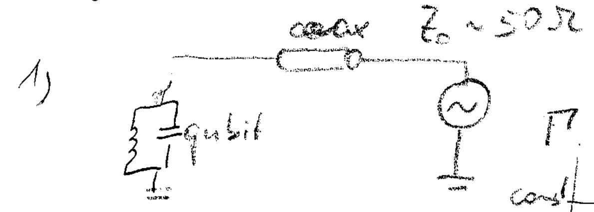
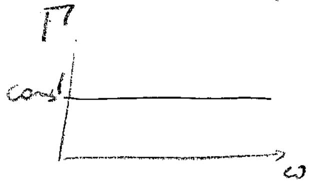
decay rate: $\Gamma = \frac{1}{T_1} = \frac{1}{RC}$ $\frac{1}{R} = Re[Z(\omega)^{-1}]$

1)





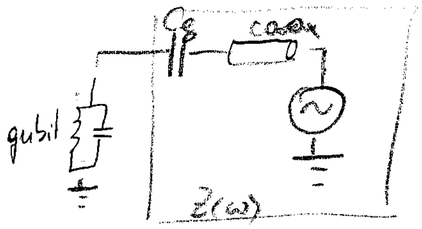
coupling via a bias wire

$Z(\omega) = Z_0 = 50\Omega$

$1/R = Re[50^{-1}] = 1/50\Omega$

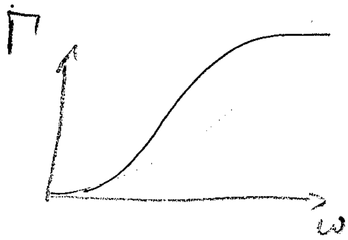
C.. qubit capacitance

2)



$$Z(\omega) = Z_0 - \frac{i}{\omega C_g}$$

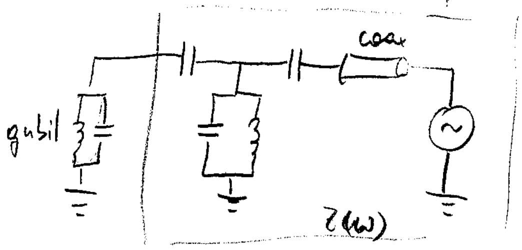
$$Z^{-1}(\omega) = \frac{Z_0 + \frac{i}{\omega C_g}}{Z_0^2 + \frac{1}{\omega C_g}^2}$$



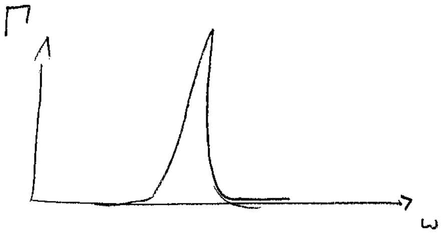
$$Re\, Z^{-1}(\omega) = \frac{Z_0}{Z_0^2 + \frac{1}{\omega C_g}^2} = \frac{1}{Z_0 + \frac{1}{Z_0 \omega^2 C_g^2}}$$

High-pass filter

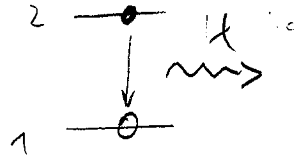
3)



$Z(\omega) = \ldots$



Spontaneous Emission: Einstein A-Coefficient.



$A_{21} \propto \Omega_{21}^2 \, \rho_0(\omega)$ ... from Fermi Golden Rule

$\Gamma = \frac{2\pi}{\hbar^2} |\langle f | H' | i \rangle|^2 \rho(\omega)$

Ω_{21} ... vacuum Rabi frequency

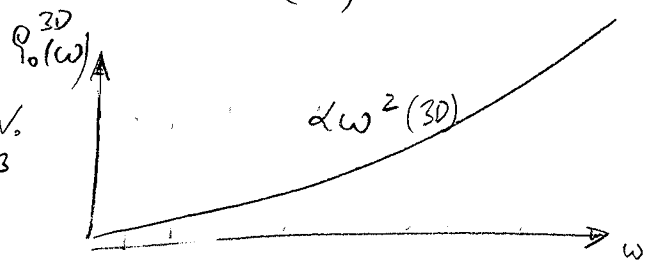
$\Omega_{21} = 2 d_{21} E_0 / \hbar$

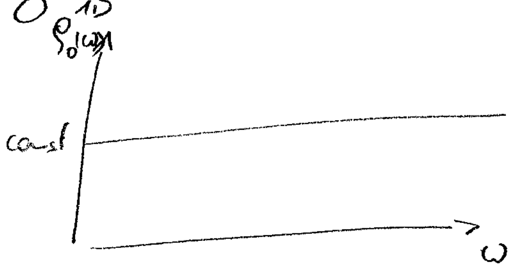
d_{21} ... dipole matrix element

E_0 ... vacuum field strength

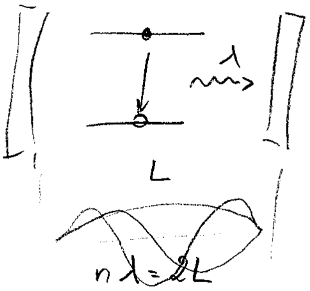
$\rho_0(\omega)$ ... mode density

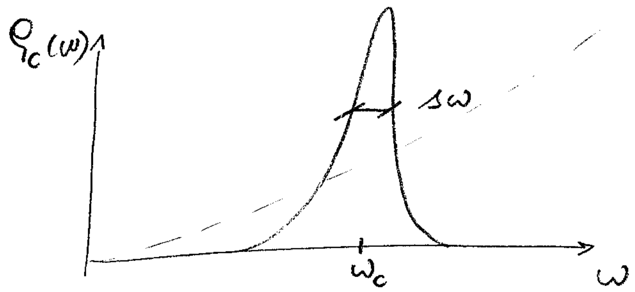
$\rho_0(\omega) = \frac{\omega^2 V_0}{\pi^2 c^3}$





emission <u>into</u> cavity:





$$\int_0^\infty g(\omega)\, d\omega = 1$$

⇒ only specific wave lengths are allowed

$$\Rightarrow g(\omega) = \frac{2}{\pi \Delta\omega} \frac{\Delta\omega_c^2}{4(\omega-\omega_c)^2 + \Delta\omega_c^2}$$

Purcell Factor: $F_P = \frac{\Gamma_{cav}}{\Gamma_{free}} \propto \frac{Q \lambda^3}{V_0}$

V_0 cavity volume

Q .. quality of cavity

λ .. wavelength

$\Rightarrow F > 1$: enhanced spont. emission (large Q, small V_0)

Comparison 1D vs. 3D cavity:

Interaction strength $\Omega_{21} \hat{=} 2g = \frac{2 d_{21} E_0}{\hbar}$ ← vacuum field → engineerable

(d_{21}: two-level system specific)

calculate E_0:

cavity mode $\hat{=}$ harmonic oscillator with annihilation (a) and creation ($a^\dagger$) operators

field operator: $\hat{E}(r) = \mathcal{E}_0 \left(f(r)\hat{a} + f^*(r) a^\dagger \right)$ $\mathcal{E}_0$ ... normalization constant

determine $\mathcal{E}_0$ for the vacuum state $f(r)$ ... spatial mode distribution

electromagnetic energy in 0-photon state is $\frac{1}{2}\hbar\omega_r$.

$\frac{1}{2}$ of it is stored in the electric field.

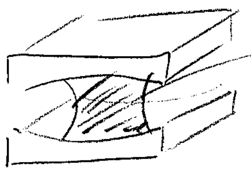
$$\frac{1}{4}\hbar\omega_r = \langle 0 | \frac{\varepsilon_0}{2} \int |\hat{E}|^2 d^3r \, |0\rangle =$$

$$= \frac{\varepsilon_0}{2} \mathcal{E}_0^2 V_0 \underbrace{\langle 0 | (a^\dagger + a)^2 | 0 \rangle} \qquad V_0 = \int |f(r)| \, d^3r$$

$$\langle 0 | a^\dagger a^\dagger + a^\dagger a + a a^\dagger + a a | 0 \rangle = 1$$

$$\Rightarrow \mathcal{E}_0 = \sqrt{\frac{\hbar\omega_r}{2\varepsilon_0 V_0}}$$

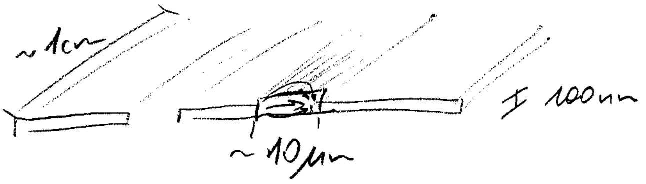
V_0 of a 3D cavity:



$\sim 700\,\mathrm{mm}^3$

$\omega_r = 2\pi\, 50\,\mathrm{GHz}$

V_0 of transmission line:



$$V_0 \sim 10^{-2}\,\mathrm{m} \cdot 10^{-5}\,\mathrm{m} \cdot 10^{-7}\,\mathrm{m} \sim 10^{-14}\,\mathrm{m}^3 \hat{=} 10^{-5}\,\mathrm{mm}^3$$

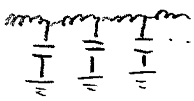
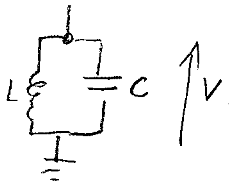
$\omega_r = 2\pi\, 10\,\mathrm{GHz}$

$$\mathcal{E}_0^{\text{3D cavity}} = c \cdot \sqrt{\frac{50}{700}} \sim c \cdot 0.3$$

$$\mathcal{E}_0^{\text{1D transm. line}} = c \cdot \sqrt{\frac{10}{10^{-5}}} \sim c \cdot 1000$$

$10^3 - 10^4$ times larger E-field

Transmission line as QM harmonic oscillator:

 $\hat{=}$ 

$$\hat{H} = \frac{1}{2} C V^2 + \frac{1}{2} \frac{\Phi^2}{L} \qquad \left(\frac{\Phi^2}{L} = \frac{1}{2} L I^2\right)$$

electrostatic energy — magnetic energy

Voltage operator: $\hat{V} = \sqrt{\frac{\hbar \omega_r}{2C}} \, (a^\dagger + a)$

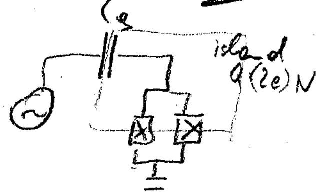
voltage across oscillator circuit in ground state $\hat{=}$ vacuum fluctuations

$\langle 0 | \hat{V} | 0 \rangle = 0$ since $\langle 0 | \hat{a} | 0 \rangle = \langle 0 | a^\dagger | 0 \rangle = 0$

$$\Delta V_0^2 = \langle \hat{V}^2 \rangle_0 - \langle V \rangle_0^2 = \langle \hat{V}^2 \rangle_0 = \langle 0 | \hat{V}^2 | 0 \rangle =$$

$$= \frac{\hbar \omega_r}{2C} \langle 0 | a^\dagger a^\dagger + a^\dagger a + a a^\dagger + a a | 0 \rangle = \frac{\hbar \omega_r}{2C}$$

$$\Delta V_0 = \sqrt{\frac{\hbar \omega_r}{2C}} \sim 1 \mu V$$



Jaynes Cummings for circuit QED:

1) Two-level approximation of CPB Hamiltonian

$$H = \sum_N \left\{ E_C (N - N_g)^2 |N\rangle\langle N| - \frac{E_J}{2} \left(|N\rangle\langle N+1| + |N+1\rangle\langle N| \right) \right\}$$

$$E_C = \frac{(2e)^2}{2C_\Sigma} \qquad E_J = E_{J0} \cos\delta$$

take only $N = 0, 1$ into account:

$$H_2 = E_C N_g^2 |0\rangle\langle 0| + E_C (1 - N_g)^2 |1\rangle\langle 1| - \frac{E_J}{2} \underbrace{\left(|0\rangle\langle 1| + |1\rangle\langle 0| \right)}_{\bar\sigma_x}$$

$$= -\frac{E_{el}}{2} |0\rangle\langle 0| + \frac{E_{el}}{2} |1\rangle\langle 1| - \frac{E_J}{2} \bar\sigma_x$$

$\Rightarrow$ shift of energy: $E_{el} = E_C (1 - 2N_g)$

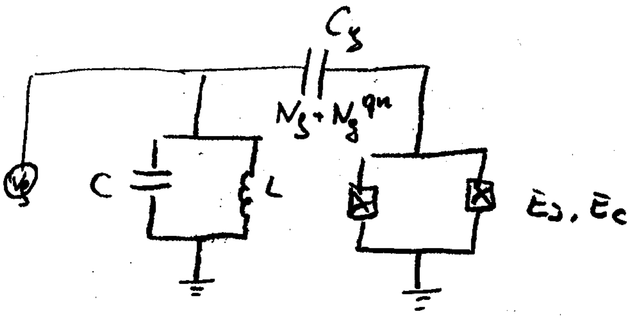
$$\Rightarrow \; = -\frac{E_{el}}{2} \bar\sigma_z - \frac{E_J}{2} \bar\sigma_x$$

$\Rightarrow$ Eigenbasis by rotation about y-axis: $e^{i\frac{\theta}{2}\sigma_y}$: $\sigma_x = \cos\theta\,\bar\sigma_x + \sin\theta\,\bar\sigma_z$, $\sigma_z = -\sin\theta\,\bar\sigma_x + \cos\theta\,\bar\sigma_z$

$$H_2 = \frac{1}{2} \begin{pmatrix} -E_{el} & -E_J \\ -E_J & E_{el} \end{pmatrix}$$

$$\Rightarrow H_2 = \frac{\hbar\Omega}{2} \sigma_z \qquad \theta = \arctan\left(\frac{E_J}{E_{el}}\right)$$

2) Coupling to gate capacitor:



N_g .. polarization charge on C_g:

$$N_g = \frac{C_g V_g}{2e}$$

$$H = \frac{1}{2} E_C \left(1 - 2(N_g + \hat N_g^{qu})\right) \bar\sigma_z - \frac{E_J}{2} \bar\sigma_x$$

For simplicity $N_g = \frac{1}{2}$: $H = \frac{E_C}{2} \left(\hat N_g^{qu} \bar\sigma_z - \frac{E_J}{2} \bar\sigma_x \right) =$
charge sweet spot

$$\hat Q^{qu} = (2e) \hat N_g^{qu} = C_g \hat V_g \qquad = \frac{E_C}{2} \underbrace{\frac{C_g}{2e} \sqrt{\frac{\hbar\omega}{2C}}}_{C_g \hat V / 2e} (\hat a^+ + \hat a) \bar\sigma_z - \frac{E_J}{2} \bar\sigma_x$$

$\theta = \arctan\left(\frac{E_J}{E_L}\right) \quad E_L \sim 0 \;@ N_g = \frac{1}{2} \quad \theta = \frac{\pi}{2}$

$E_C = \frac{(2e)^2}{2C_\Sigma}$

$\Rightarrow \quad \sigma_x = \bar{\sigma}_z \quad \sigma_z = -\bar{\sigma}_x$

$$= e \frac{C_g}{C_\Sigma} \sqrt{\frac{\hbar\omega_1}{2C}} (a^+ + a) \underbrace{\sigma_x}_{= (\sigma^+ + \sigma^-)} + \frac{E_J}{2}\sigma_z$$

$\sigma^+ = \sigma_x + i\sigma_y$

$\sigma^- = \sigma_x - i\sigma_y$

$\cancel{a^+\sigma^+} + a\sigma^+ + a^+\sigma^- + \cancel{a\sigma^-}$ energy conservation, RWA

$\Rightarrow$ full circuit Hamiltonian

$$\boxed{\hat{H} = \hbar\omega_1\left(a^+a + \frac{1}{2}\right) + \frac{E_J}{2}\sigma_z + \underbrace{\frac{C_g}{C_\Sigma} 2e\sqrt{\frac{\hbar\omega_1}{2C}}}_{\hbar g} (a\sigma^+ + a^+\sigma^-)}$$

$\hbar g \;\hat{=}\; \frac{2g}{\pi}$ = Vacuum Rabi Frequency