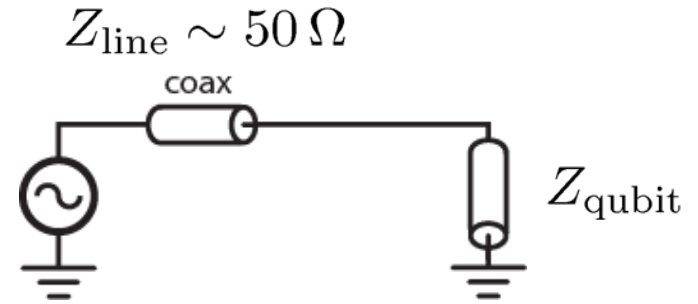


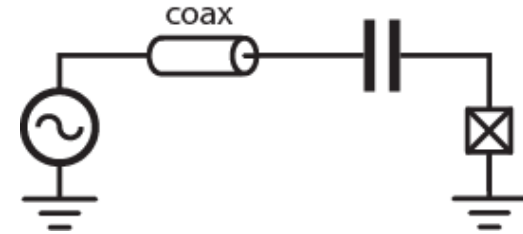
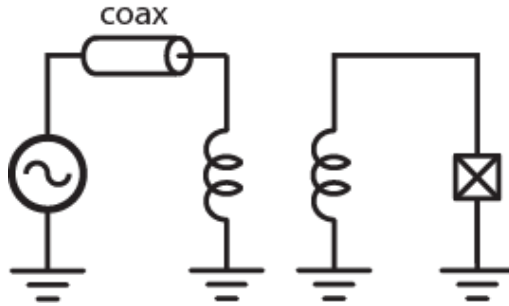
Controlling Coupling to the E.M. Environment

coupling to environment (bias wires):

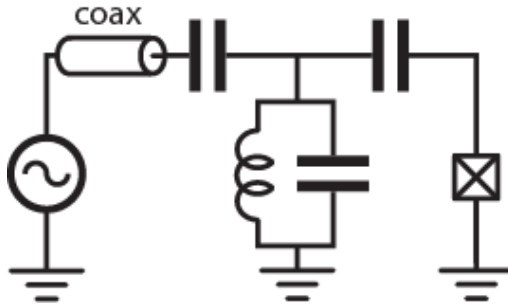
decoherence
from energy relaxation
(spontaneous emission)



decoupling using non-resonant impedance transformers:



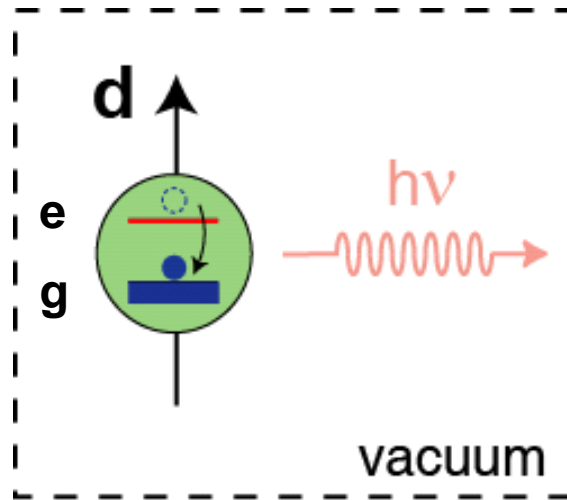
using resonant impedance transformers



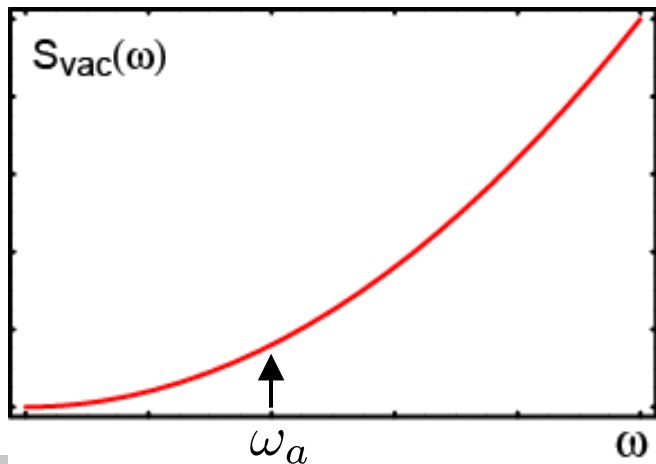
control spontaneous emission
by circuit design

Cavity QED with Electronic Circuits

Spontaneous Emission

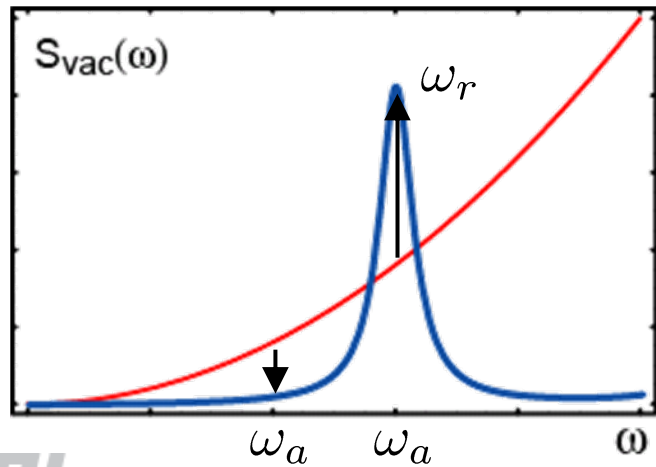
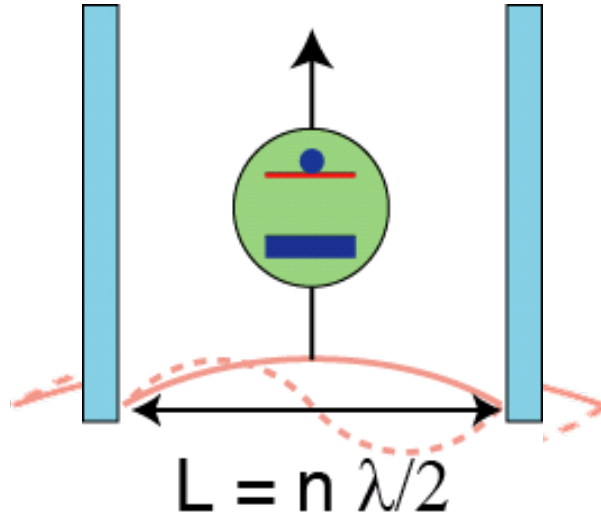


decay by dipole interaction with vacuum fluctuations



$$\gamma \sim \Omega^2 S_{\text{vac}}(\omega_a)$$

Suppression and Enhancement of Emission



engineering the vacuum

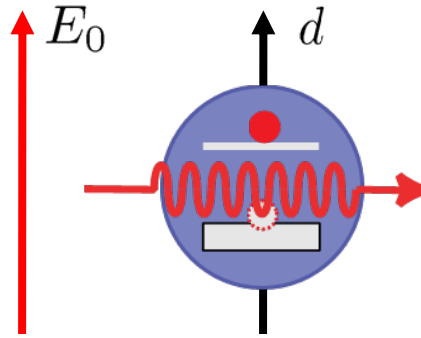
$$\gamma \sim \Omega^2 S_{\text{vac}}(\omega_a)$$

suppression of emission

enhancement of emission

Free Atom

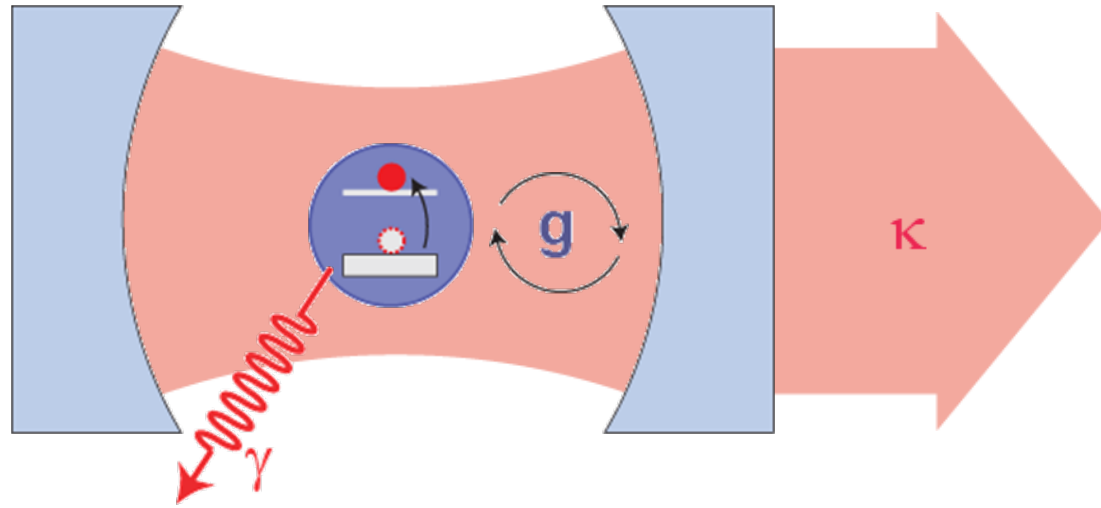
weak interaction with single photons:



- dipole moment d (usually small in atoms $\sim ea_o$)
- single photon fields E_o (small in 3D)
- photon/atom interaction $\hbar g \sim dE_o$ (usually small)

Cavity Quantum Electrodynamics

coupling single photons to single qubits:



Jaynes-Cummings Hamiltonian

$$H = \hbar\omega_r \left(a^\dagger a + \frac{1}{2} \right) + \frac{\hbar\omega_a}{2} \sigma^z + \hbar g (a^\dagger \sigma^- + a \sigma^+) + H_\kappa + H_\gamma$$

strong coupling limit ($g = dE_0/\hbar > \gamma, \kappa, 1/t_{\text{transit}}$)

Dressed States Energy Level Diagram

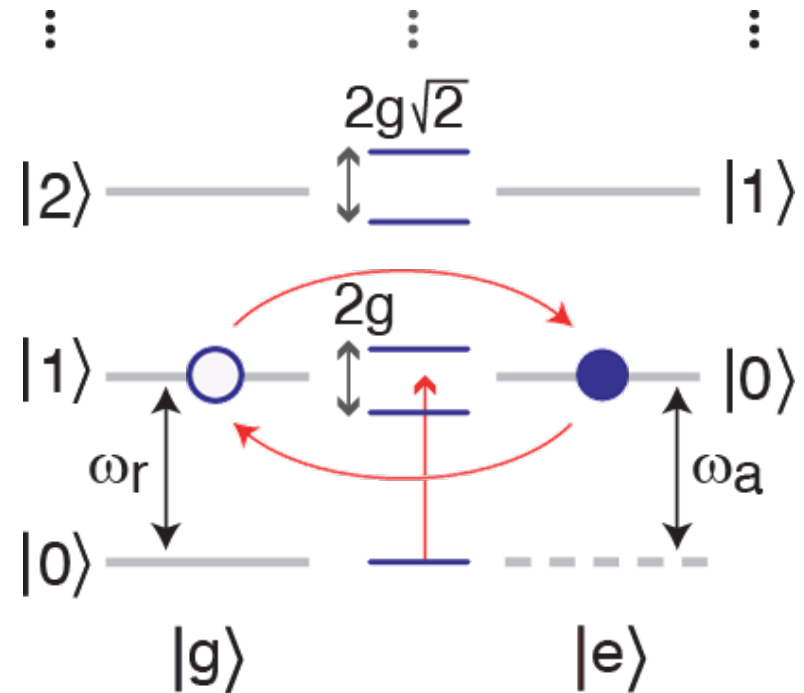
$$H = \hbar\omega_r \left(a^\dagger a + \frac{1}{2} \right) + \frac{\hbar\omega_a}{2} \sigma^z + \hbar g (a^\dagger \sigma^- + a \sigma^+)$$

in resonance:

$$\omega_a - \omega_r = \Delta = 0$$

strong coupling limit:

$$g = \frac{dE_0}{\hbar} > \gamma, \kappa$$



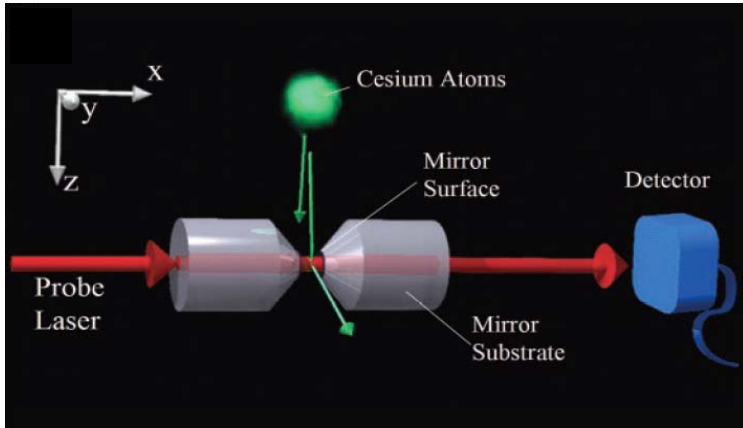
Jaynes-Cummings Ladder

Atomic cavity quantum electrodynamics reviews:

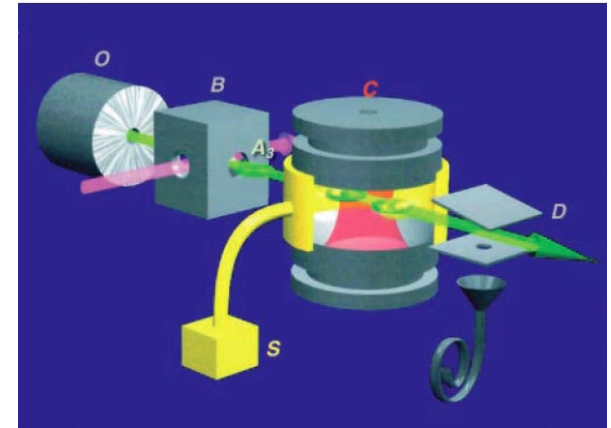
J. Ye., H. J. Kimble, H. Katori, *Science* **320**, 1734 (2008)

S. Haroche & J. Raimond, *Exploring the Quantum*, OUP Oxford (2006)

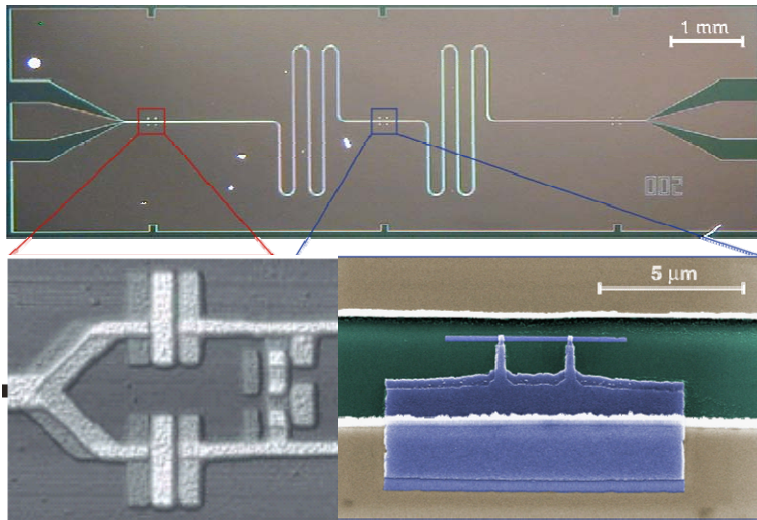
Cavity Quantum Electrodynamics (QED)



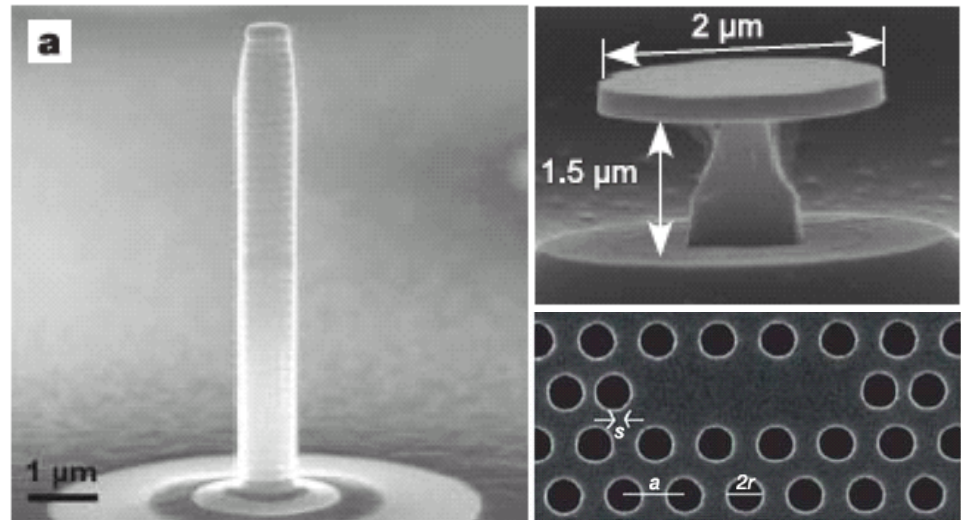
alkali atoms
MPQ, Caltech, ...



Rydberg atoms
ENS, MPQ, ...

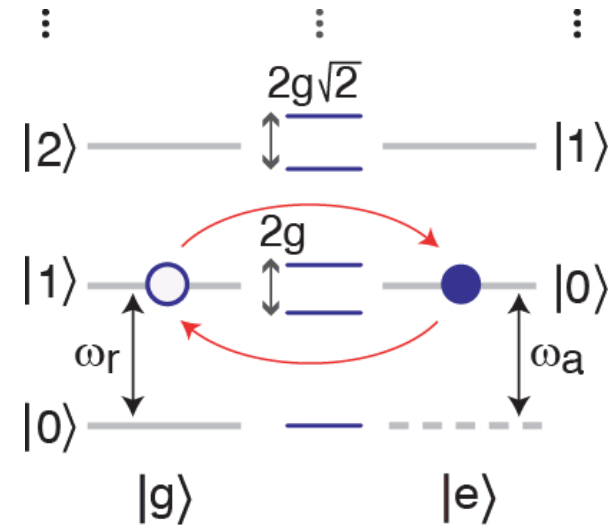
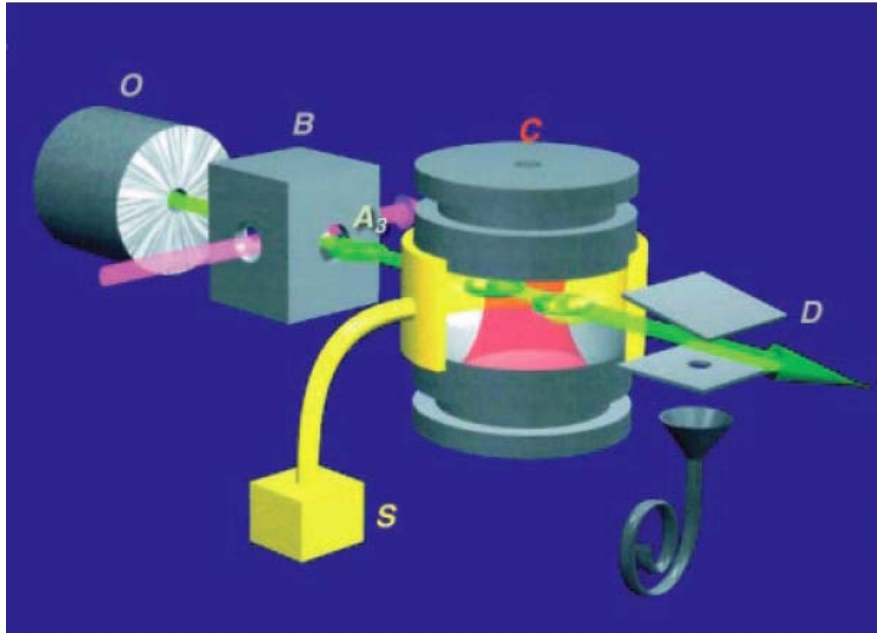


superconductor circuits
Yale, Delft, NTT, ETHZ, NIST, ...

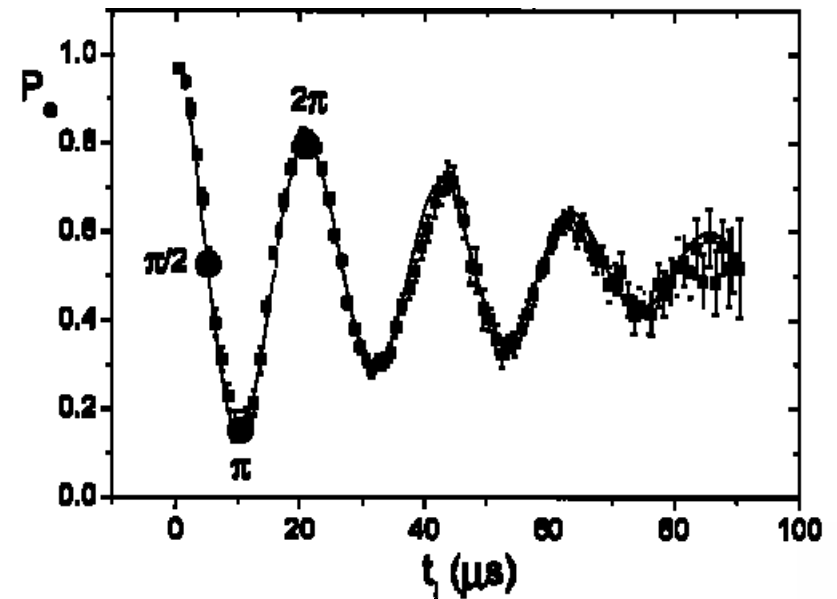


semiconductor quantum dots
Wurzburg, ETHZ, Stanford ...

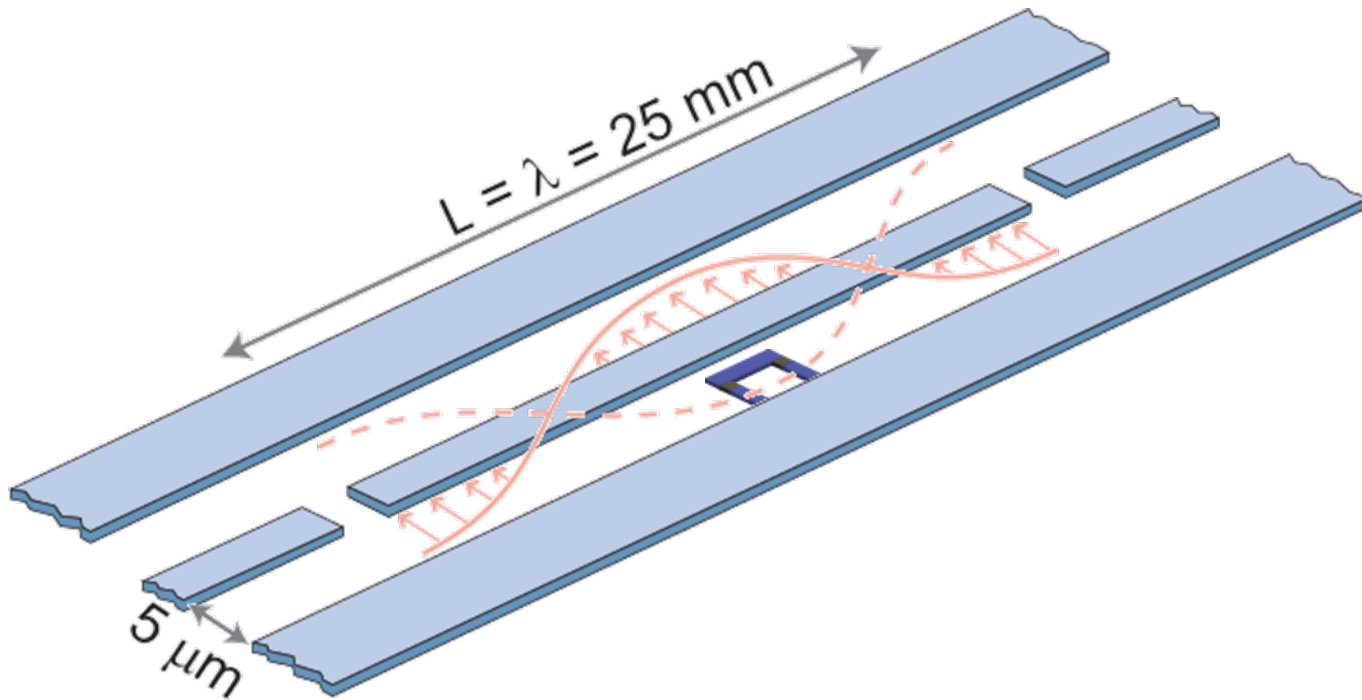
Vacuum Rabi Oscillations with Rydberg Atoms



Review: J. M. Raimond, M. Brune, and S. Haroche
Rev. Mod. Phys. **73**, 565 (2001)
 P. Hyafil, ..., J. M. Raimond, and S. Haroche,
Phys. Rev. Lett. **93**, 103001 (2004)



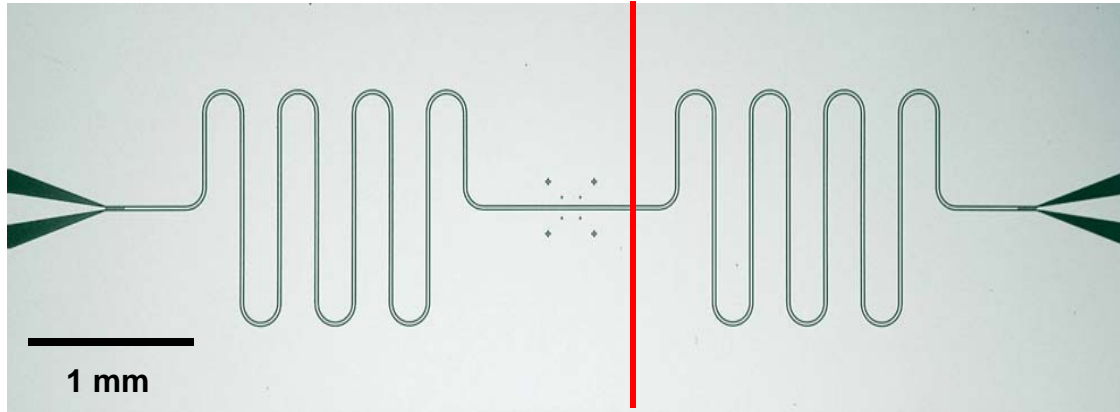
Circuit Quantum Electrodynamics



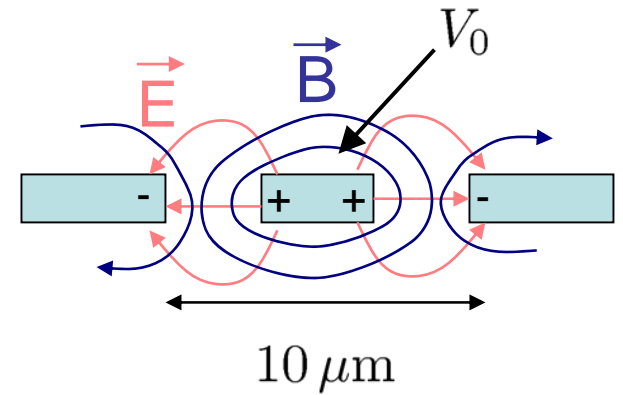
elements

- the cavity: a superconducting 1D transmission line resonator with **large vacuum field** E_0 and **long photon life time** $1/\kappa$
- the artificial atom: a Cooper pair box with **large dipole moment** d and **long coherence time** $1/\gamma$

Vacuum Field in 1D Cavity



cross-section
of transm. line (TEM mode):



voltage across resonator in vacuum state ($n = 0$)

harmonic oscillator

$$V_{0,\text{rms}} = \sqrt{\frac{\hbar\omega_r}{2C}} \approx 1 \mu\text{V}$$

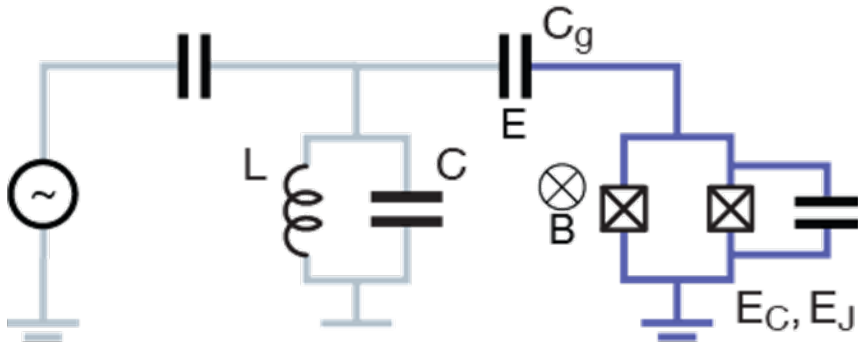
$$H_r = \hbar\omega_r \left(a^\dagger a + \frac{1}{2} \right)$$

$$E_0 = \frac{V_{0,\text{rms}}}{b} \approx 0.2 \text{ V/m}$$

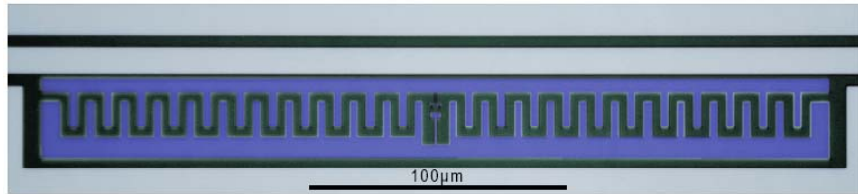
**10^3 larger than in
3D cavity**

for $\omega_r/2\pi \approx 6 \text{ GHz}$ ($C \sim 1 \text{ pF}$), $b \approx 5 \mu\text{m}$

Qubit/Photon Coupling in a Circuit



qubit coupled to resonator



coupling strength:

$$\hbar g = eV_{0,\text{rms}} \frac{C_g}{C_\Sigma}$$

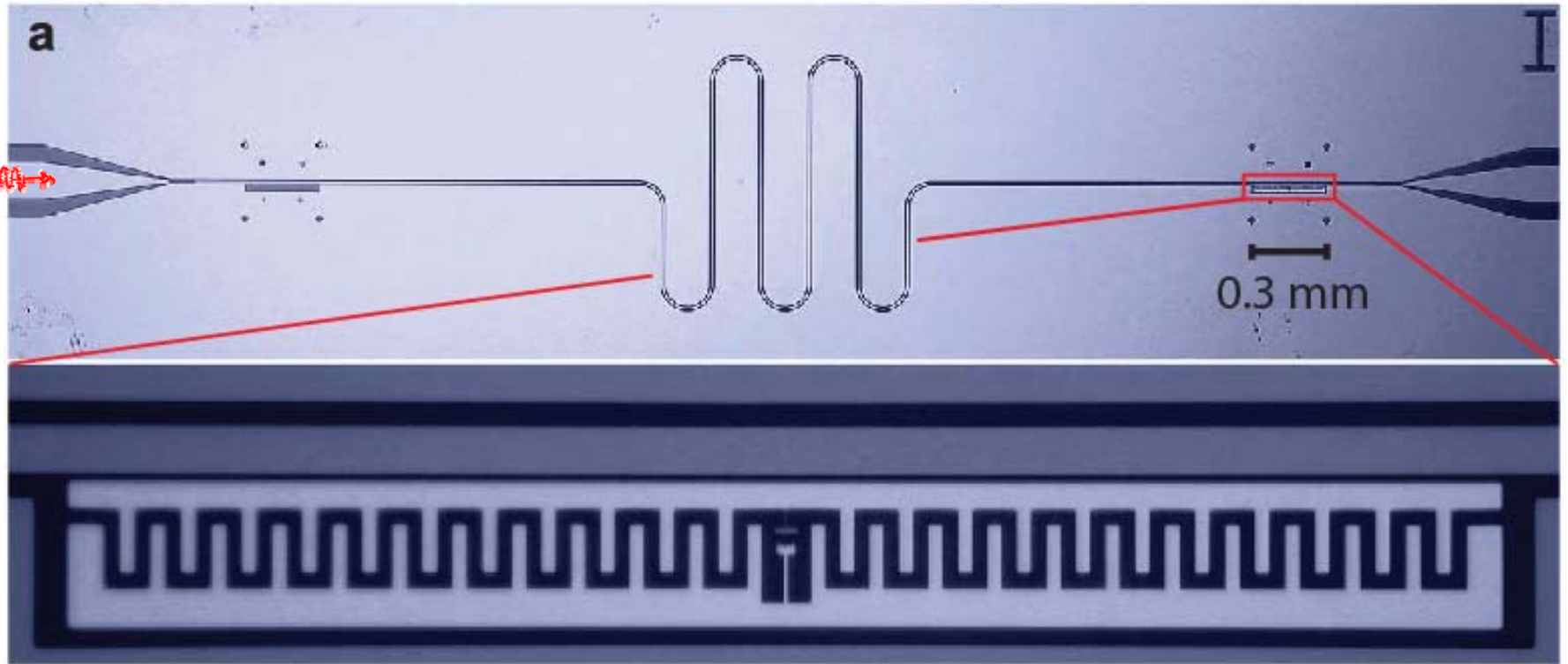
$$\Rightarrow \nu_{\text{vac}} = \frac{g}{\pi} \approx 1 \dots 300 \text{ MHz}$$

$g \gg [\kappa, \gamma]$ possible!

large effective dipole moment

$$d = \frac{\hbar g}{E_0} \sim 10^2 \dots 10^4 ea_0$$

Circuit QED with One Photon



superconducting cavity QED circuit

Resonant Vacuum Rabi Mode Splitting ...

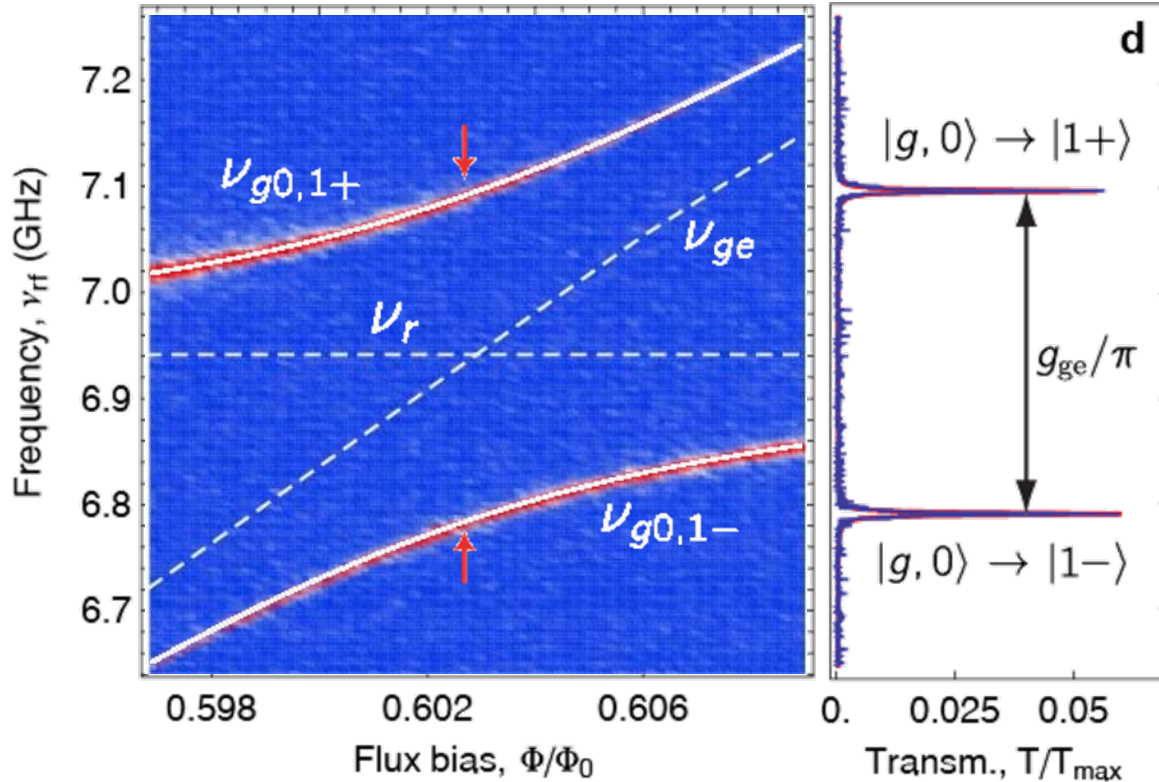
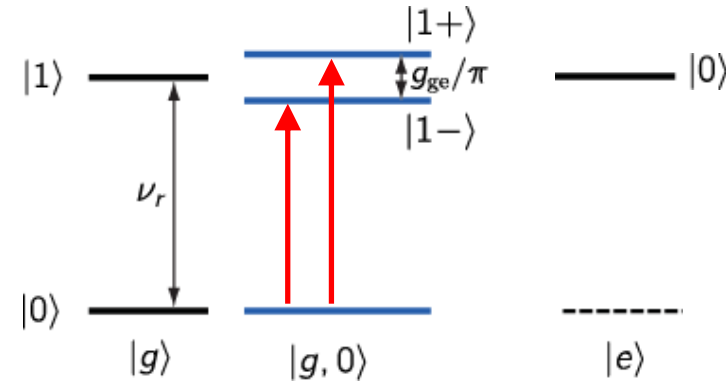
... with one photon ($n = 1$):

very strong coupling:

$$g_{ge}/\pi = 308 \text{ MHz}$$

$$\kappa, \gamma < 1 \text{ MHz}$$

$$g_{ge} \gg \kappa, \gamma$$



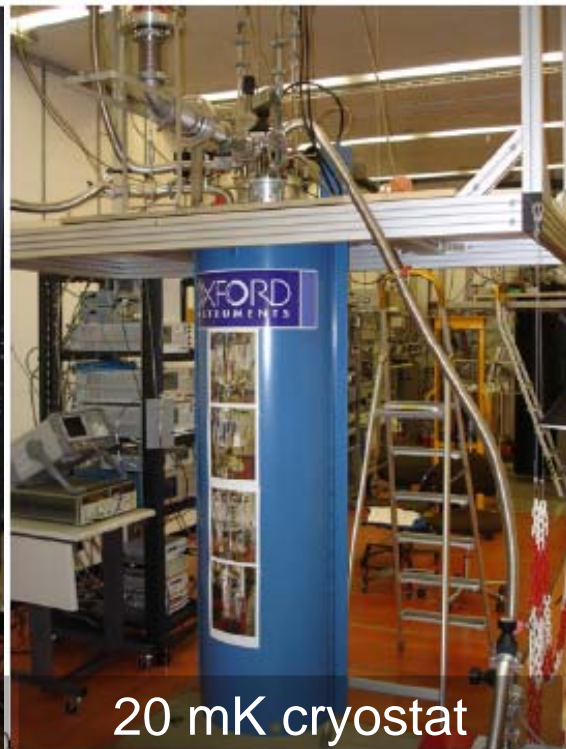
forming a 'molecule' of a qubit and a photon

$$|1\pm\rangle = (|g,1\rangle \pm |e,0\rangle) / \sqrt{2}$$

ETH first demonstration: A. Wallraff, ... and R. J. Schoelkopf, *Nature (London)* **431**, 162 (2004)

this data: J. Fink et al., *Nature (London)* **454**, 315 (2008)

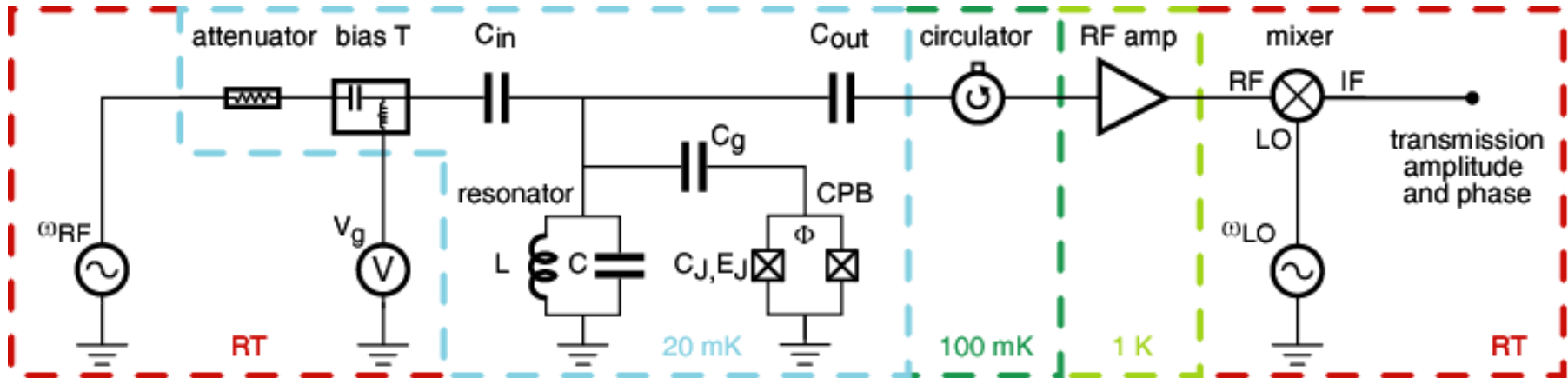
Measurement Setup



How to Measure Single Microwave Photons

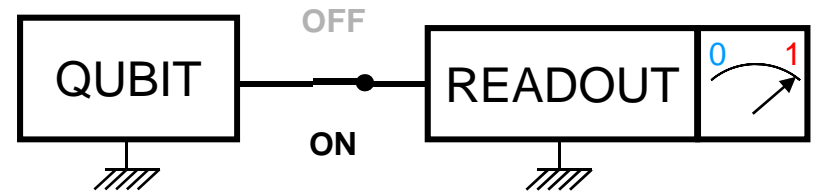
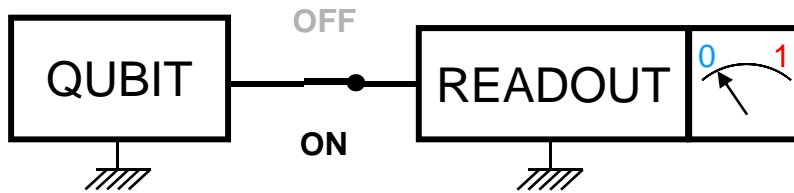
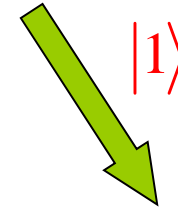
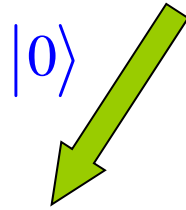
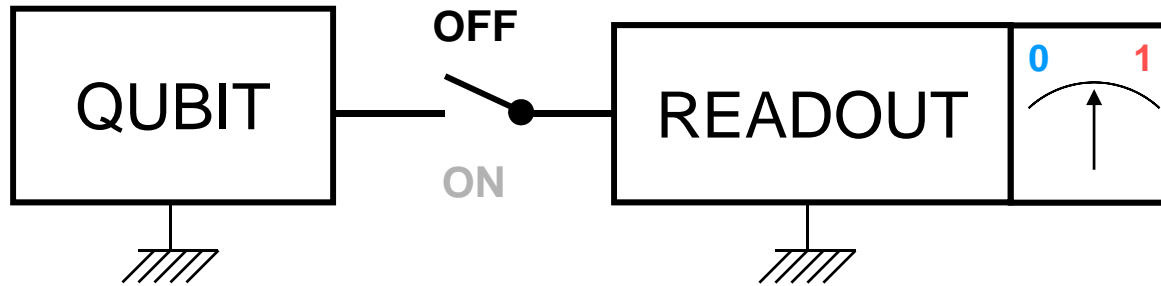
- average power to be detected

$$\rightarrow \langle n = 1 \rangle \hbar \omega_r \kappa / 2 \approx P_{RF} = -140 \text{ dBm} = 10^{-17} \text{ W}$$



- efficient with cryogenic low noise HEMT amplifier ($T_N = 6 \text{ K}$)
- prevent leakage of thermal photons (cold attenuators and circulators)

Qubit Read Out



desired: good on/off ratio
no relaxation in on state (QND)

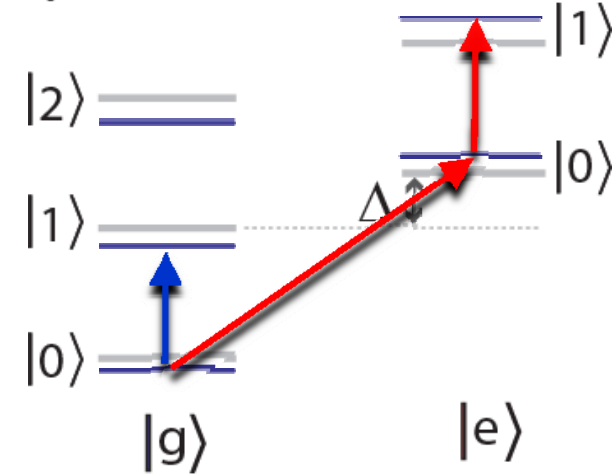
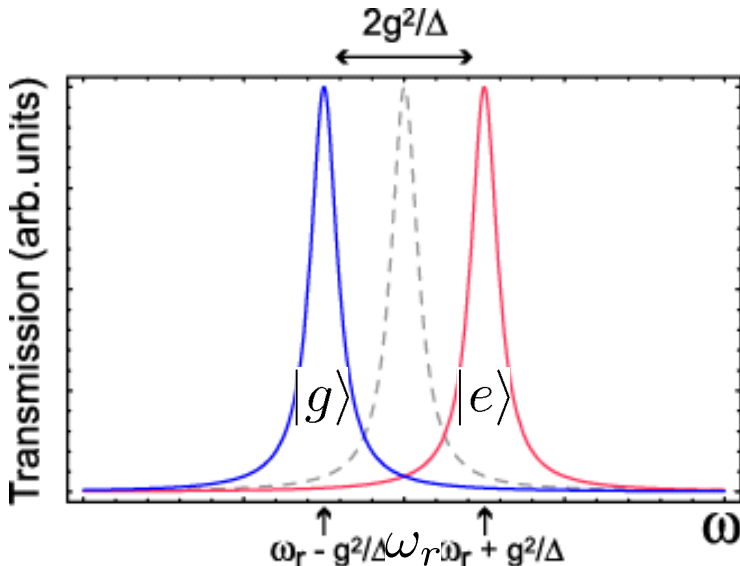
Non-Resonant (Dispersive) Interaction

approximate diagonalization: $|\Delta| = |\omega_a - \omega_r| \gg g$:

$$H \approx \hbar \left(\omega_r + \frac{g^2}{\Delta} \sigma_z \right) a^\dagger a + \frac{\hbar}{2} \left(\omega_a + \frac{g^2}{\Delta} \right) \sigma_z$$

cavity frequency shift
and qubit ac-Stark shift

Lamb Shift



qubit detuned by Δ
from resonator

A. Blais *et al.*, *PRA* 69, 062320 (2004)

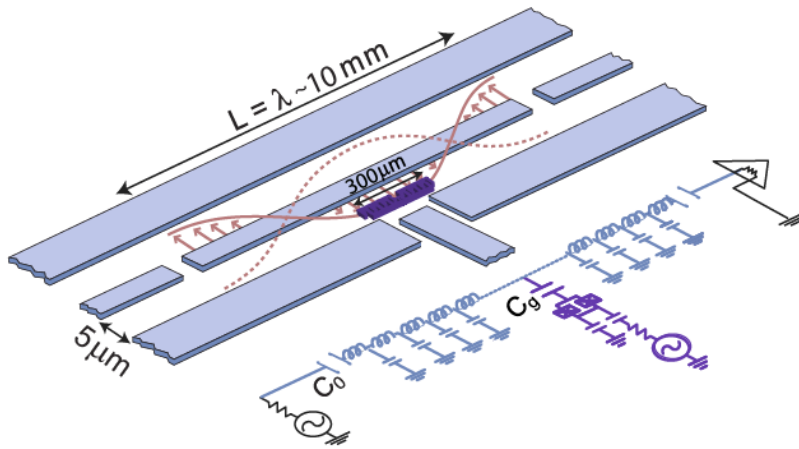
A. Wallraff *et al.*, *Nature (London)* 431, 162 (2004)

D. I. Schuster *et al.*, *Phys. Rev. Lett.* 94, 123062 (2005)

A. Fragner *et al.*, *Science* 322, 1357 (2008)

Circuit QED – read out of qubit state

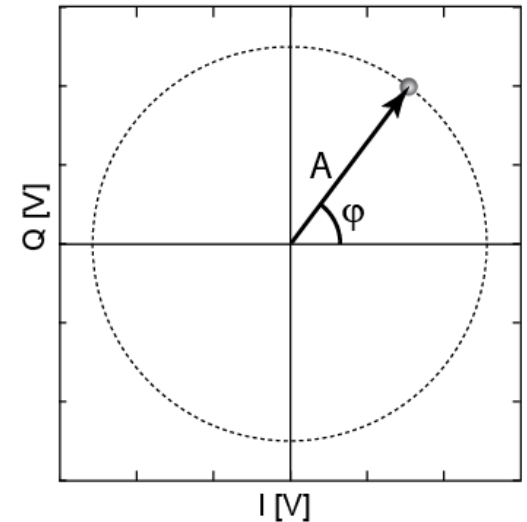
- transmission measurement to determine qubit state:



Phase sensitive measurement of transmitted microwave:

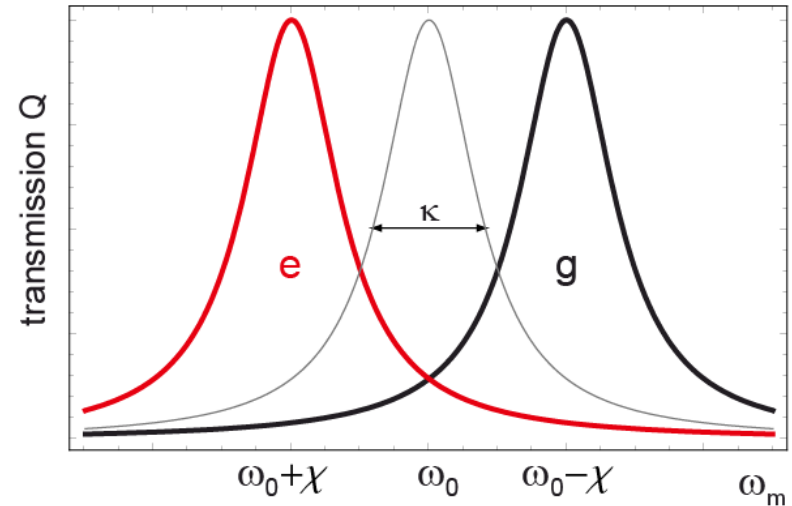
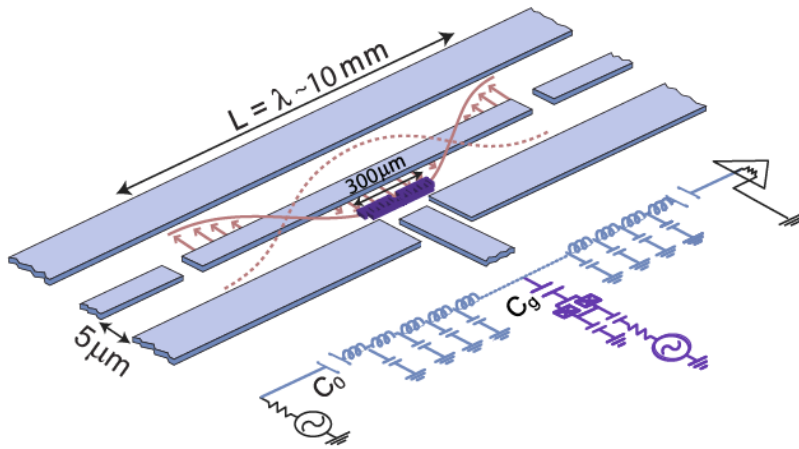
Voltage signal:

$$A(t) \sin(\omega_m t + \phi(t)) \equiv I(t) \sin \omega_m t + Q(t) \cos \omega_m t$$



Circuit QED – read out of qubit state

- transmission measurement to determine qubit state:



qubit far from resonance ($\Delta_{ar} = |\omega_a - \omega_r| \gg g$) – dispersive Hamiltonian:

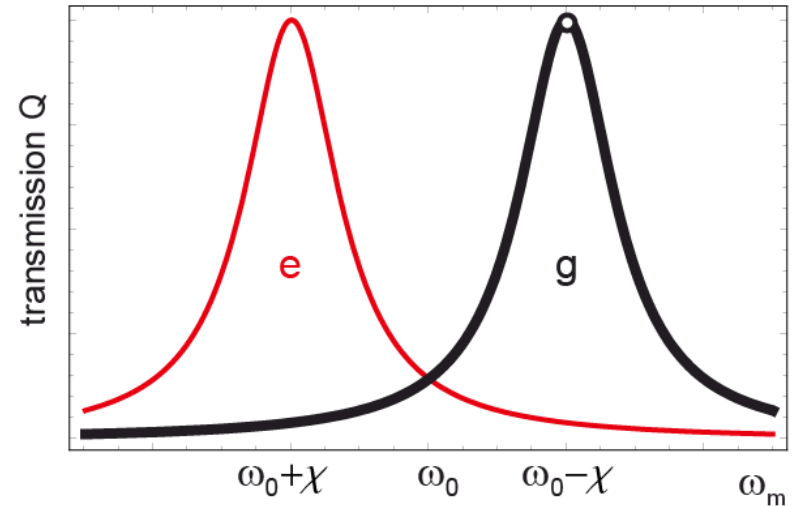
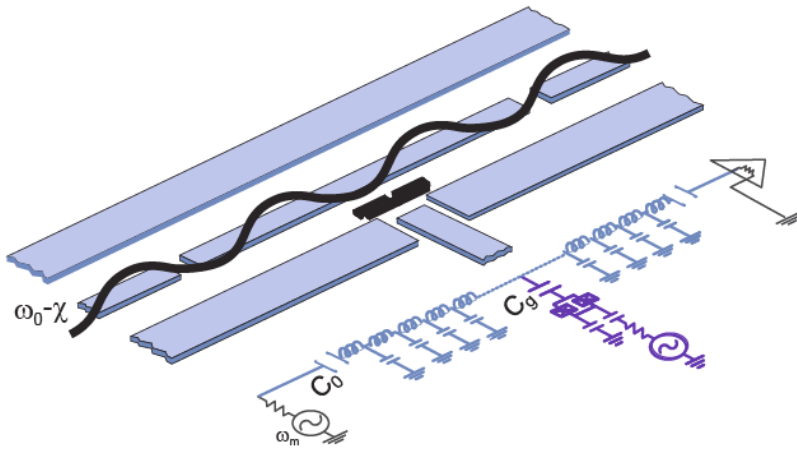
$$H = \hbar(\omega_r + \chi\sigma_z)a^\dagger a + \frac{\hbar}{2}(\omega_a + \chi)\sigma_z$$

[Blais *et al.*, PRA 69 (2004)]

← state-dependent frequency shift

Circuit QED – read out of qubit state

- transmission measurement to determine qubit state:



qubit far from resonance ($\Delta_{ar} = |\omega_a - \omega_r| \gg g$) – dispersive Hamiltonian:

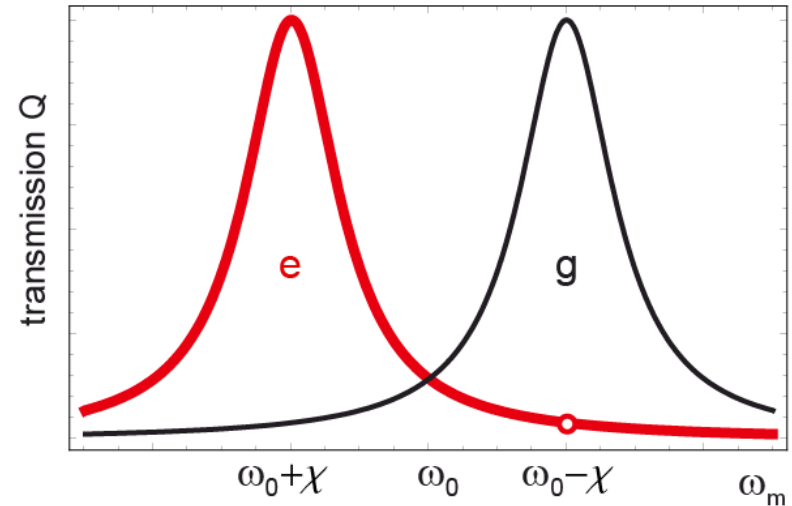
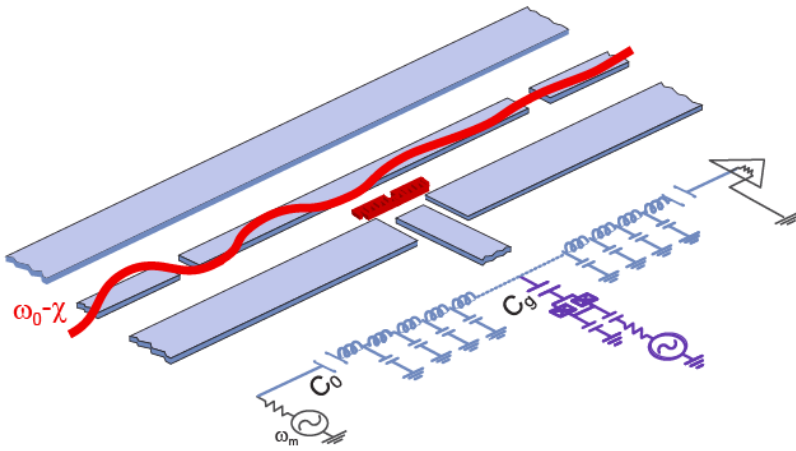
$$H = \hbar(\omega_r + \chi\sigma_z)a^\dagger a + \frac{\hbar}{2}(\omega_a + \chi)\sigma_z$$

[Blais *et al.*, PRA 69 (2004)]

← state-dependent frequency shift

Circuit QED – read out of qubit state

- transmission measurement to determine qubit state:



qubit far from resonance ($\Delta_{ar} = |\omega_a - \omega_r| \gg g$) – dispersive Hamiltonian:

$$H = \hbar(\omega_r + \chi\sigma_z)a^\dagger a + \frac{\hbar}{2}(\omega_a + \chi)\sigma_z$$

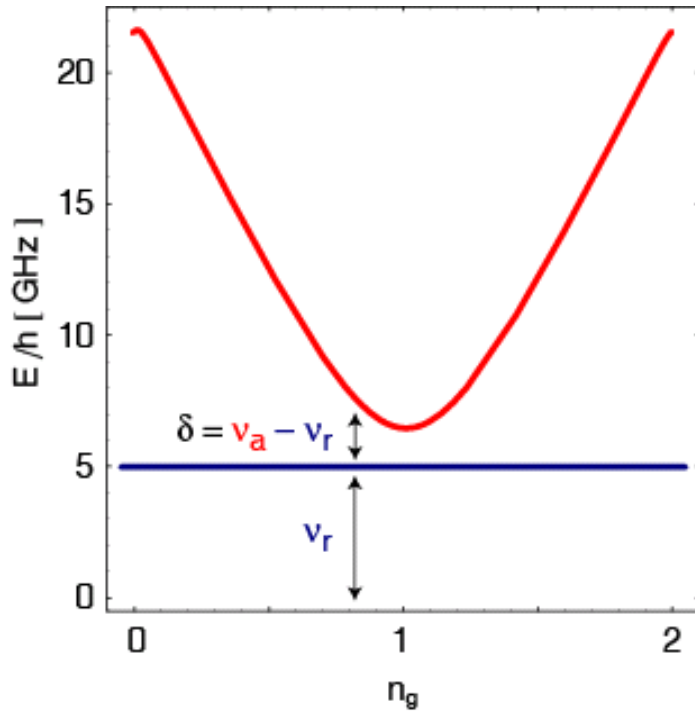
[Blais *et al.*, PRA 69 (2004)]

state-dependent frequency shift $\rightarrow \sigma_z$ determined

Dispersive Shift of Resonance Frequency

sketch of qubit level separation:

$$\Delta = 2\pi\delta > g$$

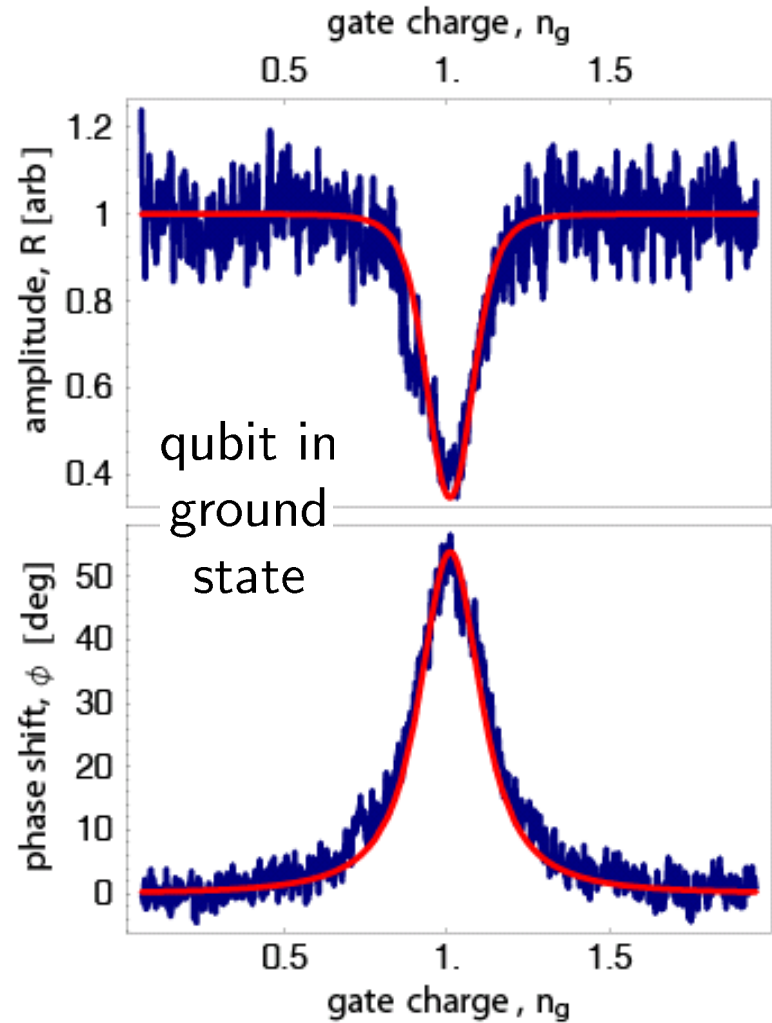


$$g/\pi = \nu_{\text{vac}} = 11 \text{ MHz}$$

$$\Delta(n_g = 1)/2\pi = 66 \text{ MHz}$$

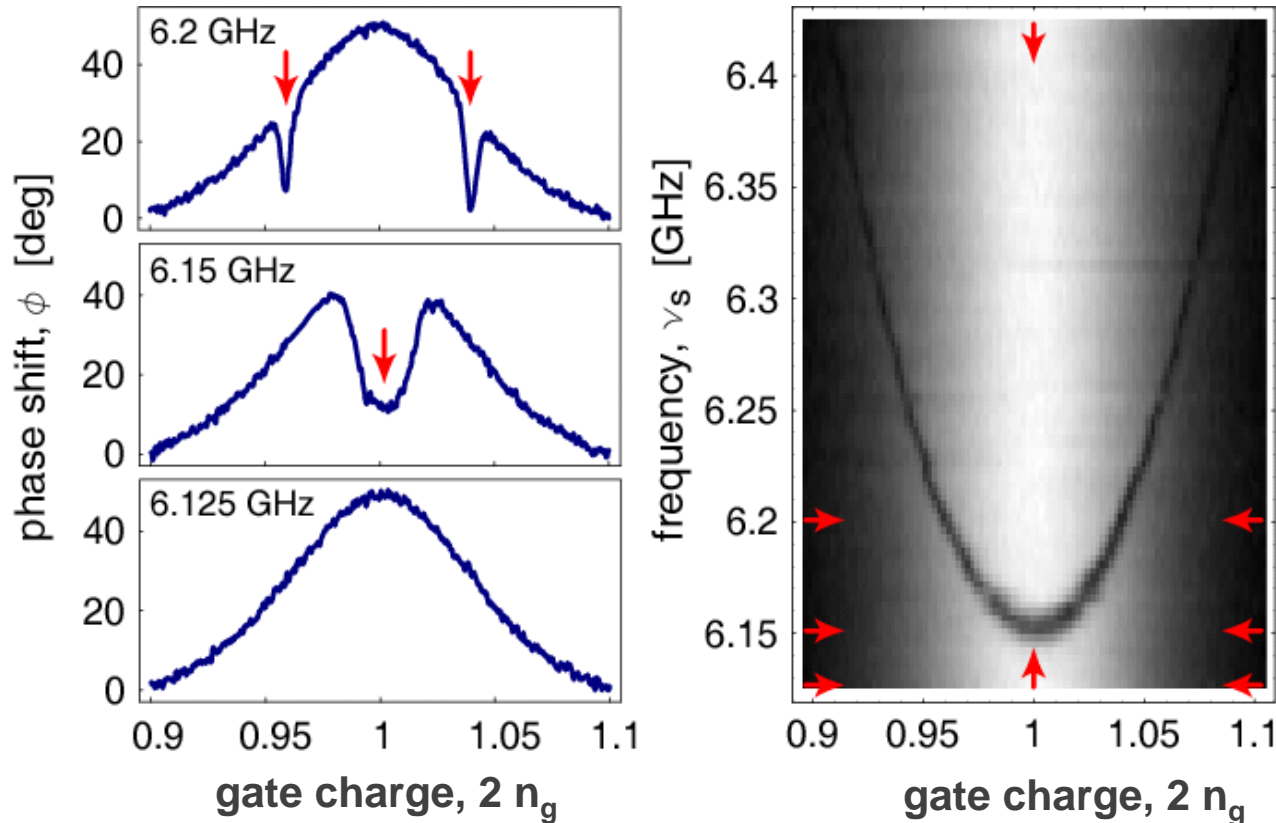
$$n = 10$$

measured resonator transmission amplitude and phase:



CW Spectroscopy of Cooper Pair Box

drive qubit transition at frequency ν_s

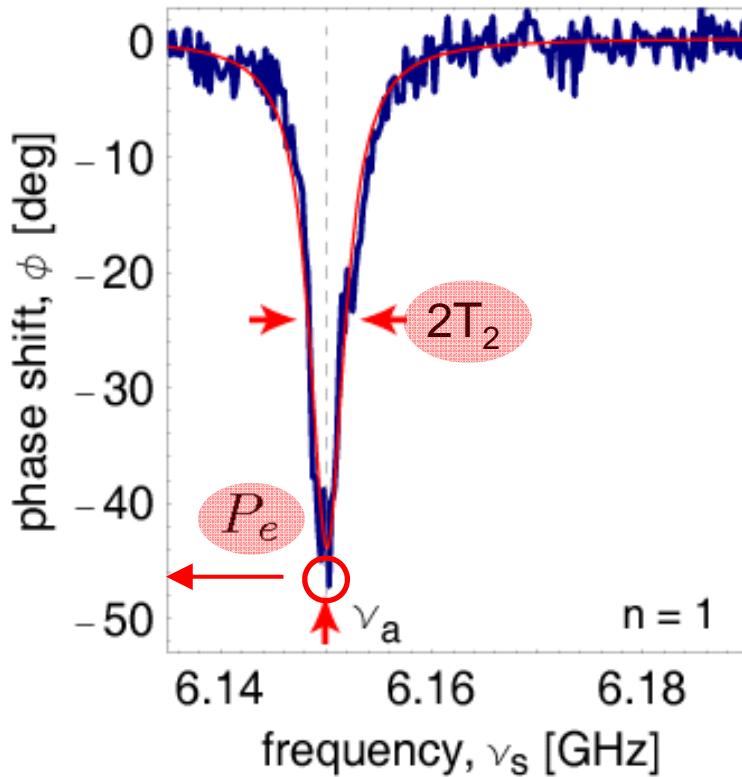


detuning $\Delta_{r,a}/2\pi \sim 100$ MHz

extracted: $E_J = 6.2$ GHz, $E_C = 4.8$ GHz

Line Shape

excited state population (steady-state Bloch equations):
 equilibrium between emission and absorption processes



$$P_e = 1 - P_g = \frac{1}{2} \frac{\Omega^2 T_1 T_2}{1 + (T_2 \Delta_{s,a})^2 + \Omega^2 T_1 T_2}$$

- drive strength Ω
- varying $\Delta_{s,a} = \omega_s - \tilde{\omega}_a$
- weak continuous measurement (approx. 1 photon in resonator)