frac{1}{\sqrt{2}}\left(|0\rangle + |1\rangle\right)$$

$$|1\rangle \;\text{———}\; \boxed{\hat{H}} \;\text{———}\; \frac{1}{\sqrt{2}}\left(|0\rangle - |1\rangle\right)$$

- matrix representation

$$\hat{H} = \frac{1}{\sqrt{2}}\left(\hat{X} + \hat{Z}\right) = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}$$

Note: - this gate is used in many quantum algorithms to prepare superposition states from basis states

Can you think of alternative ways to generate a superposition state?

Single Qubit Dynamics

magnetic field

$\vec{B} = (0, 0, B_z)$

magnetic moment

spin 1/2 particle in external field

- Hamiltonian $\qquad H = -\vec{\mu} \cdot \vec{B}$
- corresponding Operator $\qquad \hat{H} = -\dfrac{g \mu_B B_z}{2} \hat{Z}$

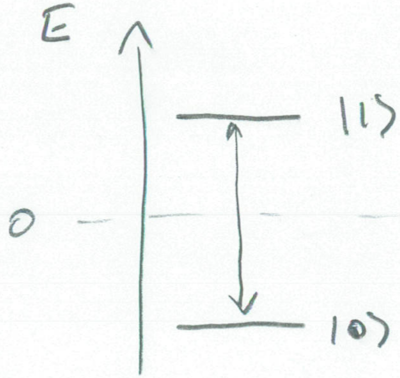
g : gyromagnetic ratio

μ_B : Bohr magneton

- time independent Schrödinger equation $\qquad \hat{H} |\psi_i\rangle = E_i |\psi_i\rangle$
- eigenstates of $\hat{H}$ are $|0\rangle$ and $|1\rangle$

$$\hat{H} |0\rangle = E_0 |0\rangle$$
$$\hat{H} |1\rangle = E_1 |1\rangle$$

- energy level diagram



$$E_1 = \frac{g \mu_B B_z}{2}$$

$$E_0 = -\frac{g \mu_B B_z}{2}$$

$$\Delta E = g \mu_B B_z = \hbar \Omega_z = E_1 - E_0$$

- time dependent Schrödinger equation

$$i\hbar \frac{d}{dt}|\psi\rangle = \hat{H}|\psi\rangle$$

How would you determine the dynamics of a system described by the operator $\hat{H}$?

- general solution for time independent $\hat{H}$

$$|\psi(t)\rangle = \exp\left(-\frac{i}{\hbar}\hat{H}t\right)|\psi(0)\rangle$$

with $$\exp(i\theta\hat{O}) = \cos\theta\,\hat{I} + i\sin\theta\,\hat{O}$$

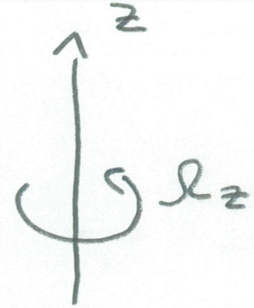
for Operators with $\hat{O}^2 = \hat{I}$ and $\theta \in \mathbb{R}$

e.g. for all Pauli matrices

- for spin ½ example

$$\hat{H} = -\frac{\hbar\Omega_z}{2}\hat{Z}$$

rotation about z-axis on Bloch sphere!



$$|\psi(t)\rangle = \exp\left(-\frac{i}{\hbar}\hat{H}t\right)|\psi(0)\rangle$$

$$= \left(\cos\frac{\theta_z}{2}\hat{I} + i\sin\frac{\theta_z}{2}\hat{Z}\right)|\psi(0)\rangle = R_z(\theta_z)|\psi(0)\rangle$$

with $\theta_z = \Omega_z t$

Dynamics of Superposition State

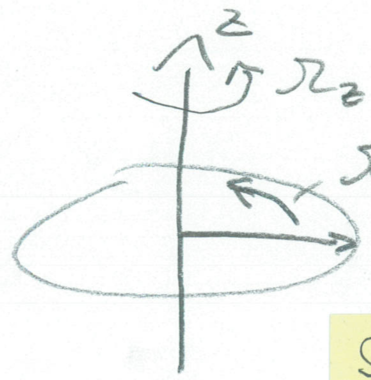
- initial state : $$|\psi(0)\rangle = \frac{1}{\sqrt{2}}\left(|0\rangle + |1\rangle\right)$$

- Hamilton operator $$\hat{H} = -\frac{\hbar \Omega_z}{2}\underbrace{\left(|0\rangle\langle 0| - |1\rangle\langle 1|\right)}_{\hat{Z}}$$

- final state $$|\psi(t)\rangle = \exp\left(-\frac{i}{\hbar}\hat{H}t\right)|\psi(0)\rangle$$
$$= \frac{1}{\sqrt{2}}\left(e^{i\frac{\Omega_z t}{2}}|0\rangle + e^{-i\frac{\Omega_z t}{2}}|1\rangle\right)$$
up to global phase $$= \frac{1}{\sqrt{2}}\left(|0\rangle + \underbrace{e^{-i\Omega_z t}}_{e^{i\varphi}}|1\rangle\right)$$

- on Bloch sphere

$$\Theta = \frac{\pi}{2}$$
$$\varphi = -\Omega_z t$$



Can you work out what dynamics the Hamiltonian $\hat{H}_x = -\frac{\hbar\Omega_x}{2}\hat{X}$ induces?

How is this useful for controlling the qubit state?

"Do the Bloch sphere dance to illustrate qubit dynamics!"

Show slides with other rotation operators.