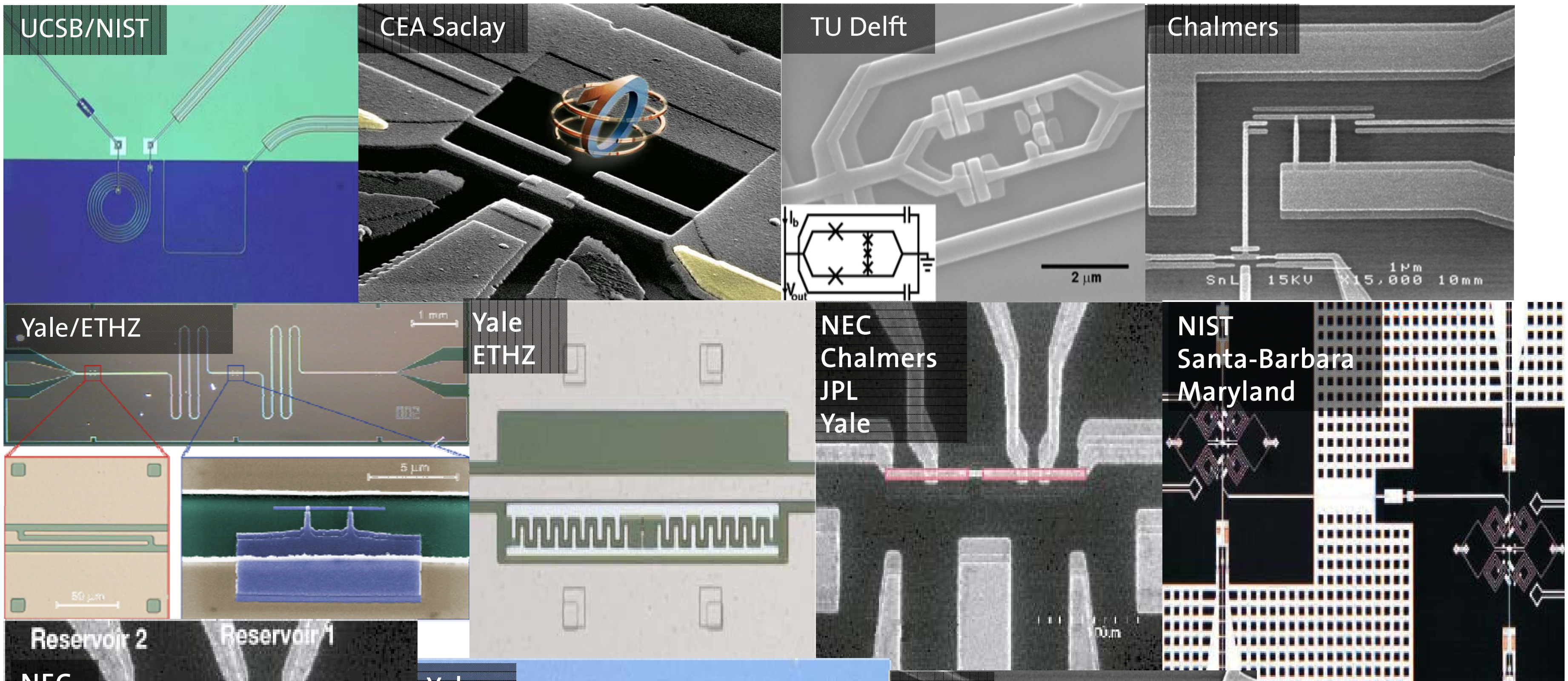
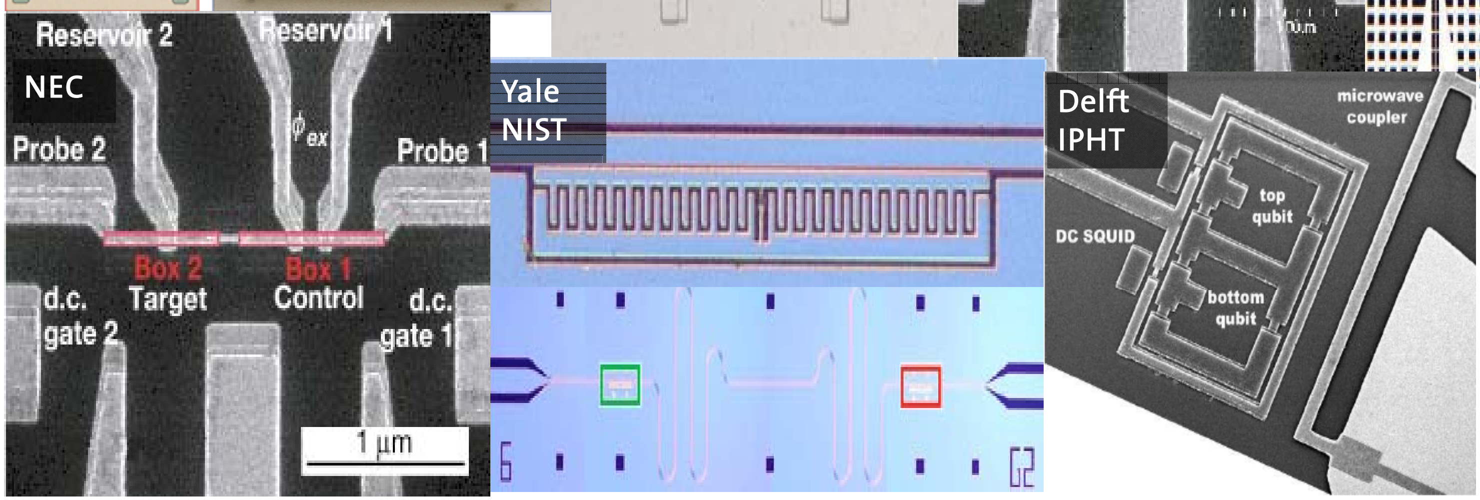
Building a Quantum Information Processor using Superconducting Circuits

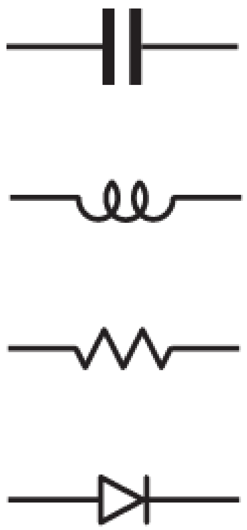




with material from
NIST, UCSB,
Berkeley, NEC,
NTT, CEA Saclay,
Yale and ETHZ

Conventional Electronic Circuits

basic circuit elements:

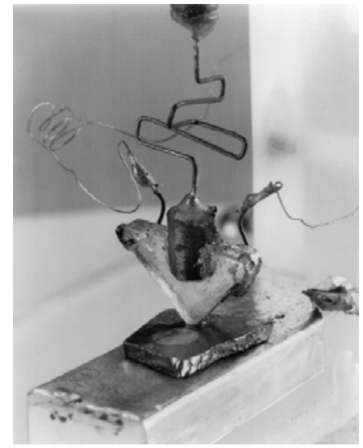


basis of modern information and communication technology

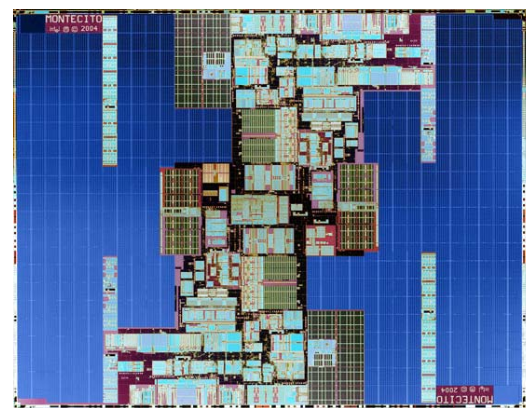
properties :

- classical physics
- no quantum mechanics
- no superposition principle
- no quantization of fields

first transistor at Bell Labs (1947)



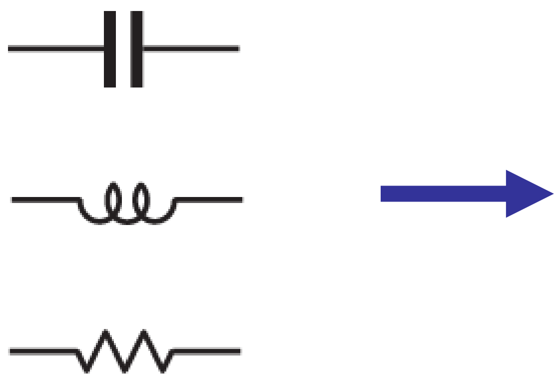
intel dual core processor (2006)



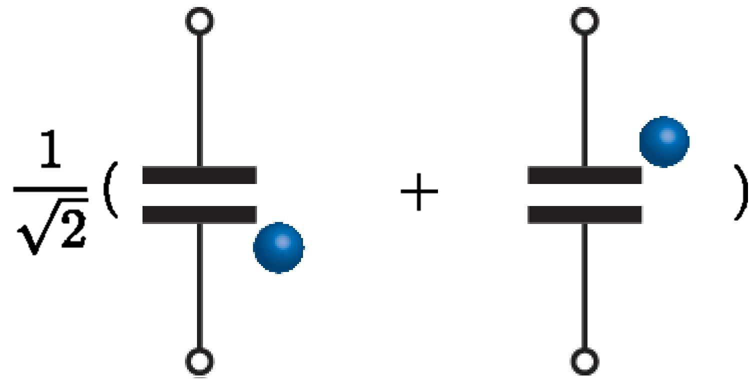
2.000.000.000 transistors
smallest feature size 65 nm
clock speed ~ 2 GHz
power consumption 10 W

Classical and Quantum Electronic Circuit Elements

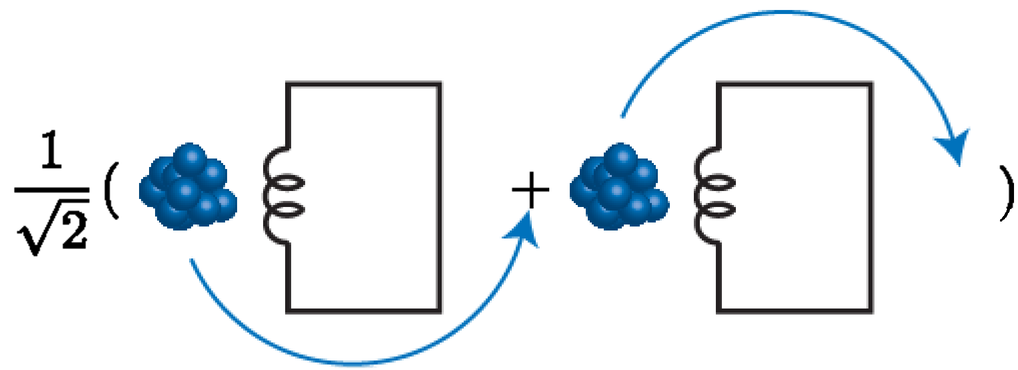
basic circuit elements:



charge on a capacitor:



current or magnetic flux in an inductor:

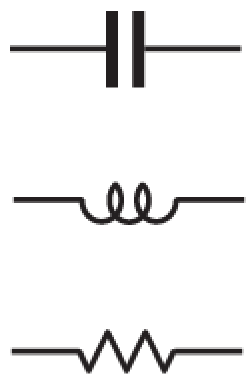


quantum superposition states:

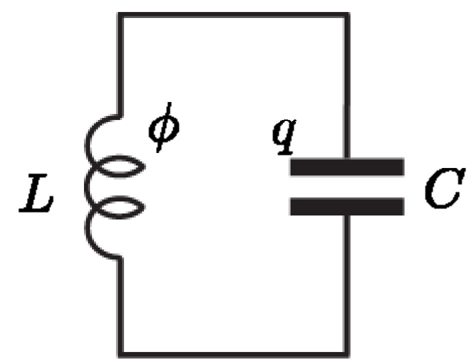
- charge q
- flux ϕ

Constructing Linear Quantum Electronic Circuits

basic circuit elements:



harmonic LC oscillator:



$$\omega = \frac{1}{\sqrt{LC}} \sim 5\,\mathrm{GHz}$$

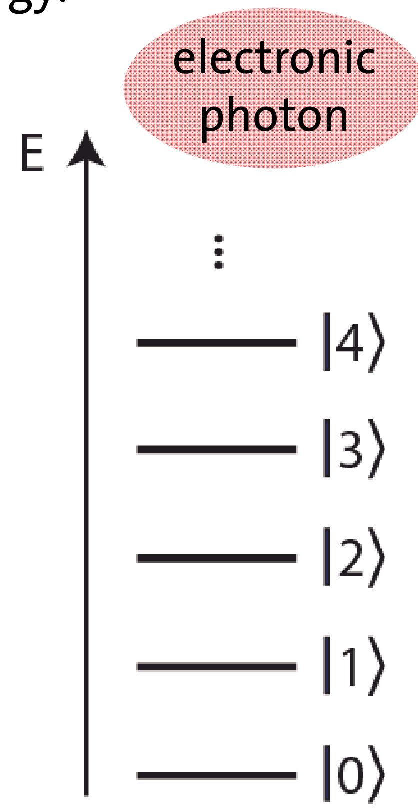
classical physics:

$$H = \frac{\phi^2}{2L} + \frac{q^2}{2C}$$

quantum mechanics:

$$\hat{H} = \frac{\hat{\phi}^2}{2L} + \frac{\hat{q}^2}{2C} = \hbar\omega(\hat{a}^\dagger\hat{a} + \frac{1}{2}) \qquad \left[\hat{\phi}, \hat{q}\right] = i\hbar$$

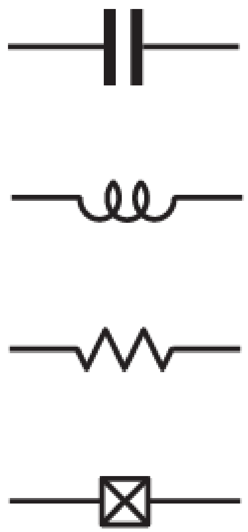
energy:



Review: M. H. Devoret, A. Wallraff and J. M. Martinis, *condmat/0411172* (2004)

Constructing Non-Linear Quantum Electronic Circuits

circuit elements:



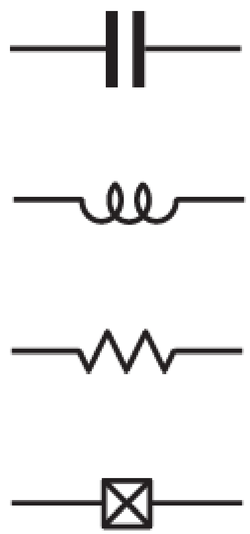
Josesphson junction:

a non-dissipative nonlinear element (inductor)

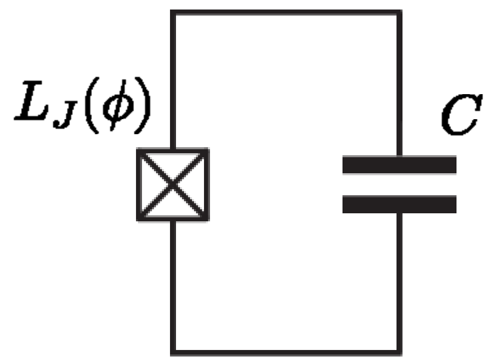
Review: M. H. Devoret, A. Wallraff and J. M. Martinis, *condmat/0411172* (2004)

Constructing Non-Linear Quantum Electronic Circuits

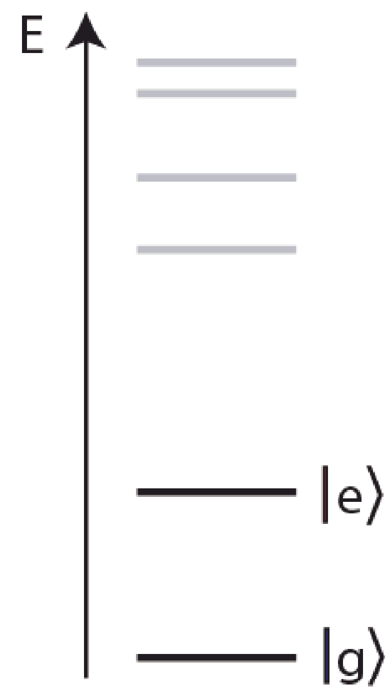
circuit elements:



anharmonic oscillator:



non-linear energy level spectrum:



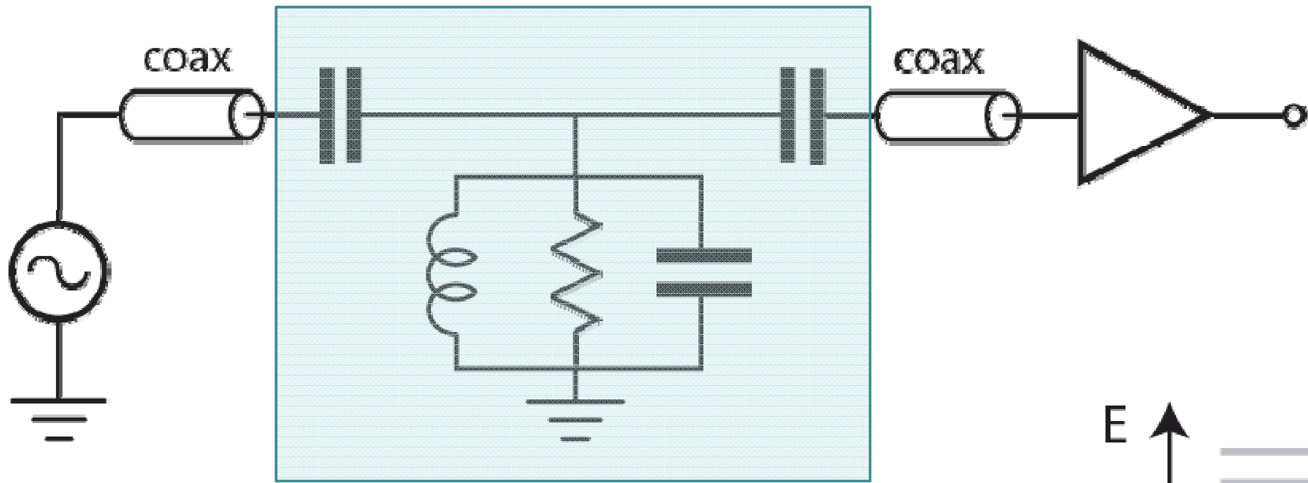
Josesphson junction:
a non-dissipative nonlinear element (inductor)

$$L_J(\phi) = \left(\frac{\partial I}{\partial \phi}\right)^{-1} = \frac{\phi_0}{2\pi I_c} \frac{1}{\cos(2\pi\phi/\phi_0)}$$

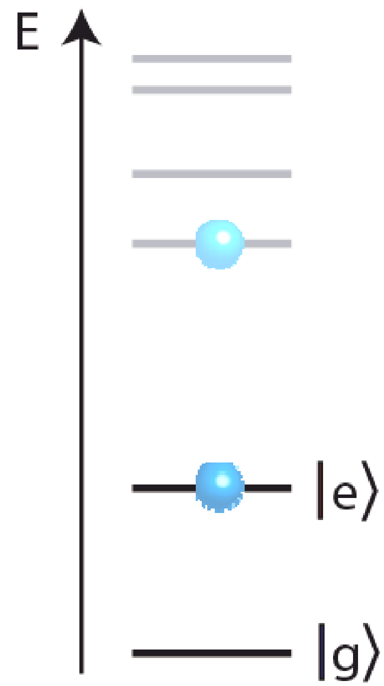
electronic artificial atom

Review: M. H. Devoret, A. Wallraff and J. M. Martinis, *condmat/0411172* (2004)

How to Operate Circuits Quantum Mechanically?



$T \sim 0.01\,\mathrm{K}$



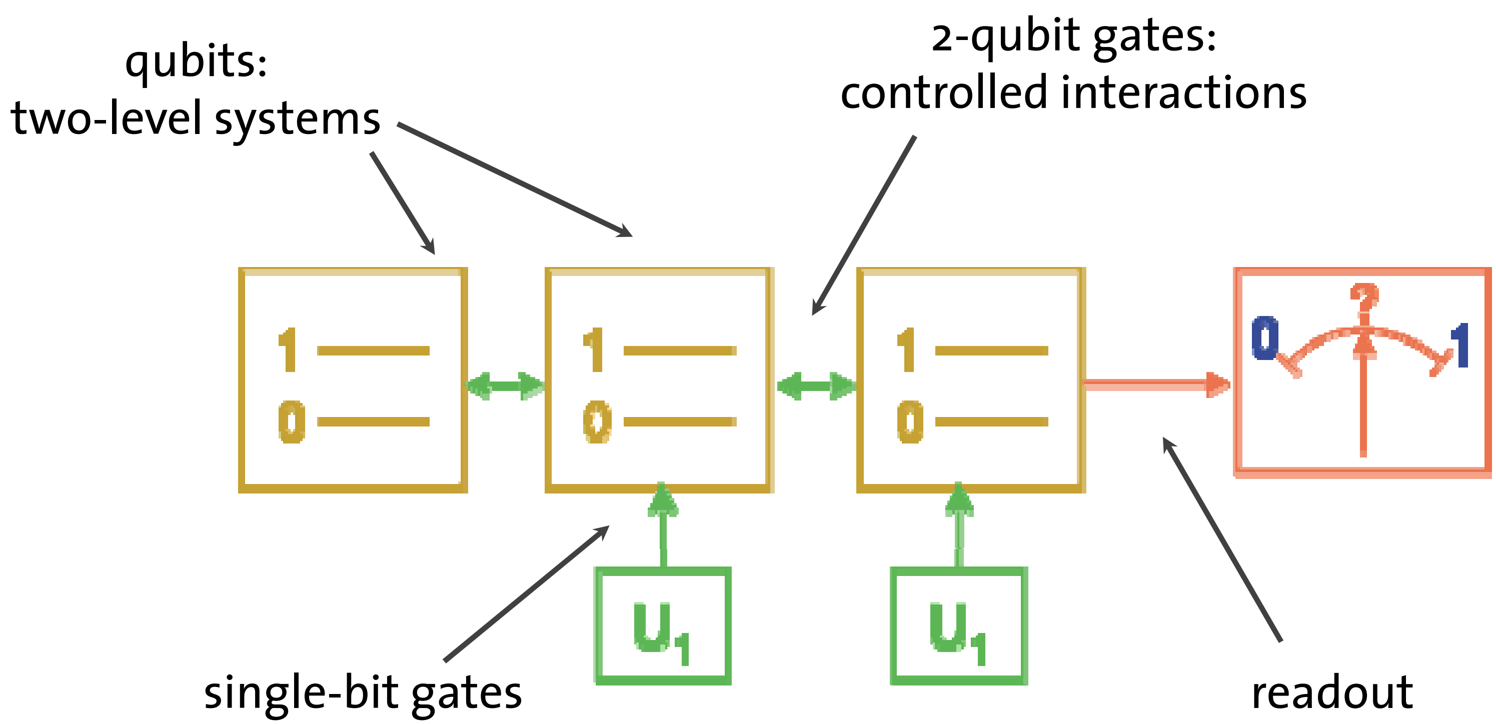
recipe:

- avoid dissipation
- work at low temperatures
- isolate quantum circuit from environment

Review: M. H. Devoret, A. Wallraff and J. M. Martinis, *condmat/0411172* (2004)

Generic Quantum Information Processor

The challenge:



- Quantum information processing requires excellent qubits, gates, ...
- Conflicting requirements: good isolation from environment while maintaining good addressability

M. Nielsen and I. Chuang,
Quantum Computation and Quantum Information (Cambridge, 2000)

The DiVincenzo Criteria

for Implementing a Quantum Computer in the standard (circuit approach) to quantum information processing (QIP):

#1. A scalable physical system with well-characterized qubits.
#2. The ability to initialize the state of the qubits.
#3. Long (relative) decoherence times, much longer than the gate-operation time.
#4. A universal set of quantum gates.
#5. A qubit-specific measurement capability.

plus two criteria requiring the possibility to transmit information:

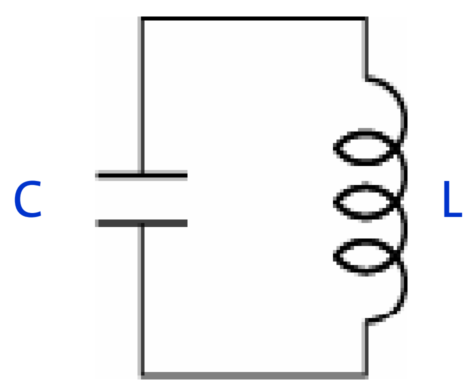
#6. The ability to interconvert stationary and mobile (or flying) qubits.
#7. The ability to faithfully transmit flying qubits between specified locations.

Outline

- realization of superconducting quantum electronic circuits
  - the harmonic oscillator
  - the current biased phase qubit
  - the charge qubit
- controlled qubit/photon interactions
  - cavity quantum electrodynamics with circuits
- qubit read-out
- single qubit control
- decoherence
- two-qubit interactions
  - generation of entanglement
  - realization of quantum algorithms

Superconducting Harmonic Oscillator

a simple electronic circuit:

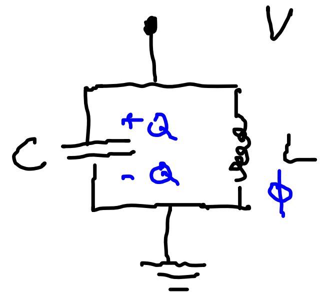


- typical inductor: L = 1 nH
- a wire in vacuum has inductance ~ 1 nH/mm
- typical capacitor: C = 1 pF
- a capacitor with plate size 10 μm x 10 μm and dielectric AlOx (ε = 10) of thickness 10 nm has a capacitance C ~ 1 pF
- resonance frequency

$$\frac{1}{2\pi\sqrt{LC}} \sim 5\ \text{GHz}$$

Quantization of the electrical LC harmonic oscillator:

parallel LC oscillator circuit:



voltage across the oscillator:

$$V = \frac{Q}{C} = -L \frac{\partial I}{\partial t}$$

total energy (Hamiltonian): $H = \frac{1}{2} C V^2 + \frac{1}{2} L I^2 = \frac{1}{2} \frac{Q^2}{C} + \frac{1}{2} \frac{\phi^2}{L}$

with the charge Q stored on the capacitor $\quad Q = V C$

a flux ϕ stored in the inductor $\quad \phi = L I$

properties of Hamiltonian written in variables Q and ϕ:

$$\frac{\partial H}{\partial Q} = \frac{Q}{C} = -L \frac{\partial I}{\partial t} = -\dot{\phi}$$

$$\frac{\partial H}{\partial \phi} = \frac{\phi}{L} = I = \dot{Q}$$

Q and ϕ are canonical variables

see e.g.: Goldstein, Classical Mechanics, Chapter 8, Hamilton Equations of Motion

Quantum version of Hamiltonian

$$\hat{H} = \frac{\hat{Q}^2}{2C} + \frac{\hat{\phi}^2}{2L}$$

with commutation relation

$$[\hat{\phi}, \hat{Q}] = i\hbar$$

compare with particle in a harmonic potential:

$$\hat{H} = \frac{\hat{p}^2}{2m} + \frac{1}{2} m \omega^2 \hat{x}^2$$

analogy with electrical oscillator:

- charge Q corresponds to momentum p
- flux ϕ corresponds to position x

$$[\hat{x}, \hat{p}] = [\hat{x}, -i\hbar \frac{\partial}{\partial x}] = i\hbar$$

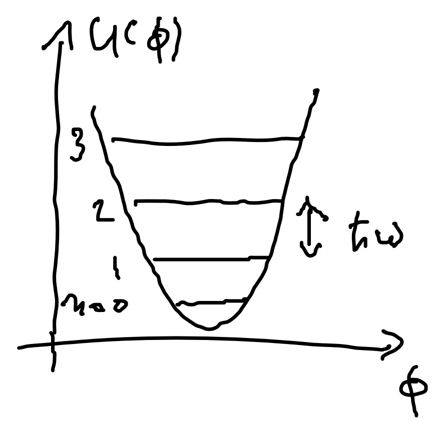
$$[\hat{\phi}, \hat{Q}] = [\hat{\phi}, -i\hbar \frac{\partial}{\partial \phi}] = i\hbar$$

Hamiltonian in terms of raising and lowering operators:

$$\hat{H} = \hbar\omega \left(a^\dagger a + \frac{1}{2}\right)$$

with oscillator resonance frequency:

$$\omega = \frac{1}{\sqrt{LC}}$$



Raising and lowering operators:

$$a^{\dagger} |n\rangle = \sqrt{n+1}\, |n+1\rangle \quad ; \quad \hat{a}\, |n\rangle = \sqrt{n}\, |n-1\rangle$$

$$a^{\dagger} a\, |n\rangle = n\, |n\rangle$$ number operator

in terms of Q and ϕ:

$$\hat{a} = \frac{1}{\sqrt{2\hbar Z_c}} \left(Z_c \hat{Q} + i \hat{\phi} \right)$$

with Z_c being the characteristic impedance of the oscillator $Z_c = \sqrt{\frac{L}{C}}$

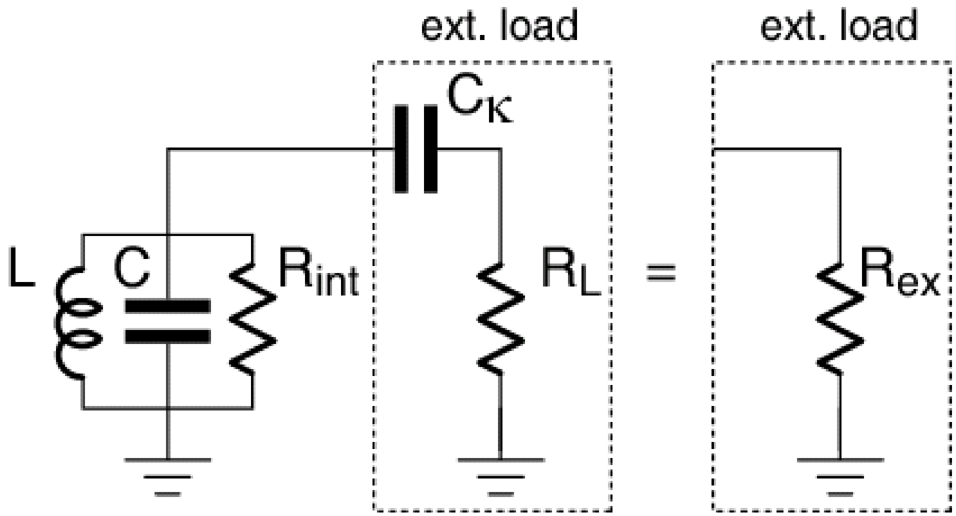
charge Q and flux ϕ operators can be expressed in terms of raising and lowering operators:

$$\hat{Q} = \sqrt{\frac{\hbar}{2 Z_c}} \left(a^{\dagger} + a \right)$$

$$\hat{\phi} = i \sqrt{\frac{\hbar Z_c}{2}} \left(a^{\dagger} - a \right)$$

Exercise: Making use of the commutation relations for the charge and flux operators, show that the harmonic oscillator Hamiltonian in terms of the raising and lowering operators is identical to the one in terms of charge and flux operators.

Internal and External Dissipation in an LC Oscillator



internal losses: R_{int}
conductor, dielectric

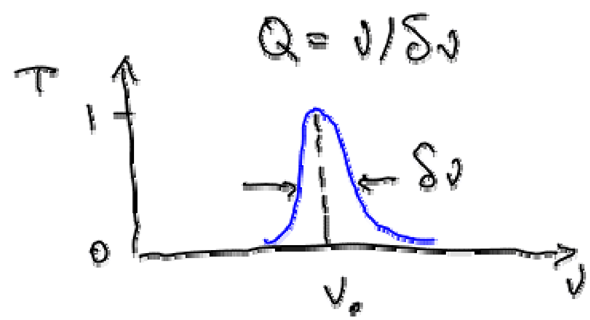
external losses: R_{ext}
radiation, coupling

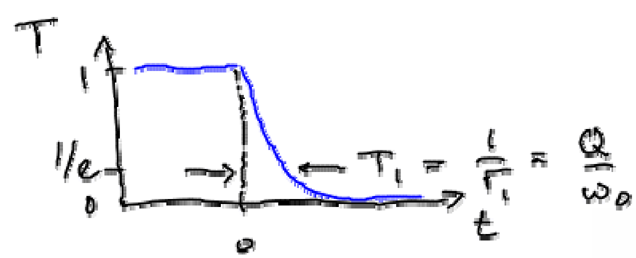
total losses $$\frac{1}{R} = \frac{1}{R_{\text{int}}} + \frac{1}{R_{\text{ext}}}$$

impedance $$Z = \sqrt{\frac{L}{C}}$$

quality factor $$Q = \frac{R}{Z} = \omega_0 RC$$

excited state decay rate $$\Gamma_1 = \frac{\omega_0}{Q} = \frac{1}{RC}$$





problem 2: internal and external dissipation