2.7.6 Super Dense Coding

task: Try to transmit two bits of classical information between Alice (A) and Bob (B) using only one qubit.

- As Alice and Bob are living in a quantum world they are allowed to use one pair of entangled qubits that they have prepared ahead of time.

protocol:
A) Alice and Bob each have one qubit of an entangled pair in their possession

$$|\psi\rangle = 1/\sqrt{2}\,(|00\rangle + |11\rangle)$$

B) Alice does a quantum operation on her qubit depending on which 2 classical bits she wants to communicate
C) Alice sends her qubit to Bob
D) Bob does one measurement on the entangled pair

shared entanglement $\frac{1}{\sqrt{2}}|00\rangle + \frac{1}{\sqrt{2}}|11\rangle$

local operations $X_1 \,;\, Y_1 \,;\, Z_1 \,;\, I_1$

send Alices qubit to Bob

Bob measures

bits to be transferred:	Alice's operation	resulting 2-qubit state	Bob's measurement
00	I_1	$I_1 \vert\psi\rangle = \frac{1}{\sqrt{2}}\left(\vert 00\rangle + \vert 11\rangle\right)$	
01	Z_1	$Z_1 \vert\psi\rangle = \frac{1}{\sqrt{2}}\left(\vert 00\rangle - \vert 11\rangle\right)$	measure in Bell basis
10	X_1	$X_1 \vert\psi\rangle = \frac{1}{\sqrt{2}}\left(\vert 10\rangle + \vert 01\rangle\right)$	
11	iY_1	$iY_1 \vert\psi\rangle = \frac{1}{\sqrt{2}}\left(\vert 10\rangle - \vert 01\rangle\right)$	

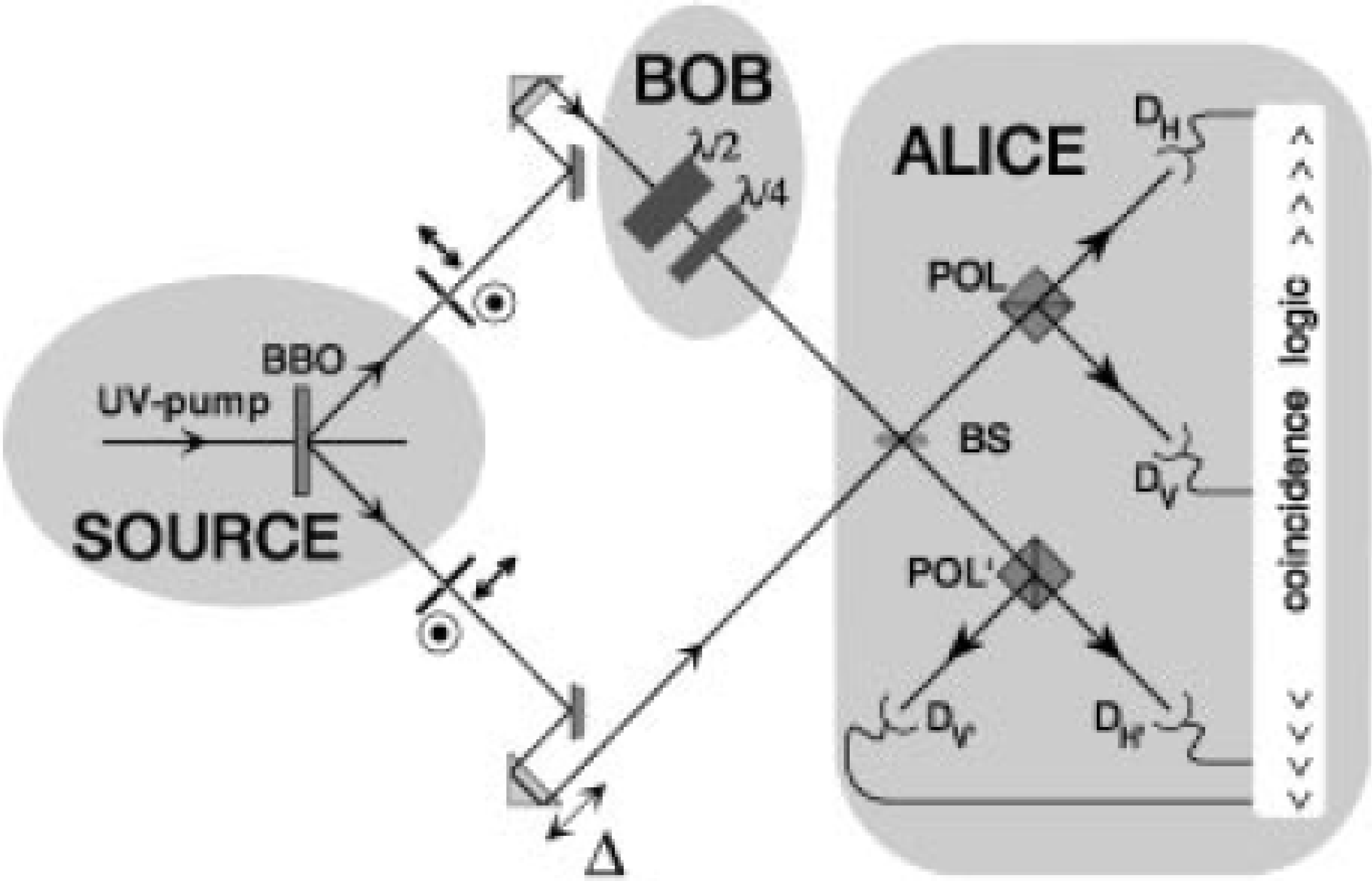
- all these states are entangled (try!)
- they are called the Bell states

comments:

- two qubits are involved in protocol BUT Alice only interacts with one and sends only one along her quantum communications channel
- two bits cannot be communicated sending a single classical bit along a classical communications channel

original proposal of super dense coding: Charles H. Bennett and Stephen J. Wiesner, Communication via one- and two-particle operators on Einstein-Podolsky-Rosen states, **Phys. Rev. Lett. 69, 2881(1992)**

2.7.7 Experimental demonstration of super dense coding using photons



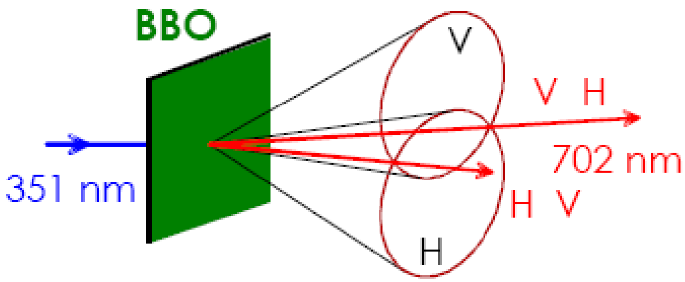
Generating polarization entangled photon pairs using **Parametric Down Conversion**:

parametric down-conversion

- 1 UV-photon ➔ 2 "red" photons
- conservation of

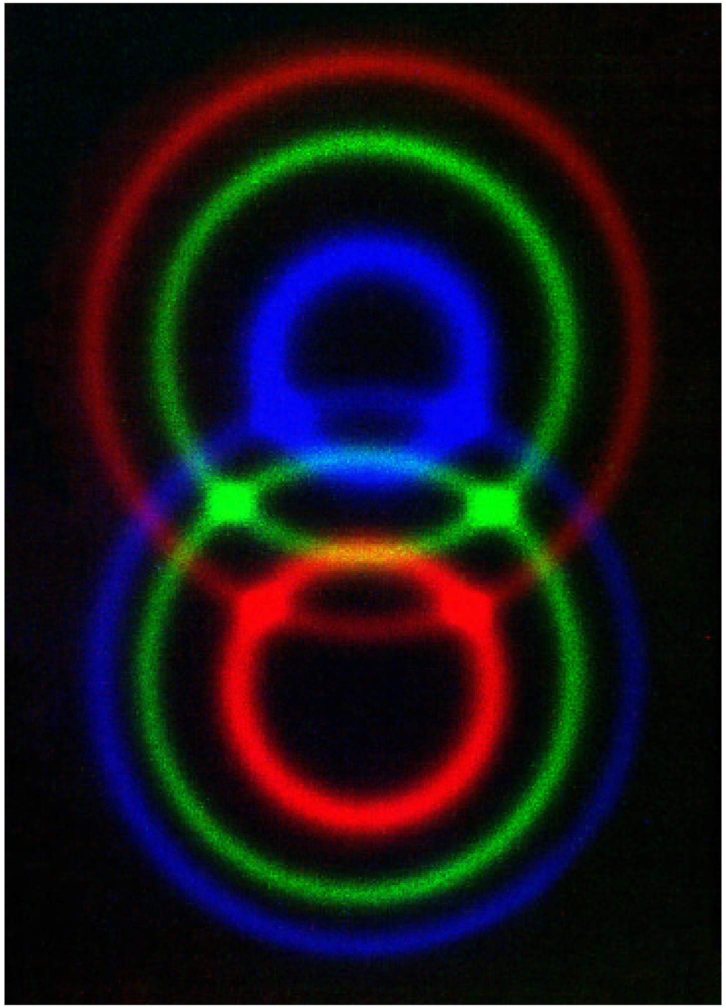
  energy $\omega_p = \omega_s + \omega_i$

  momentum $\vec{k}_p = \vec{k}_s + \vec{k}_i$
- Polarisationskorrelationen (typ II)



$$|\Psi^-\rangle = \tfrac{1}{\sqrt{2}}\left(|H\rangle|V\rangle - |V\rangle|H\rangle\right)$$

optically nonlinear medium: BBO (BaB_2O_4) beta barium borate



Half Wave Plate

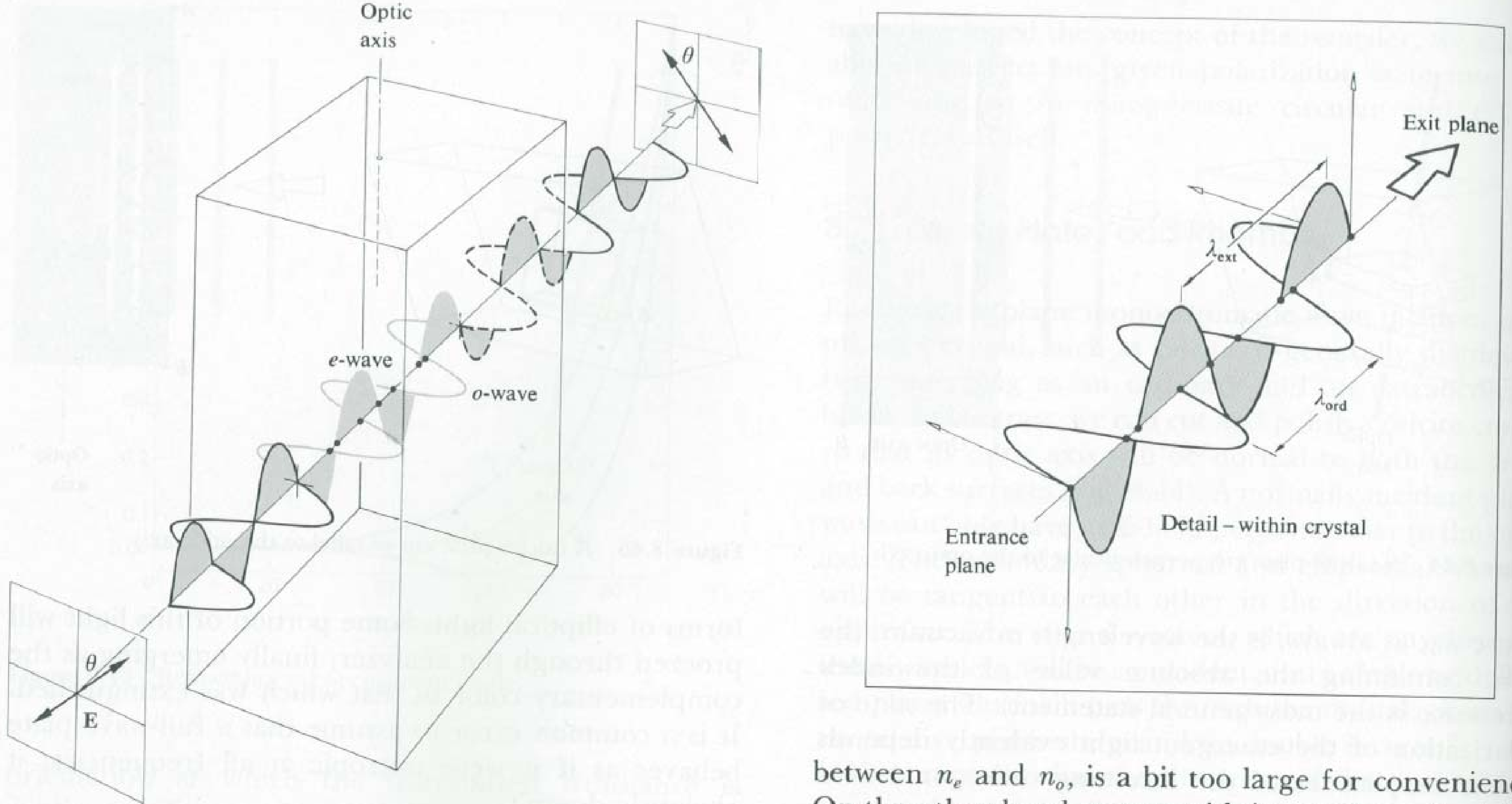
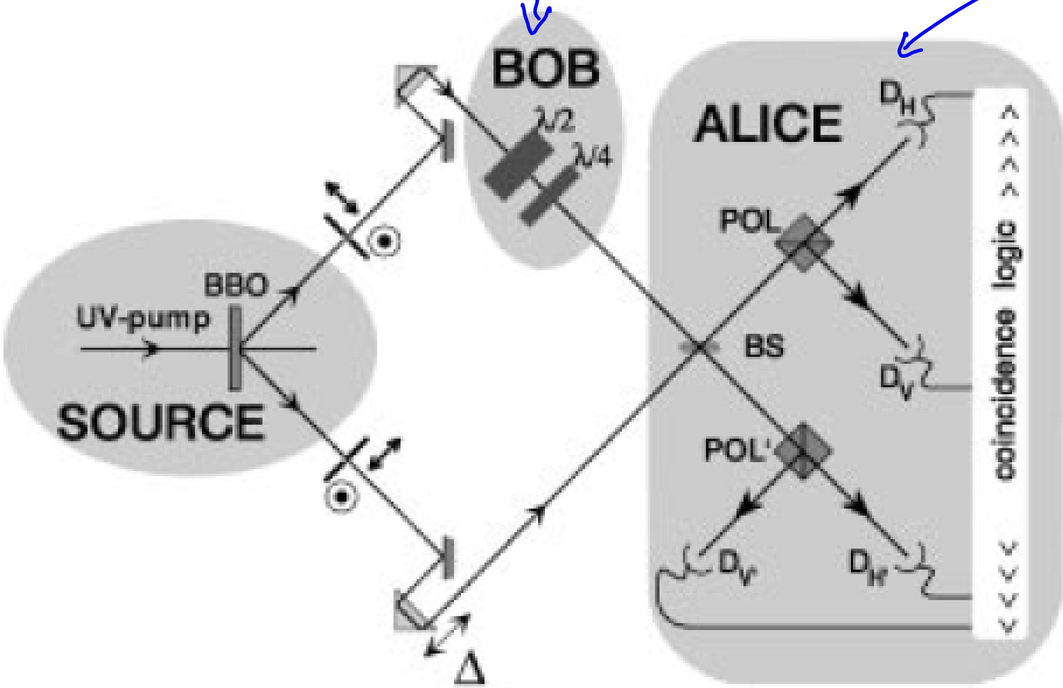


Figure 8.46 A half-wave plate.

between n_e and n_o, is a bit too large for convenience. On the other hand, quartz with its much smaller birefringence is frequently used, but it has no natural cleavage planes and must be cut, ground, and polished.

state manipulation

Bell state measurement



$$\Psi^- = \frac{1}{\sqrt{2}}\left(|HV\rangle - |VH\rangle\right) \quad \text{asym.}$$

$$\left.\begin{array}{l} \Psi^+ = \frac{1}{\sqrt{2}}\left(|HV\rangle + |VH\rangle\right) \\ \phi^+ = \frac{1}{\sqrt{2}}\left(|HH\rangle + |VV\rangle\right) \\ \phi^- = \frac{1}{\sqrt{2}}\left(|HH\rangle - |VV\rangle\right) \end{array}\right\} \text{sym.}$$

H = horizontal polarization
V = vertical polarization

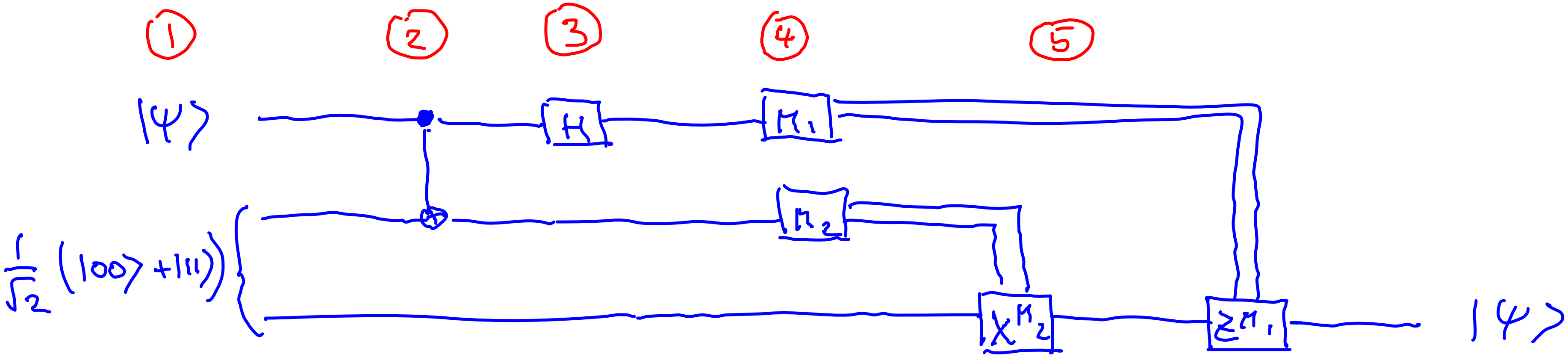
Klaus Mattle, Harald Weinfurter, Paul G. Kwiat, and Anton Zeilinger, Dense coding in experimental quantum communication, Phys. Rev. Lett.76, 4656 (1996)

2.9 Quantum Teleportation

Task: Alice wants to transfer an unknown quantum state ψ to Bob only using **one entangled pair** of qubits and **classical information** as a resource.

note:
- Alice does not know the state to be transmitted
- Even if she knew it the classical amount of information that she would need to send would be infinite.

The **teleportation circuit**:



original article: Bennett, C. H. et al., Teleporting an unknown quantum state via dual classical and Einstein-Podolsky-Rosen channels, *Phys. Rev. Lett.* **70**, 1895-1899 (1993)

2.9.1 How does it work?

① $$|\psi\rangle \otimes \frac{1}{\sqrt{2}}\left(|00\rangle + |11\rangle\right) = \frac{1}{\sqrt{2}}\left(\alpha|000\rangle + \alpha|011\rangle + \beta|100\rangle + \beta|111\rangle\right)$$

CNOT between qubit to be teleported and one bit of the entangled pair:

② $$\xrightarrow{CNOT_{12}} \frac{1}{\sqrt{2}}\left(\alpha|000\rangle + \alpha|011\rangle + \beta|110\rangle + \beta|101\rangle\right)$$

Hadamard on qubit to be teleported:

③ $$\xrightarrow{H_1} \frac{1}{2}\Big[\left(|00\rangle(\alpha|0\rangle + \beta|1\rangle\right) + |10\rangle(\alpha|0\rangle - \beta|1\rangle) + |01\rangle(\alpha|1\rangle + \beta|0\rangle) + |11\rangle(\alpha|1\rangle - \beta|0\rangle)\Big]$$

measurement of qubit 1 and 2, classical information transfer and single bit manipulation on target qubit 3:

④ $\xrightarrow{M_1 \otimes M_2}$ ⑤

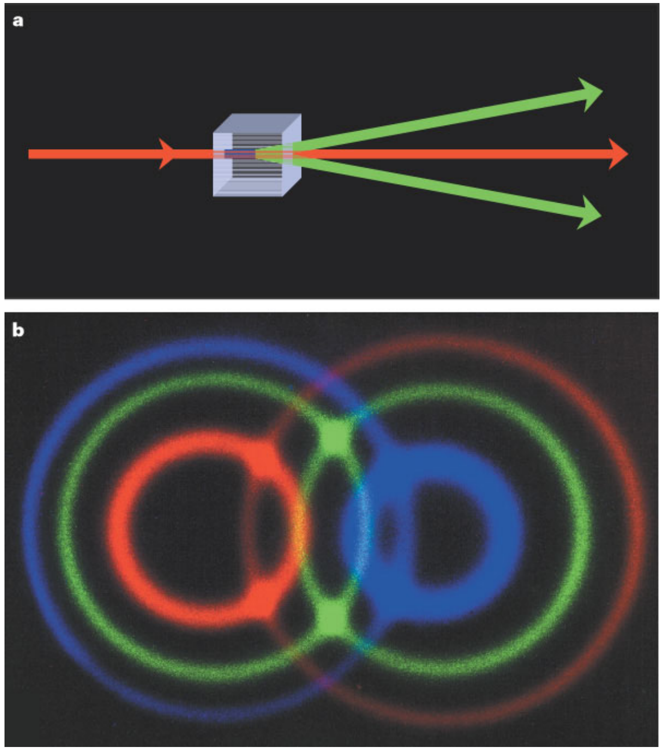
$$P_{00} = \frac{1}{4}\;;\quad |\psi_3\rangle = \alpha|0\rangle + \beta|1\rangle \xrightarrow{I} |\psi\rangle$$

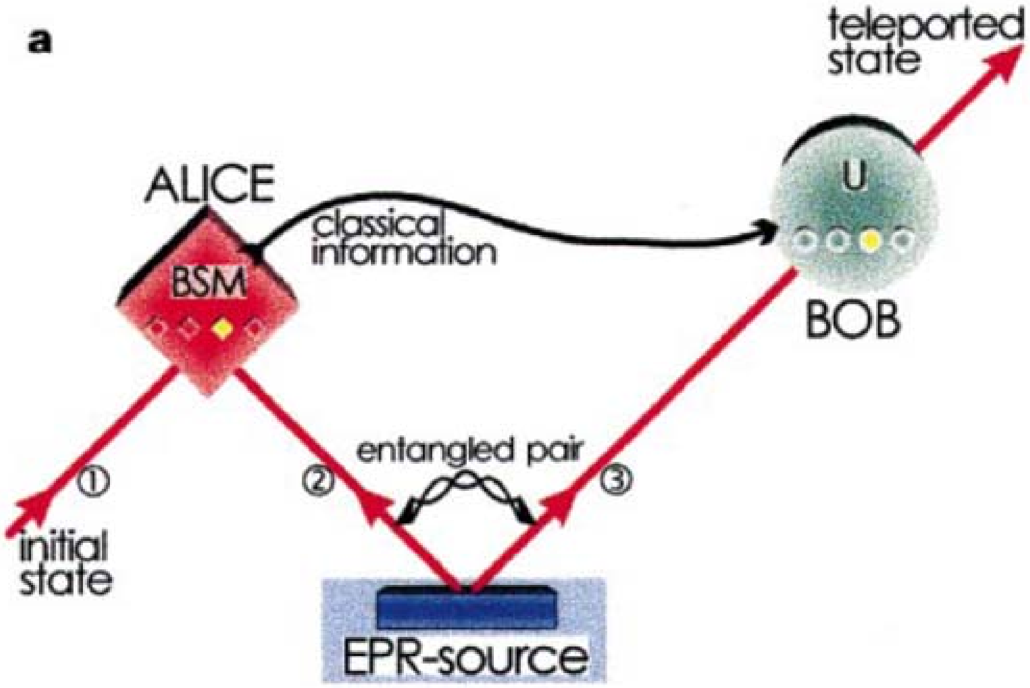
$$P_{10} = \frac{1}{4}\;;\quad |\psi_3\rangle = \alpha|0\rangle - \beta|1\rangle \xrightarrow{Z} |\psi\rangle$$

$$P_{01} = \frac{1}{4}\;;\quad |\psi_3\rangle = \alpha|1\rangle + \beta|0\rangle \xrightarrow{X} |\psi\rangle$$

$$P_{11} = \frac{1}{4}\;;\quad |\psi_3\rangle = \alpha|1\rangle - \beta|0\rangle \xrightarrow{XZ} |\psi\rangle$$

2.9.2 (One) Experimental Realization of Teleportation using Photon Polarization:

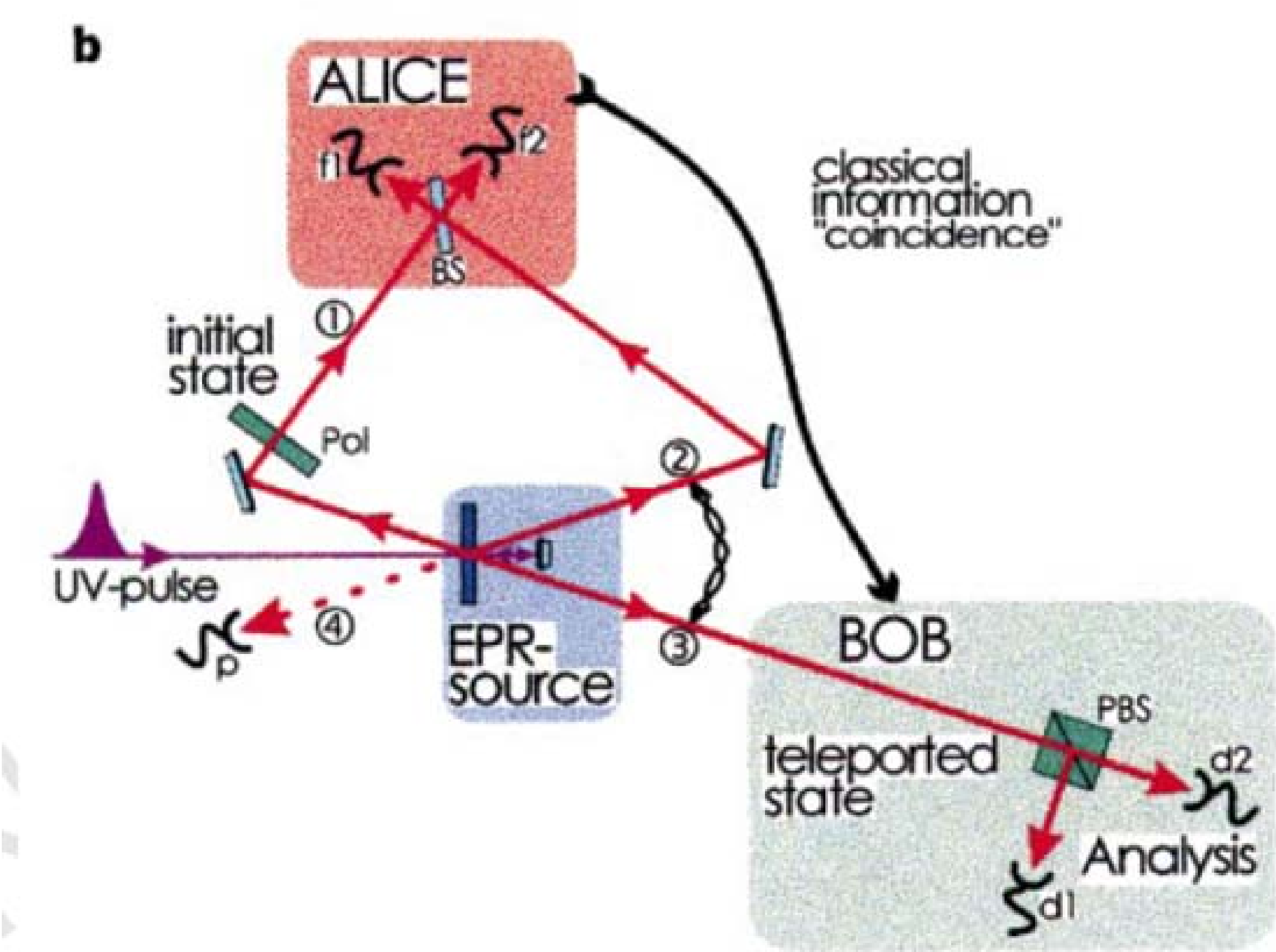




- parametric down conversion (PDC)
  source of entangled photons
- qubits are polarization encoded

Dik Bouwmeester, Jian-Wei Pan, Klaus Mattle, Manfred Eibl, Harald Weinfurter, Anton Zeilinger, Experimental quantum teleportation *Nature* **390**, 575 (1997)

Experimental Setup

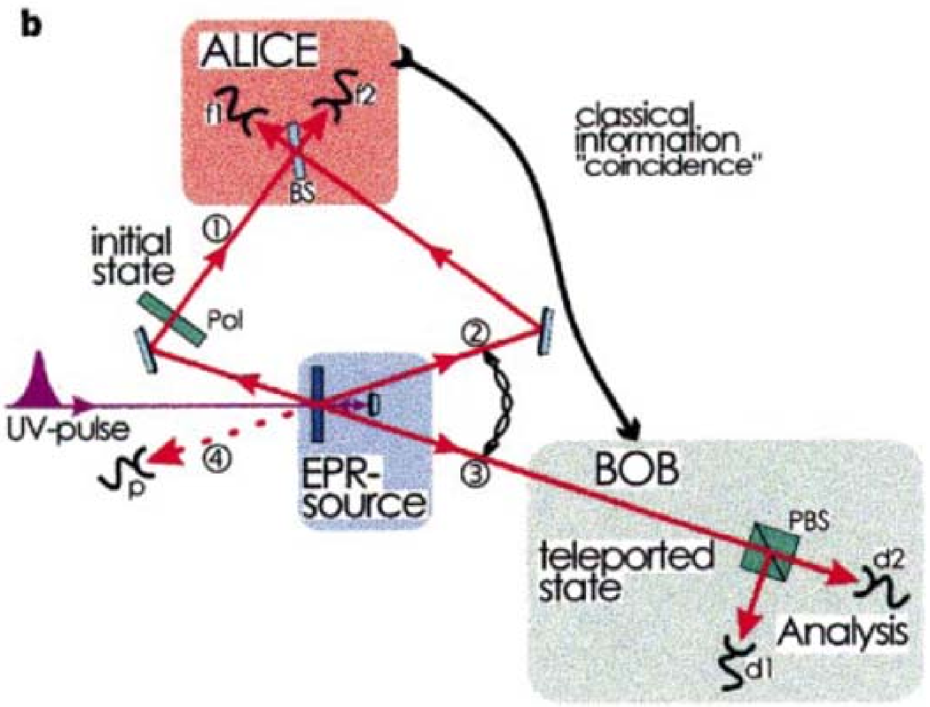


Experimental Implementation

start with states

$$|\psi_1\rangle = \alpha\, |H\rangle + \beta\, |V\rangle$$

$$|\psi_{23}\rangle = \frac{1}{\sqrt{2}} \left(|HV\rangle - |VH\rangle \right)$$



combine photon to be teleported (1) and one photon of entangled pair (2) on a 50/50 beam splitter (BS) and measure (at Alice) resulting state in Bell basis.

analyze resulting teleported state of photon (3) using polarizing beam splitters (PBS) single photon detectors

- polarizing beam splitters (PBS) as detectors of teleported states teleportation papers for you to present:

Experimental Realization of Teleporting an Unknown Pure Quantum State via Dual Classical and Einstein-Podolsky-Rosen Channels
D. Boschi, S. Branca, F. De Martini, L. Hardy, and S. Popescu
Phys. Rev. Lett. **80**, 1121 (1998) [PROLA Link]

Unconditional Quantum Teleportation
A. Furusawa, J. L. Sørensen, S. L. Braunstein, C. A. Fuchs, H. J. Kimble, and E. S. Polzik
Science 23 October 1998 282: 706-709 [DOI: 10.1126/science.282.5389.706] (in Research Articles)
Abstract » Full Text » PDF »

Complete quantum teleportation using nuclear magnetic resonance
M. A. Nielsen, E. Knill, R. Laflamme
Nature 396, 52 - 55 (05 Nov 1998) Letters to Editor
Abstract | Full Text | PDF | Rights and permissions | Save this link

Deterministic quantum teleportation of atomic qubits
M. D. Barrett, J. Chiaverini, T. Schaetz, J. Britton, W. M. Itano, J. D. Jost, E. Knill, C. Langer, D. Leibfried, R. Ozeri, D. J. Wineland
Nature 429, 737 - 739 (17 Jun 2004) Letters to Editor
Abstract | Full Text | PDF | Rights and permissions | Save this link

Deterministic quantum teleportation with atoms
M. Riebe, H. Haeffner, C. F. Roos, W. Haensel, J. Benhelm, G. P. T. Lancaster, T. W. Koerber, C. Becher, F. Schmidt-Kaler, D. F. V. James, R. Blatt
Nature 429, 734 - 737 (17 Jun 2004) Letters to Editor
Abstract | Full Text | PDF | Rights and permissions | Save this link

Quantum teleportation between light and matter
Jacob F. Sherson, Hanna Krauter, Rasmus K. Olsson, Brian Julsgaard, Klemens Hammerer, Ignacio Cirac, Eugene S. Polzik
Nature 443, 557 - 560 (05 Oct 2006) Letters to Editor
Full Text | PDF | Rights and permissions | Save this link

John Bell's thought experiment

- Charlie simultaneously prepares two particles having physical properties **Q, R, S, T** and gives one particle each to Alice and Bob.

- Alice measures the properties **Q** and **R** of her particle with the possible outcomes **q = ±1** and **r = ±1.**

- Bob simultaneously measures the properties **S** and **T** of his particles with the possible outcomes **s = ±1** and **t = ±1**.

consider the quantity:

$$QS + RS + RT - QT$$

$$= (R+Q)S + (R-Q)T = \pm 2$$

since $R+Q = 0$
or $R-Q = 0$

the probability of the system being in state $Q = q\,,\ R = r\,,\ S = s\,,\ T = t$

is given by : $p(q, r, s, t)$

we also denote **E(x)** as the mean of the quantity **x** : $E(x) = \sum_{q,r,s,t} p(q,r,s,t)\, x$

Now, Alice and Bob perform measurements on the two particles and record their outcomes. Then they meet up and perform the multiplications (e.g. **q s**) and calculate the average values **E(QS)**.

What are the possible outcomes of measuring the quantity **E(QS+RS+RT-QT)**?

find an upper bound:

$$E(QS+RS+RT-QT) = \sum_{q,r,s,t} p(q,r,s,t)\underbrace{(qs+rs+rt-qt)}_{\le 2}$$

$$\le \underbrace{\sum_{q,r,s,t} p(q,r,s,t)}_{=1}\, 2$$

$$= 2$$

also:

$$\boxed{E(QS+RS+RT-QT)} = \sum_{q,r,s,t} p(q,r,s,t)(qs+rs+rt-qt)$$

$$= E(QS)+E(RS)+E(RT)-E(QT)$$

$$\boxed{\le 2}$$

Bell inequality

measure this quantity for a Bell state: $|\psi\rangle = \frac{1}{\sqrt{2}}\left(|01\rangle - |10\rangle\right)$

Alice measures: $Q = Z_1$ $\quad$ $R = X_1$

Bob measures: $S = \frac{1}{\sqrt{2}}\left(-Z_2 - X_2\right)$ $\quad$ $T = \frac{1}{\sqrt{2}}\left(Z_2 - X_2\right)$

determine expectation values of joint measurements:

$$\langle QS\rangle = \langle\psi|\, Z_1 \frac{1}{\sqrt{2}}\left(-Z_2 - X_2\right)|\psi\rangle = \frac{1}{\sqrt{2}}\langle\psi| - Z_1 Z_2 - Z_1 X_2 |\psi\rangle$$
$$= \frac{1}{\sqrt{2}}\frac{1}{2}\left(\langle 01| - \langle 10|\right)\left[\left(|01\rangle - |10\rangle\right) - \left(|00\rangle + |11\rangle\right)\right] = \frac{1}{\sqrt{2}}\frac{1}{2}(1+1)$$
$$= \frac{1}{\sqrt{2}}$$

$$\langle RS\rangle = \frac{1}{\sqrt{2}} \;;\quad \langle RT\rangle = \frac{1}{\sqrt{2}} \;;\quad \langle QT\rangle = -\frac{1}{\sqrt{2}}$$

determine value of Bell inequality:

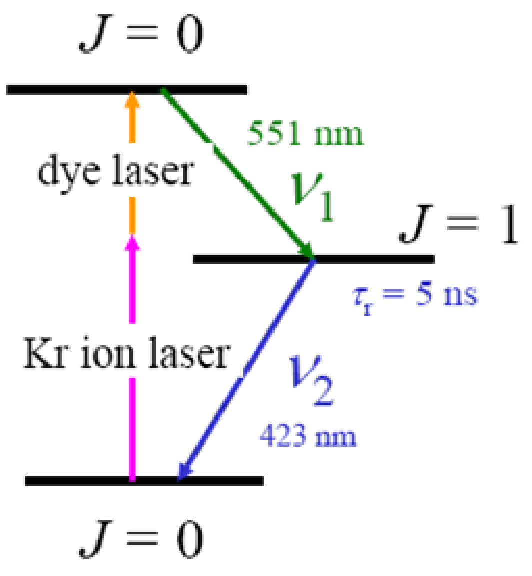
$$\langle QS\rangle + \langle RS\rangle + \langle RT\rangle - \langle QT\rangle = 2\sqrt{2} > 2 \quad !!!$$

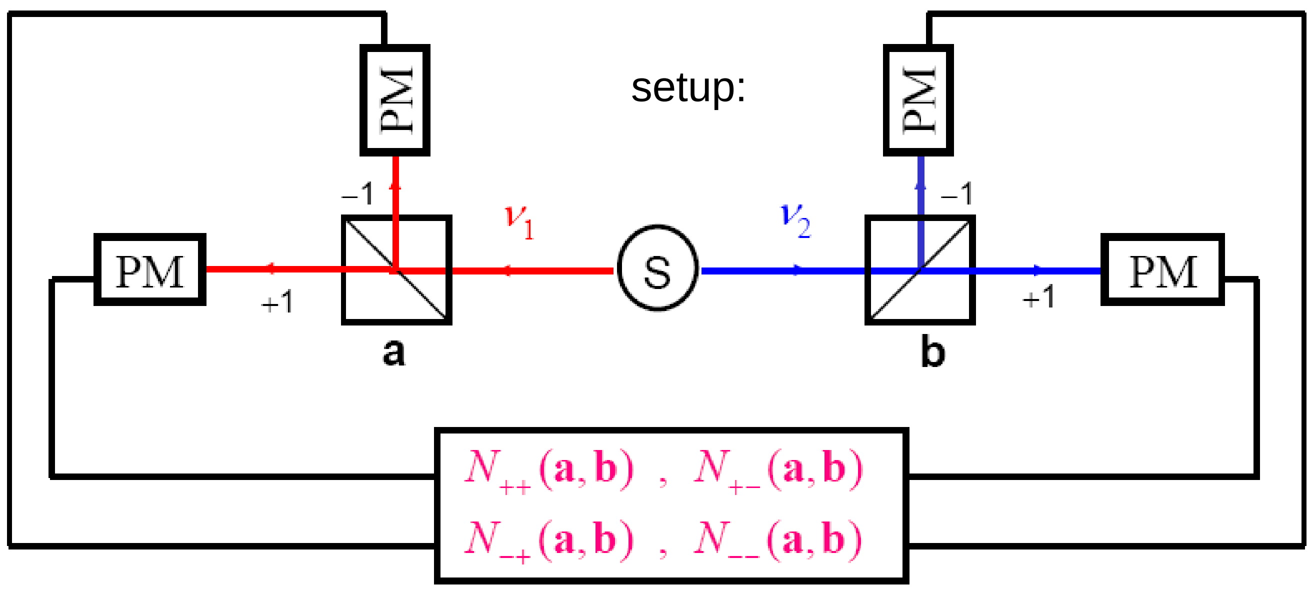
Bell states maximally violate the Bell inequality!

Experimental violation of Bell Inequality (Alain Aspect):

generation of polarization entangled photons:

$$|\Psi(\nu_1,\nu_2)\rangle = \frac{1}{\sqrt{2}}\{|x,x\rangle + |y,y\rangle\}$$





$$4p^2\ {}^1S_0 - 4s4p\ {}^1P_1 - 4p^2\ {}^1S_0$$

radiative cascade in calcium 40

measure coincidences and calculate correlation coefficient:

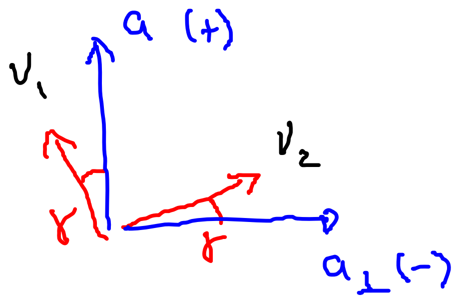
$$E(\mathbf{a},\mathbf{b}) = \frac{N_{++}(\mathbf{a},\mathbf{b}) - N_{+-}(\mathbf{a},\mathbf{b}) - N_{-+}(\mathbf{a},\mathbf{b}) + N_{--}(\mathbf{a},\mathbf{b})}{N_{++}(\mathbf{a},\mathbf{b}) + N_{+-}(\mathbf{a},\mathbf{b}) + N_{-+}(\mathbf{a},\mathbf{b}) + N_{--}(\mathbf{a},\mathbf{b})}$$

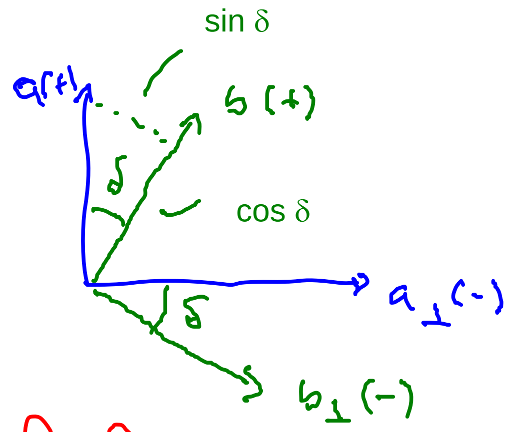
Experimental Realization of Einstein-Podolsky-Rosen-Bohm Gedankenexperiment: A New Violation of Bell's Inequalities
A. Aspect, P. Grangier, and G. Roger
Phys. Rev. Lett. 49, 91-94 (1982)
[PDF (682 kB)]

Experimental Tests of Realistic Local Theories via Bell's Theorem
A. Aspect, P. Grangier, and G. Roger
Phys. Rev. Lett. 47, 460-463 (1981)
[PDF (665 kB)]

if (a,b)=0 (parallel polarizers) then E(a,b) = 1, i.e. perfect correlation of results quantum mechanical prediction:

$$|\Psi(\nu_1,\nu_2)\rangle = \frac{1}{\sqrt{2}}\{|x,x\rangle + |y,y\rangle\}$$





probability of individual photon measurements

$$P_+(\mathbf{a}) = P_-(\mathbf{a}) = \frac{1}{2} \quad ; \quad P_+(\mathbf{b}) = P_-(\mathbf{b}) = \frac{1}{2}$$

$$\frac{1}{2}\left(\cos^2\gamma + \sin^2\gamma\right) = \frac{1}{2} = P_+ = P_-$$

for any γ

probabilities of joint measurements on both photons:

$$P_{++}(\mathbf{a},\mathbf{b}) = P_{--}(\mathbf{a},\mathbf{b}) = \frac{1}{2}\cos^2(\mathbf{a},\mathbf{b})$$

$$P_{+-}(\mathbf{a},\mathbf{b}) = P_{-+}(\mathbf{a},\mathbf{b}) = \frac{1}{2}\sin^2(\mathbf{a},\mathbf{b})$$

easy to see for $\gamma = 0$

$$E(\mathbf{a},\mathbf{b}) = P_{++} + P_{--} - P_{+-} - P_{-+}$$

$$E_{MQ}(\mathbf{a},\mathbf{b}) = \cos 2(\mathbf{a},\mathbf{b}) = \cos^2(a,b) - \sin^2(a,b)$$

$$= \frac{1}{\sqrt{2}} \qquad \text{for } (a,b) = \frac{\pi}{8}$$

measure Bell inequality:

$$S = E(\mathbf{a},\mathbf{b}) - E(\mathbf{a},\mathbf{b}') + E(\mathbf{a}',\mathbf{b}) + E(\mathbf{a}',\mathbf{b}')$$

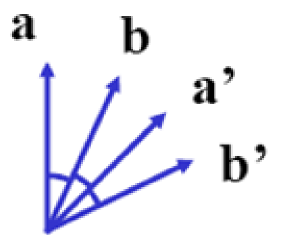
with:

$$\boxed{E(\mathbf{a},\mathbf{b}) = \frac{N_{++}(\mathbf{a},\mathbf{b}) - N_{+-}(\mathbf{a},\mathbf{b}) - N_{-+}(\mathbf{a},\mathbf{b}) + N_{--}(\mathbf{a},\mathbf{b})}{N_{++}(\mathbf{a},\mathbf{b}) + N_{+-}(\mathbf{a},\mathbf{b}) + N_{-+}(\mathbf{a},\mathbf{b}) + N_{--}(\mathbf{a},\mathbf{b})}}$$

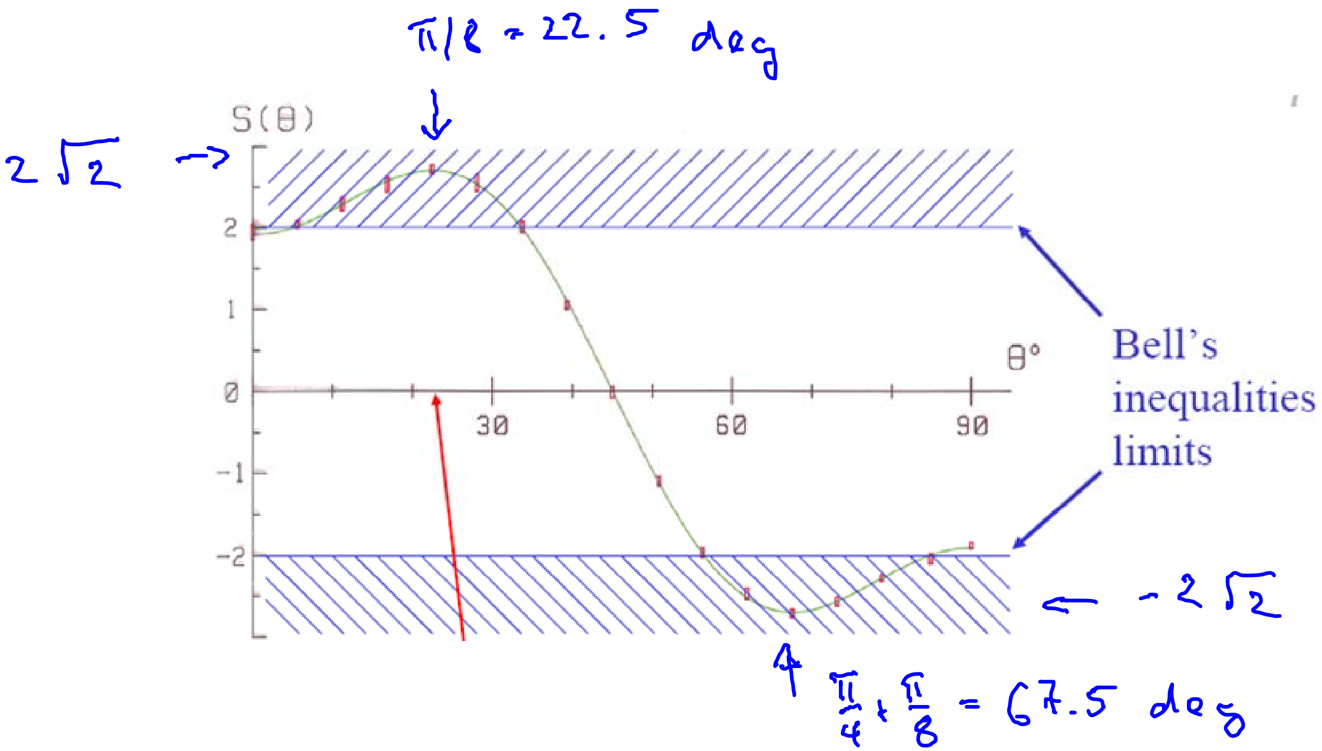
repeat for different angles between polarizer (a,b) = θ:

$$(\mathbf{a},\mathbf{b}) = (\mathbf{b},\mathbf{a}') = (\mathbf{a}',\mathbf{b}) = \frac{\pi}{8}$$

= 22.5 deg



experimental result:



comments:

Consequences of violation of Bell inequalities:

- The assumption that physical properties (e.g. **Q, R, S, T)** of systems have values which exists independent of observation (the **Realism Assumption**) is wrong.

- The assumption that experiments performed at one point in time and space (at Alices) cannot be influenced by experiments at another point in time and space (at Bobs, in a different light cone) (the **Locality Assumption**) is wrong.

Both of the above assumptions are sometimes called **Local Realism**.

Quantum mechanics violates these assumptions, as shown in experiments!

Test of Locality: The Innsbruck Experiment

Violation of Bell's Inequality under Strict Einstein Locality Conditions
G. Weihs, T. Jennewein, C. Simon, H. Weinfurter, and A. Zeilinger
Phys. Rev. Lett. 81, 5039-5043 (1998)
[PDF (195 kB)]