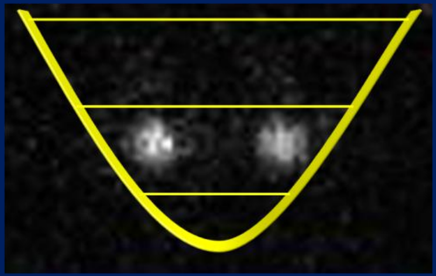
Eidgenössische Technische Hochschule Zürich
Swiss Federal Institute of Technology Zurich

Quantum computing with trapped ions



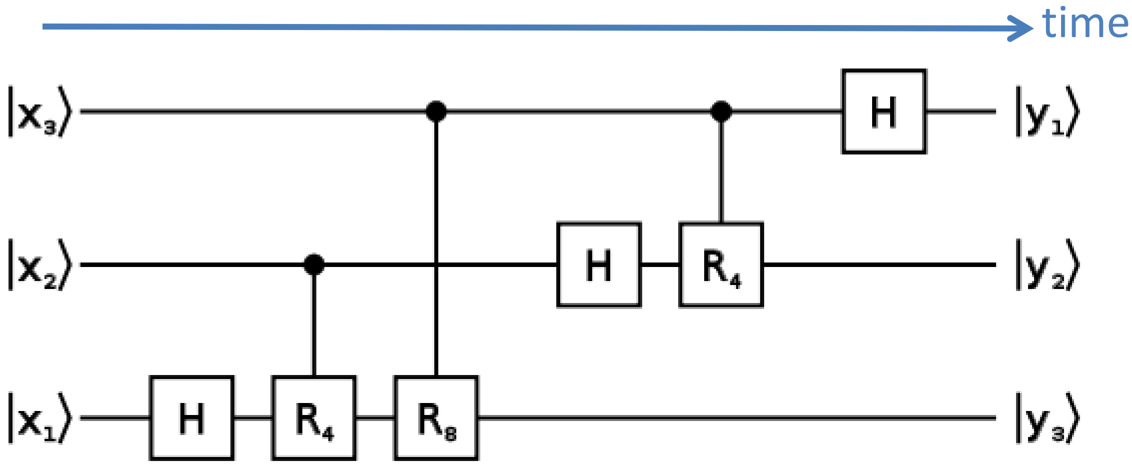
Jonathan Home

Trapped Ion Quantum Information Group
www.tiqi.ethz.ch

Pre-requisites for quantum computation

Collection of two-state quantum systems (qubits) – Deutsch 1985

Initialise qubit to single state



Detect qubit state

Operations which manipulate isolated qubits or pairs of qubits

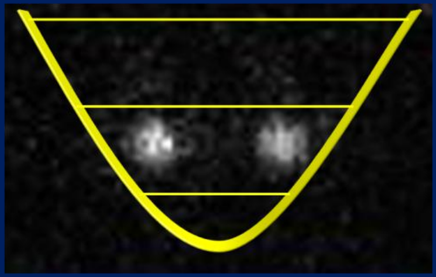
Large scale device:

Transport information around processor/distribute entangled states

Perform operations accurately enough to achieve fault-tolerant error-correction

(accuracy ~ 0.9999 required)

Trapping Charged Particles



Isolating single charged atoms

Laplace‘s equation
– no chance to trap with static fields

$$\frac{\partial^2 V}{\partial x^2} + \frac{\partial^2 V}{\partial y^2} + \frac{\partial^2 V}{\partial z^2} = 0$$

Paul trap: Use a ponderomotive potential – change potential fast compared to speed of ion

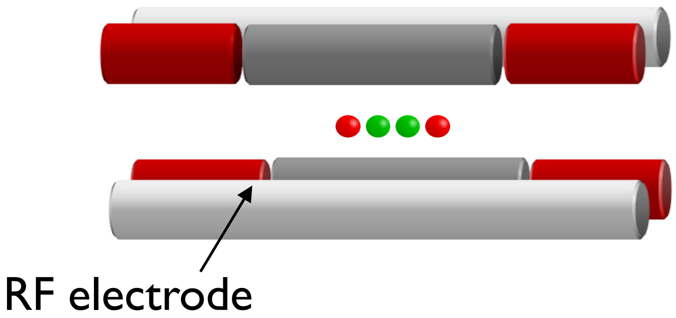
$$\frac{\partial^2 V}{\partial x^2} + \left(\frac{\partial^2 V}{\partial y^2} + \frac{\partial^2 V}{\partial z^2}\right)\cos(\Omega t)$$

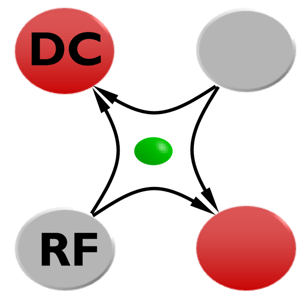
$$M\frac{d^2x}{dt^2} = qE\cos\Omega t \qquad \frac{1}{2}M\left(\frac{dx}{dt}\right)^2 = U_{\mathrm{PP}} = \frac{q^2E^2}{2M\Omega^2}\sin^2\Omega t$$

Time average - Effective potential energy which is minimal at minimum *E*

Penning trap: Add a homogeneous magnetic field – overides the electric repulsion

Traps – traditional style





Trap Frequencies

Axial : < 3 MHz

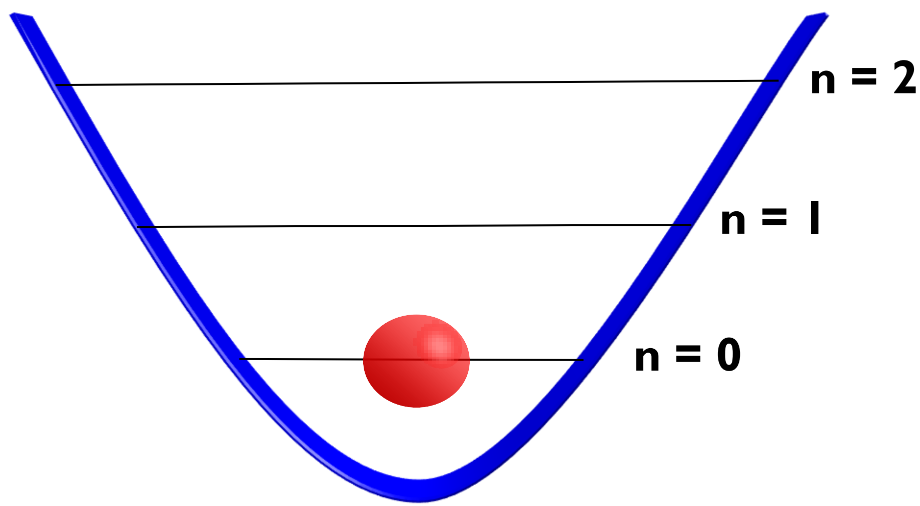
Radial: < 20 MHz

Radial Freq ∝ 1/Mass

Axial potential gives almost ideal harmonic behaviour

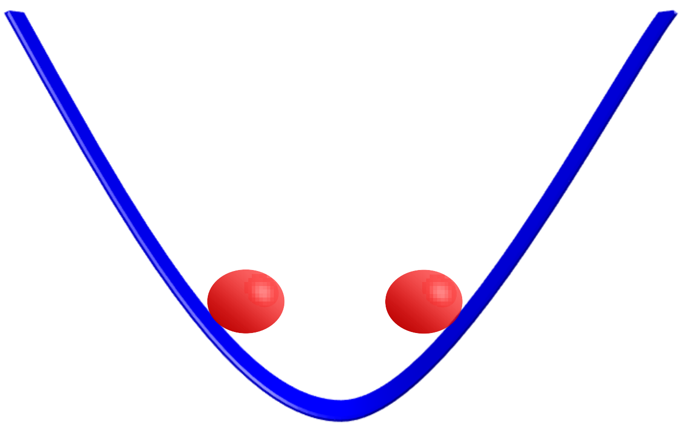
Single ion

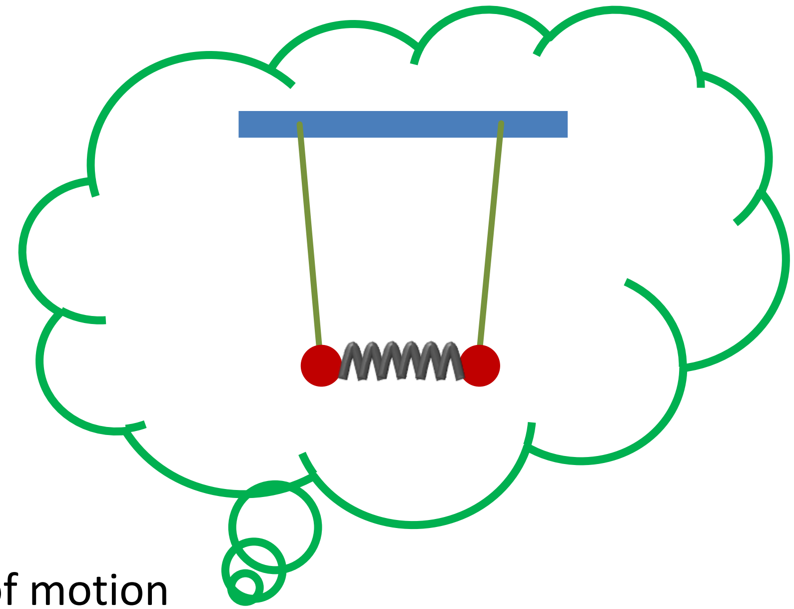
$$\hat{H} = \hbar\omega(\hat{a}^\dagger\hat{a} + 1/2)$$



Multiple ions: coupled harmonic oscillators

$$V = \frac{k}{2}z_1^2 + \frac{k}{2}z_2^2 + \frac{\alpha}{|z_1 - z_2|}$$



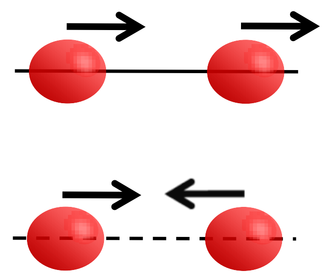


Expand about equilibrium – equation of motion

$$\begin{pmatrix} \ddot{\epsilon}_1 \\ \ddot{\epsilon}_2 \end{pmatrix} = -\omega_z^2 \begin{pmatrix} \epsilon_1 \\ \epsilon_2 \end{pmatrix} = \begin{pmatrix} k & \alpha \\ \alpha & k \end{pmatrix} \begin{pmatrix} \epsilon_1 \\ \epsilon_2 \end{pmatrix}$$

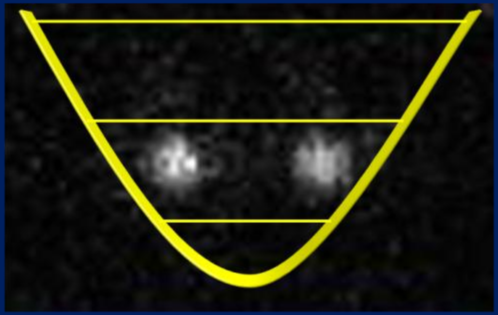
$\hbar\omega_c$

$\hbar\omega_s$



Independent oscillators
- shared motion

Internal electronic states

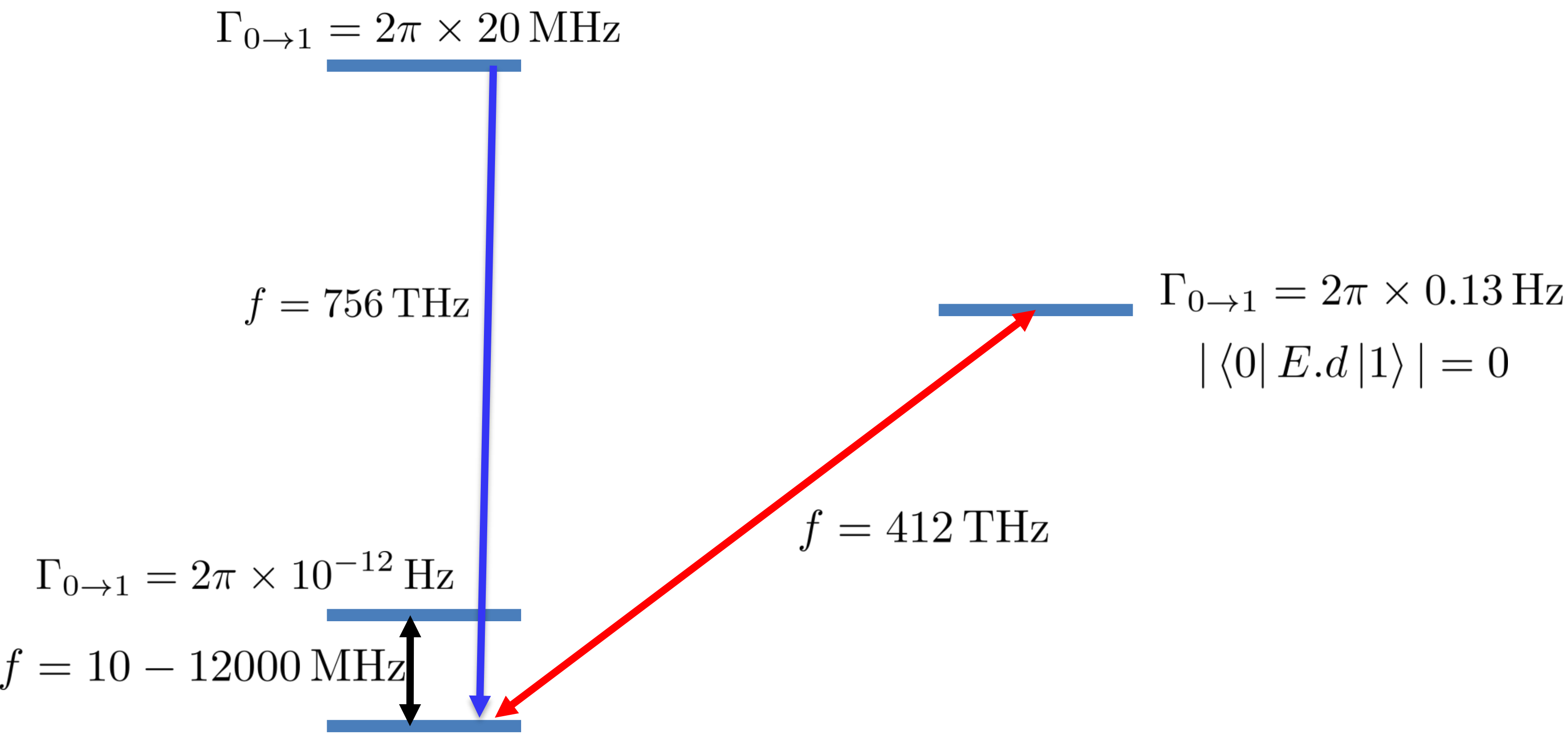


Storing qubits in an atom

$$|\psi\rangle = (a|0\rangle + b|1\rangle)$$

Requirement: long decay time for upper level.

$$\Gamma_{0\to1} \propto \omega^3 |\langle 0| E.d |1\rangle|^2$$

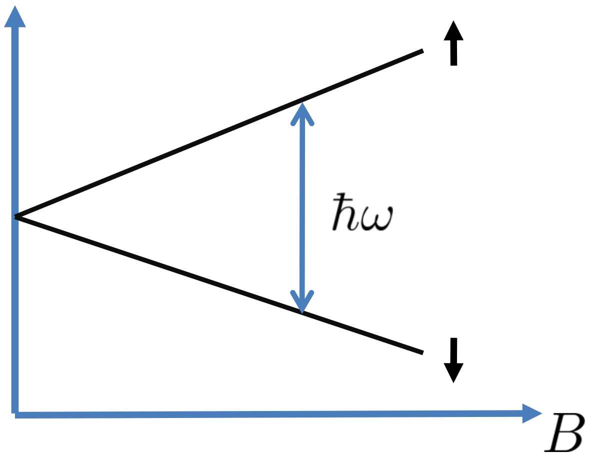


Storing qubits in an atom - phase coherence

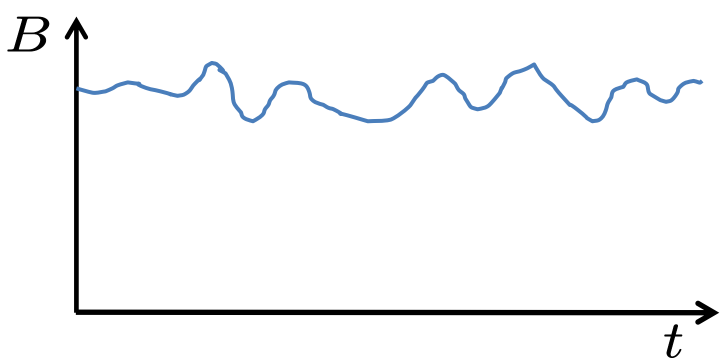
$$|\psi\rangle = (a|0\rangle + be^{i\phi}|1\rangle)$$

Noise! – mainly from classical fields

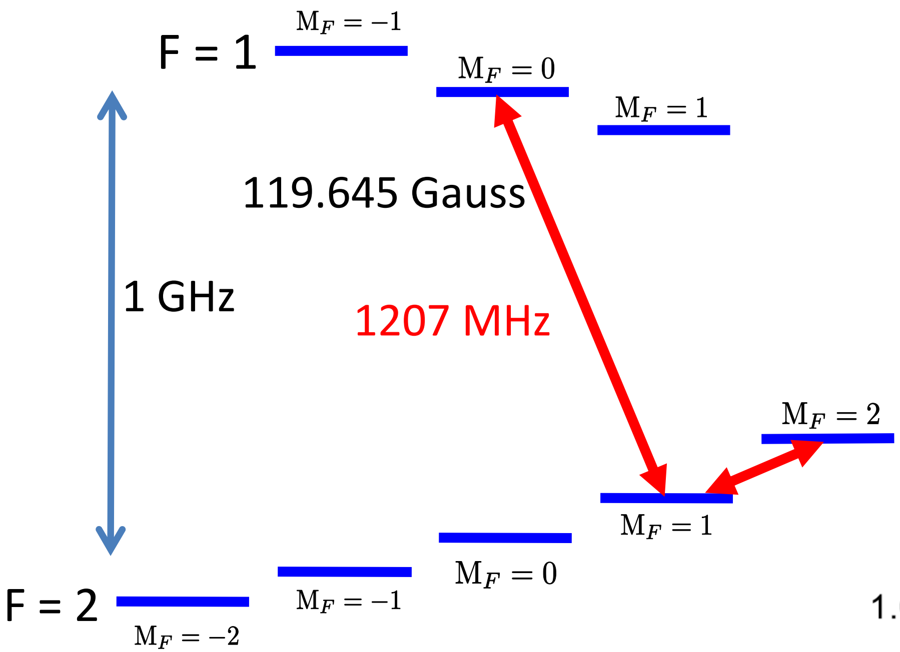
$$E = -\mu \cdot B$$



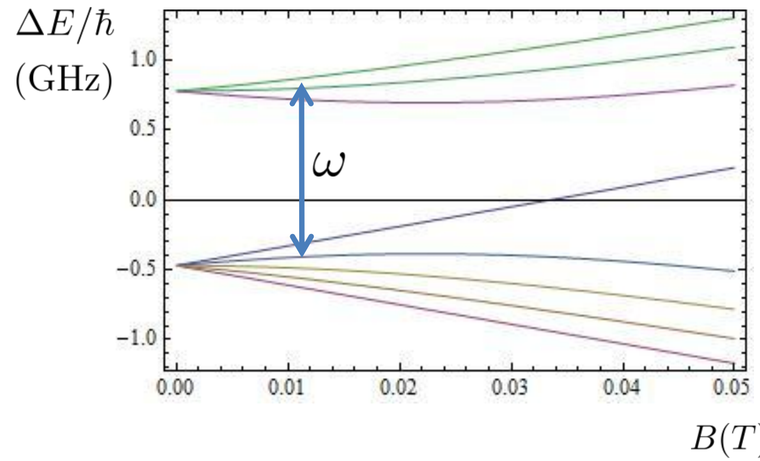
$$\phi \propto \int B(t)dt$$

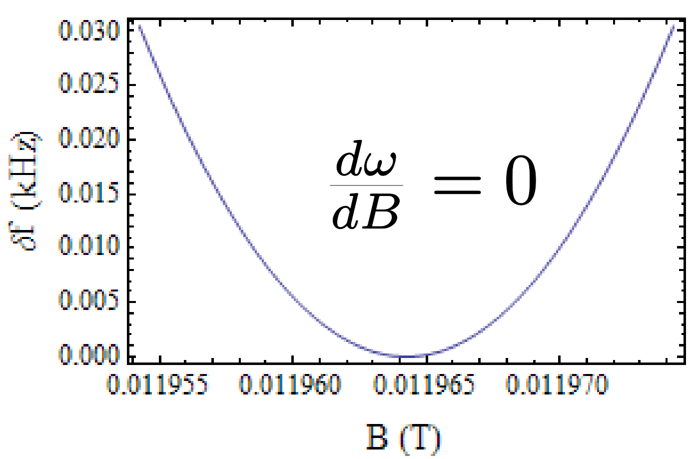


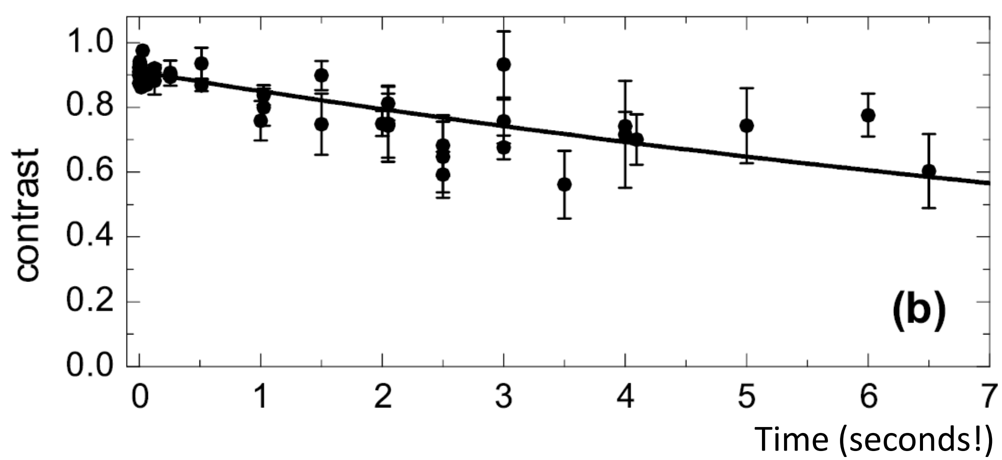
Storing qubits in an atom
Field-independent transitions



$$H = A\hat{I} \cdot \hat{J} + \hat{\mu} \cdot B$$







Langer et al. PRL 95, 060502 (2005)

Entanglement for protection

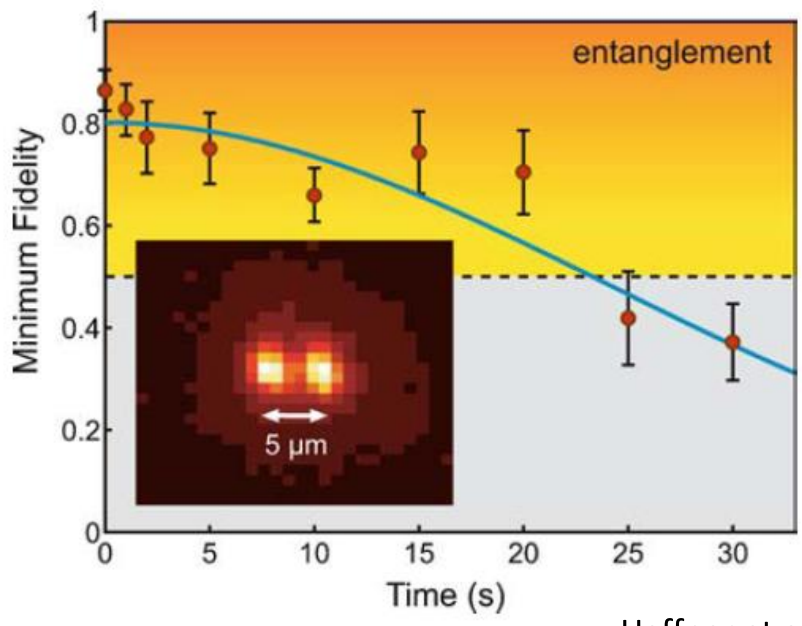
Decoherence-Free Subspaces for common-mode noise

$$|0\rangle + e^{i\omega'(t)t}|1\rangle \qquad |0\rangle + e^{i\omega(t)t}|1\rangle$$

Now consider an entangled state

$$e^{i\omega(t)t}|01\rangle + e^{i\omega'(t)t}|10\rangle = e^{i\omega(t)t}\left(|01\rangle + e^{i(\omega'(t)-\omega(t))t}|10\rangle\right)$$

If noise is common mode, entangled states can have very long coherence times (NIST, Innsbruck)

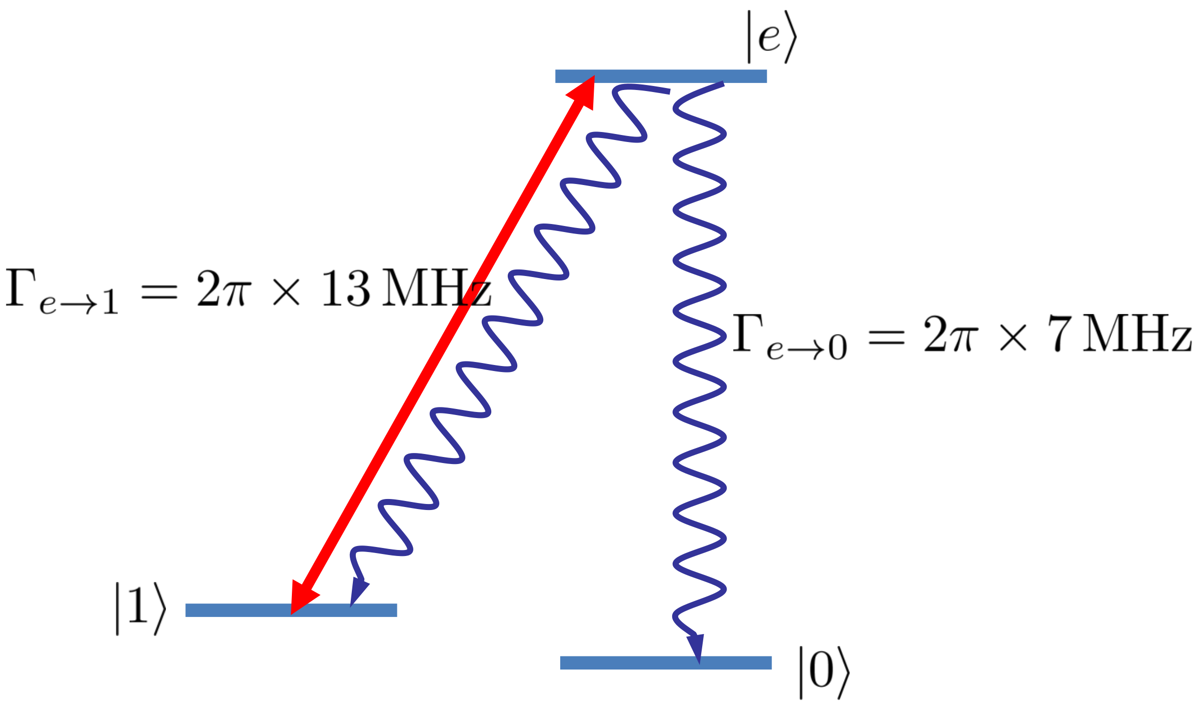


Haffner et al., Appl. Phys. B 81, 151-153 (2005)

Preparing the states of ions

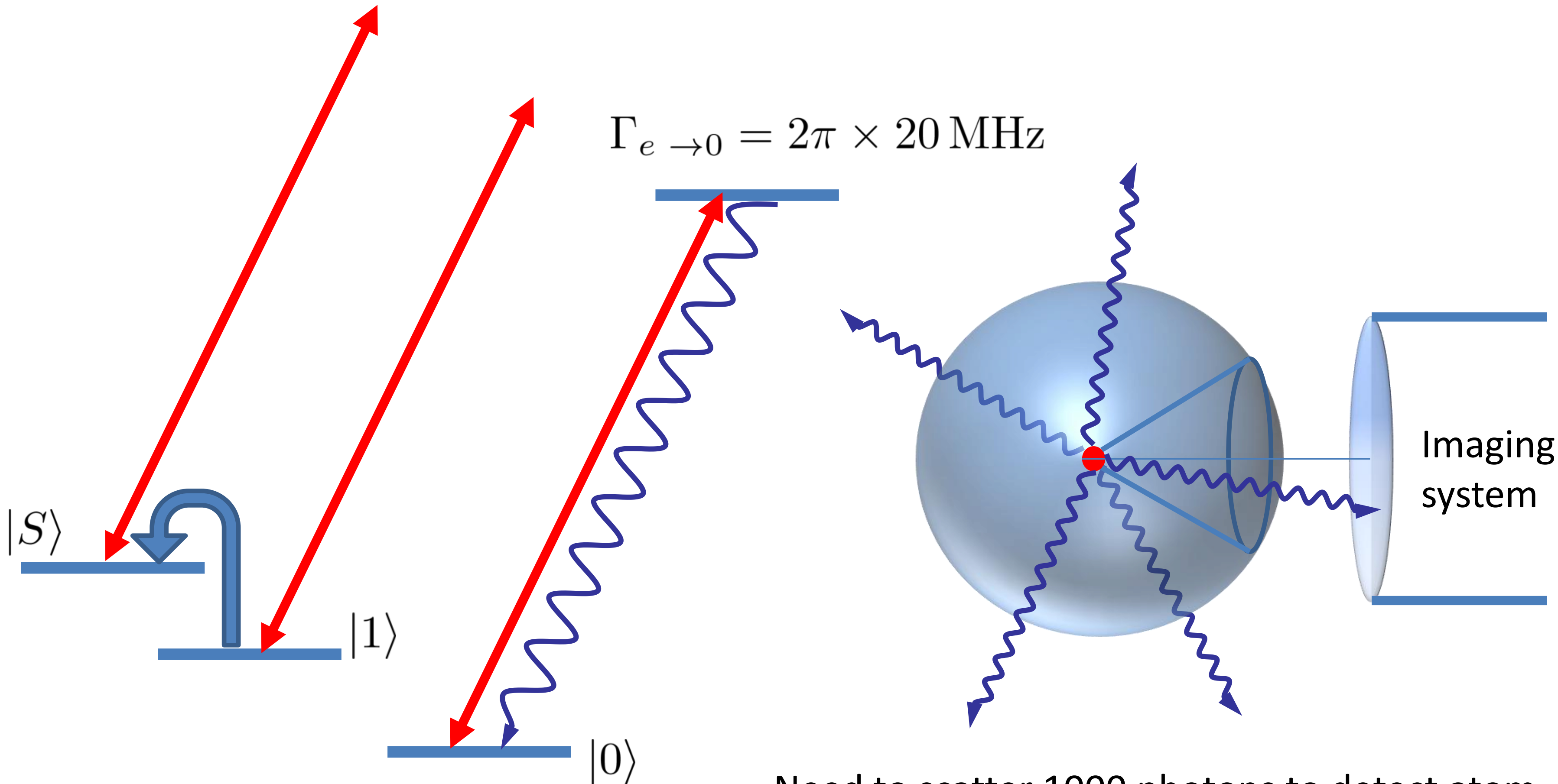
Optical pumping – state initialisation

Use a dipole transition for speed



Calcium: scatter around 3 photons to prepare $|0\rangle$ $\tau_{\mathrm{prep}} \sim 50\,\mathrm{ns}$

Reading out the quantum state

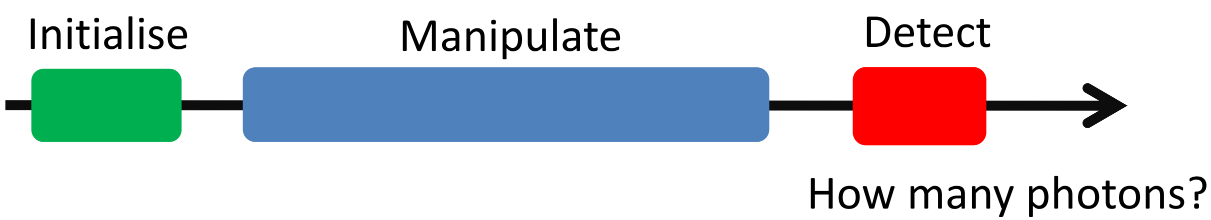


Need to scatter 1000 photons to detect atom

$$T_{\mathrm{readout}} \sim 100 \rightarrow 1000\,\mu\mathrm{s}$$

Measurement – experiment sequence

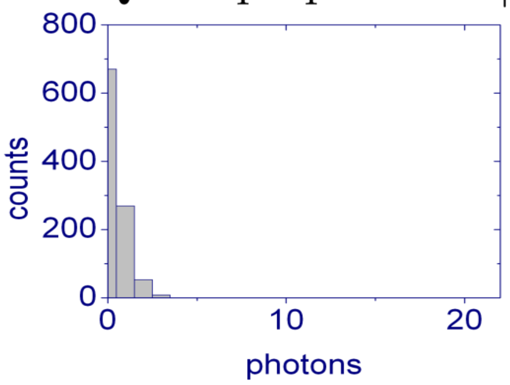
Typical sequence, single qubit:



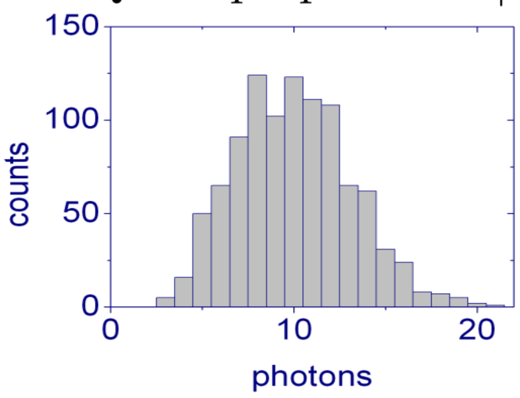
Repeat the experiment many times

Number of photons = 8, 4, 2, 0, 0, 1, 5, 0, 0, 8 ....

Qubit prepared in $|0\rangle$

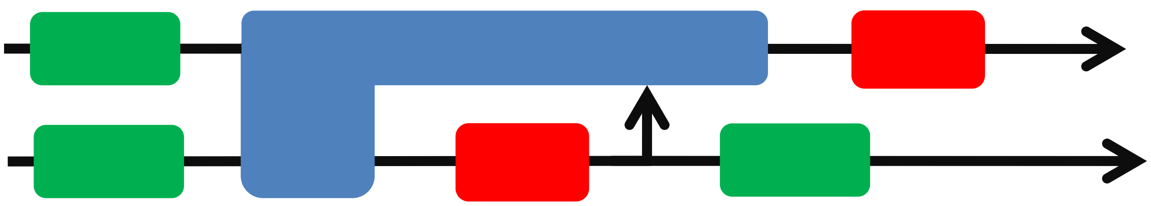


Qubit prepared in $|1\rangle$



Single shot measurement

Typical sequence with quantum error correction, teleportation

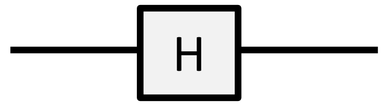


Require fast, single shot measurement,
“8 counts, that‘s a 1!“

(also classical computation to decide “what next?”)

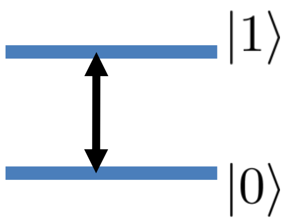
Readout extremely good – accuracy of 0.9999 achieved (Oxford 2008)
- good enough for fault-tolerant error-correction

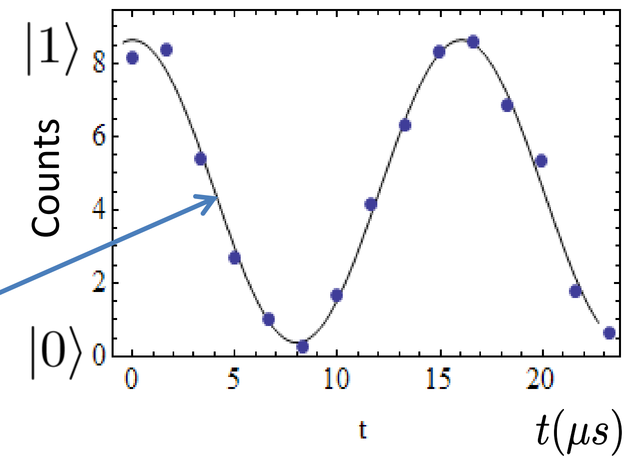
Manipulating single qubits



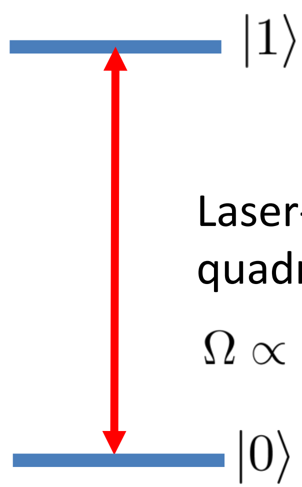
$$H = \Omega \left(|0\rangle \langle 1| + |1\rangle \langle 0| \right) \cos \left(\omega t + \phi \right)$$

Resonant microwaves





$$|\psi\rangle = (|0\rangle + e^{i\phi}|1\rangle)/\sqrt{2}$$

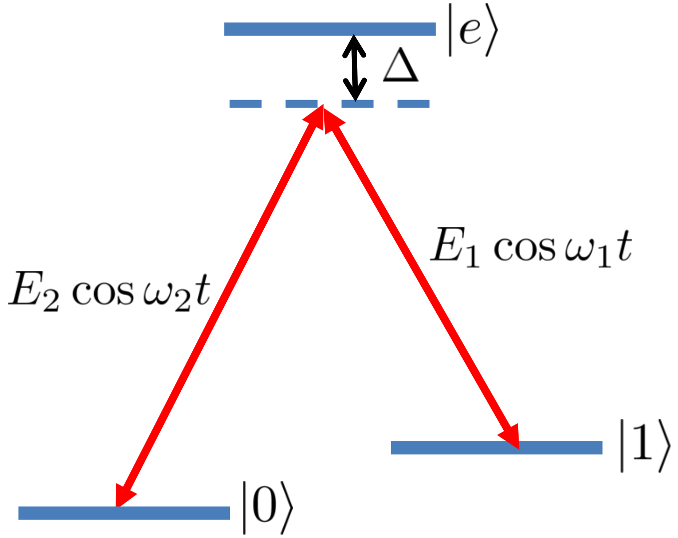


Laser-driven
quadrupole transition

$$\Omega \propto \langle 0| \, (E.r)(k.r) \, |1\rangle$$

Manipulating single qubits

Raman transition



$$\Omega \propto \frac{\langle 0| (E_1.r) |e\rangle \langle e| (E_2.r) |1\rangle}{\Delta}$$

$$\omega = \omega_1 - \omega_2$$

$$\phi = \phi_1 - \phi_2$$

Addressing individual qubits

Intensity addressing

Shine laser beam at one ion in string

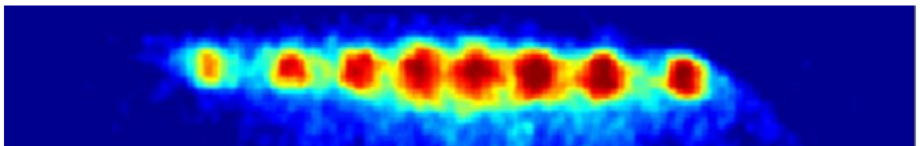
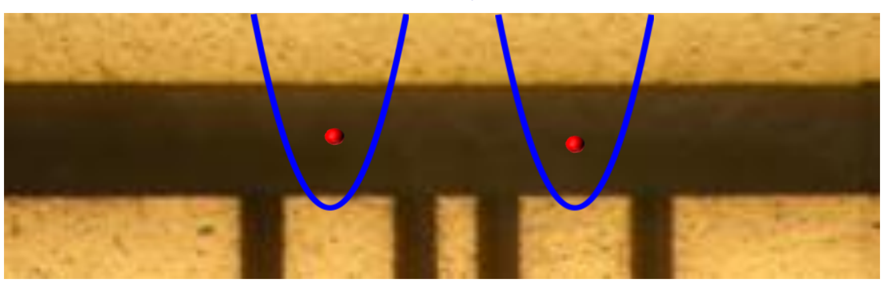


Image: Roee Ozeri

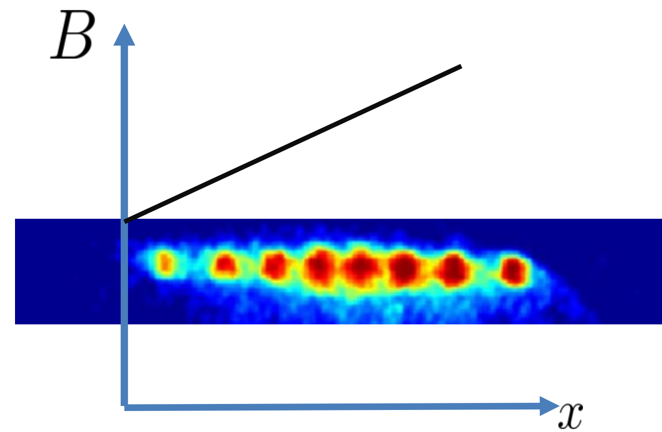
2-4 µm

Separate ions by a distance much larger than laser beam size

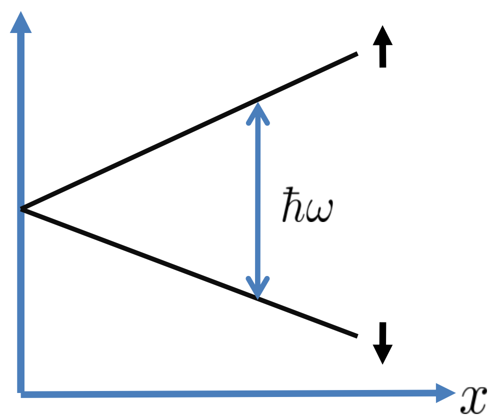
240 µm



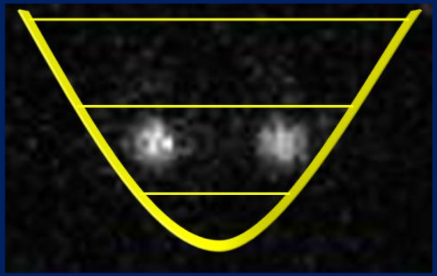
Frequency addressing



$$E = -\mu \cdot B$$



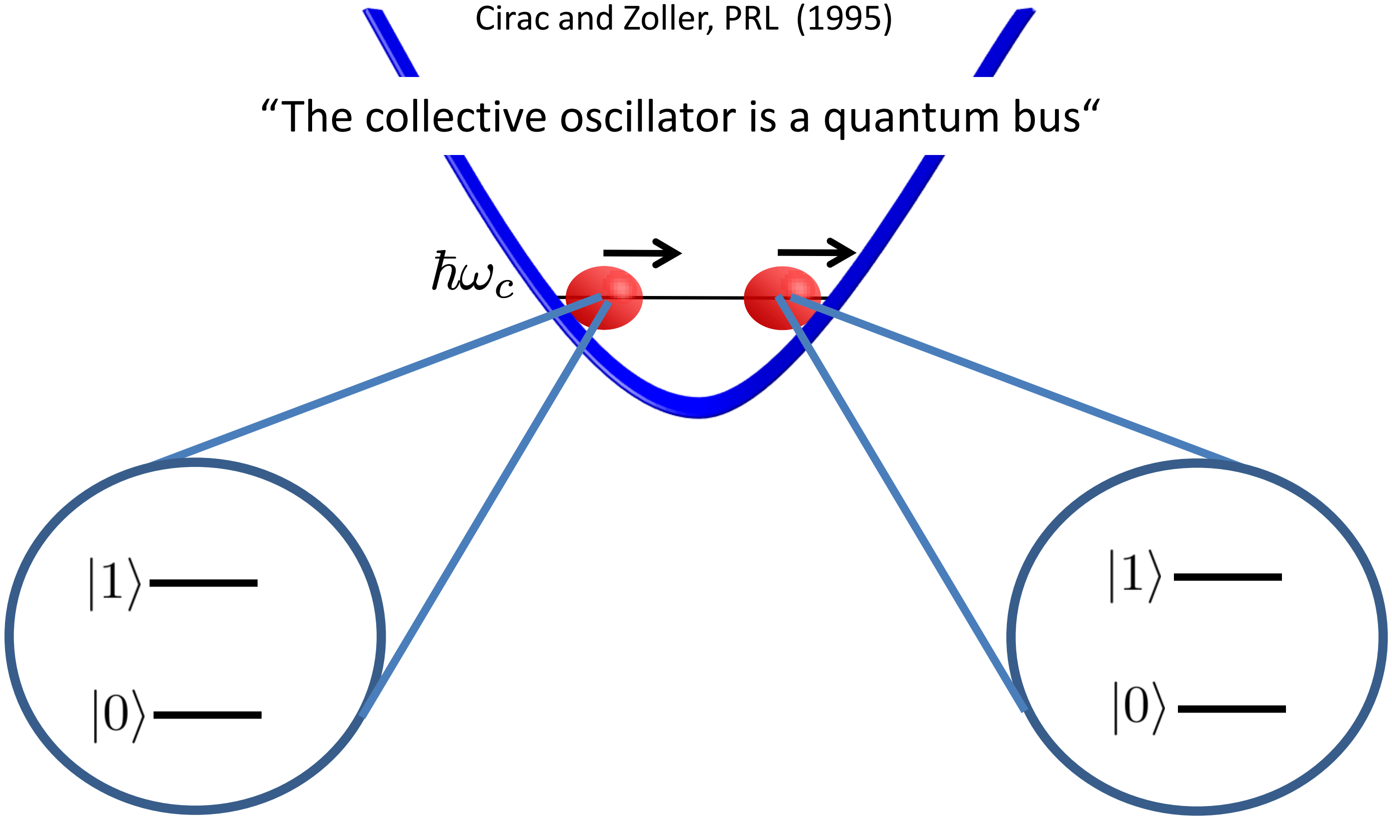
Interactions and Entanglement Generation

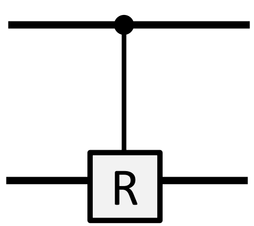


The original thought

Cirac and Zoller, PRL (1995)

“The collective oscillator is a quantum bus“

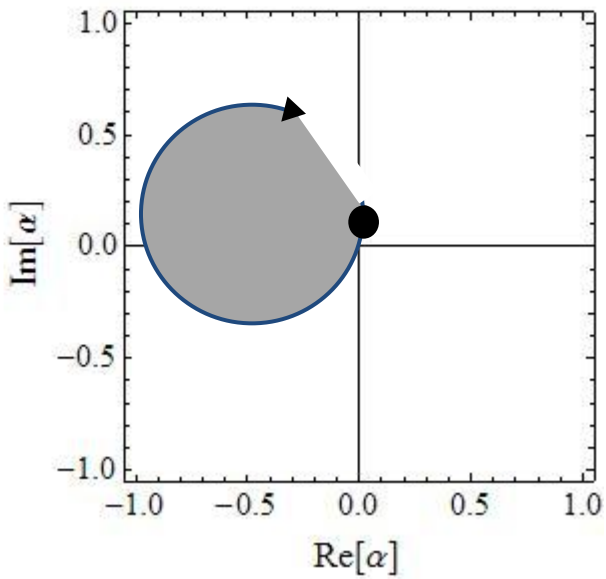




$$G = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & i & 0 & 0 \\ 0 & 0 & i & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} |0_\phi 0_\phi\rangle \\ |1_\phi 0_\phi\rangle \\ |0_\phi 1_\phi\rangle \\ |1_\phi 1_\phi\rangle \end{pmatrix}$$

The forced harmonic oscillator

Classical forced oscillator $$\frac{d^2x}{dt^2} = -\omega_z^2 x + \frac{F}{m}\cos(\omega t + \phi)$$



"returns" after $t = \frac{2\pi}{\delta}$

Radius of loop $\propto \frac{F}{\delta}$

Reminder – interaction picture

Hamiltonian for unperturbed oscillator

$$H_0(t) = \hbar\omega\left(a^\dagger a + \frac{1}{2}\right)$$

We don't want to worry about states evolving in time under this Hamiltonian, so we move into an interaction picture, with operators transformed according to

$$\frac{dO(t)}{dt} = \frac{-i}{\hbar}\left[H_0, O(t)\right]$$

For $a, a^\dagger$ $\quad a(t) = e^{i\omega t}a(0)$

Therefore $\quad z(t) = z_0\left(ae^{-i\omega t} + a^\dagger e^{i\omega t}\right)$

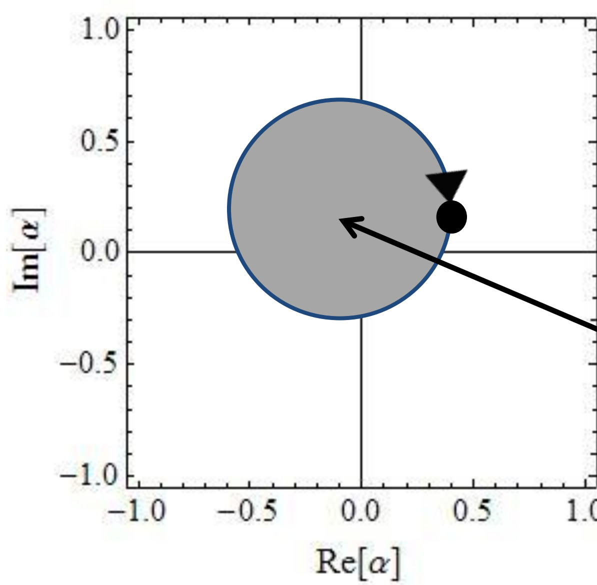
Forced quantum oscillators

$$H(t) \quad = \quad \Omega\cos(\omega t) z = \Omega \cos(\omega t) z_0 \left(\hat{a} e^{i\omega_z t} + \hat{a}^\dagger e^{-i\omega_z t}\right)$$

$$[H(t), H(t')] \neq 0$$

$$U = \exp\left(\frac{i}{\hbar}\int^t H(t')dt' - \frac{1}{2\hbar^2}\int^t\int^{t'}[H(t'), H(t'')]dt'dt'' + \ldots\right)$$

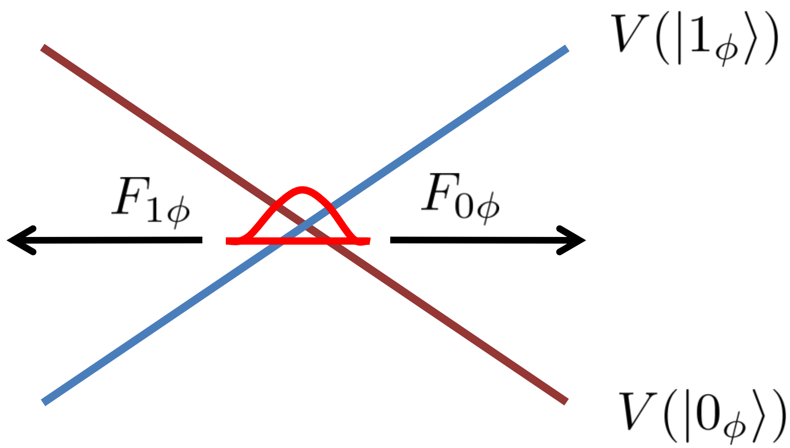
Transient excitation, phase acquired



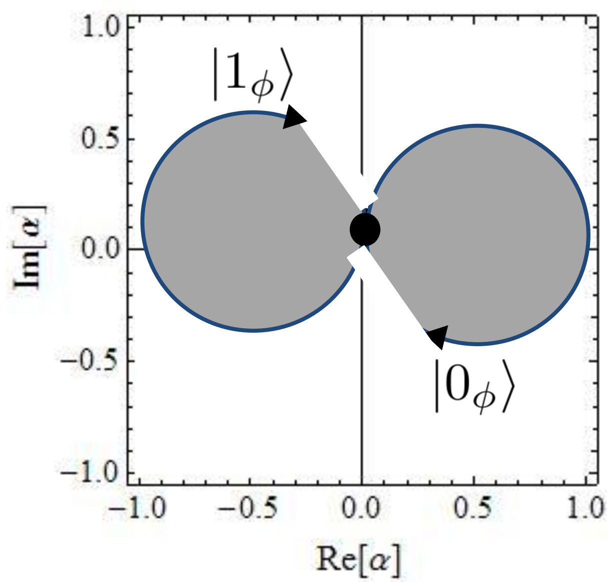
$$|1\rangle\,|\alpha_0\rangle \rightarrow e^{i\Phi}\,|1\rangle\,|\alpha_0\rangle$$

$$\Phi \propto A$$

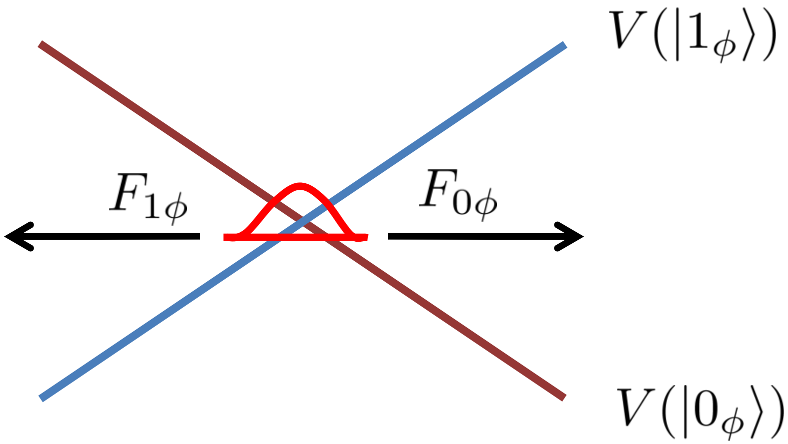
State-dependent excitation

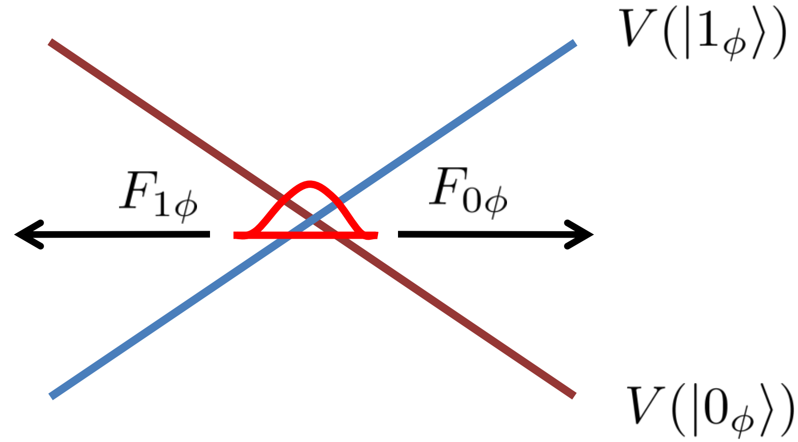


$$|\psi\rangle = |1_\phi\rangle\,|\alpha\rangle + |0_\phi\rangle\,|{-\alpha}\rangle$$



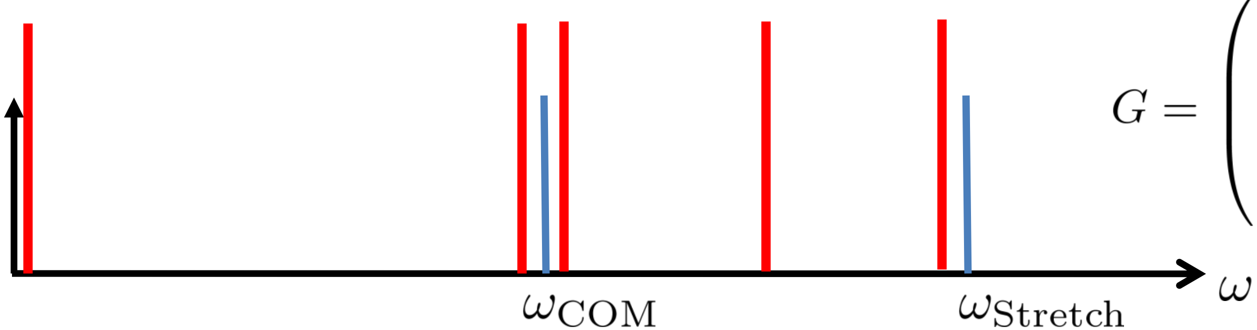
Two-qubit gate, state-dependent excitation





$|0_\phi 1_\phi\rangle\,,|1_\phi 0_\phi\rangle$ ⟶ Force is out of phase; excite Stretch mode

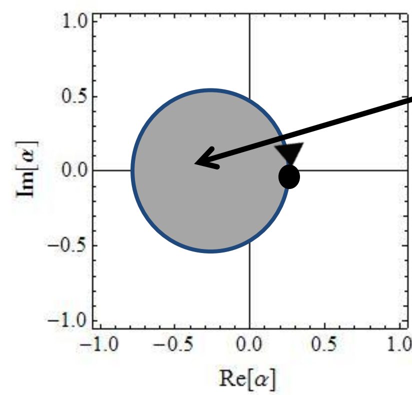
$|1_\phi 1_\phi\rangle\,,|0_\phi 0_\phi\rangle$ ⟶ Force is in-phase; excite COM mode



$$G = \begin{pmatrix} e^{i\phi_{00}} & 0 & 0 & 0 \\ 0 & e^{i\phi_{01}} & 0 & 0 \\ 0 & 0 & e^{i\phi_{01}} & 0 \\ 0 & 0 & 0 & e^{i\phi_{01}} \end{pmatrix}$$

Examples: trapped-ion quantum computing

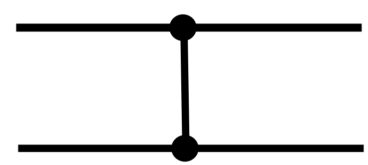
Choose the duration and power: $t_g = 2\pi/\delta \sim 7 \rightarrow 100\mu s$



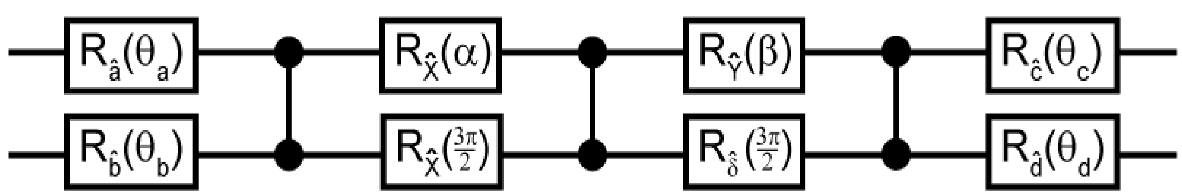
$$\Phi = \frac{\pi}{2}$$

$$G = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & i & 0 & 0 \\ 0 & 0 & i & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

G + single qubit gates is universal – can create any unitary operation.



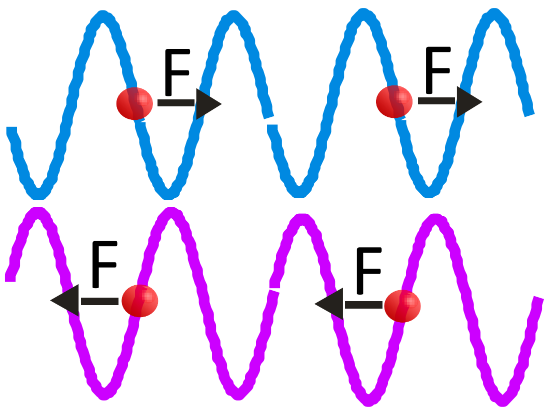
$$\hat{U} =$$



Universal two-qubit ion trap quantum processor: Hanneke et al. Nature Physics 6, 13-16 (2010)

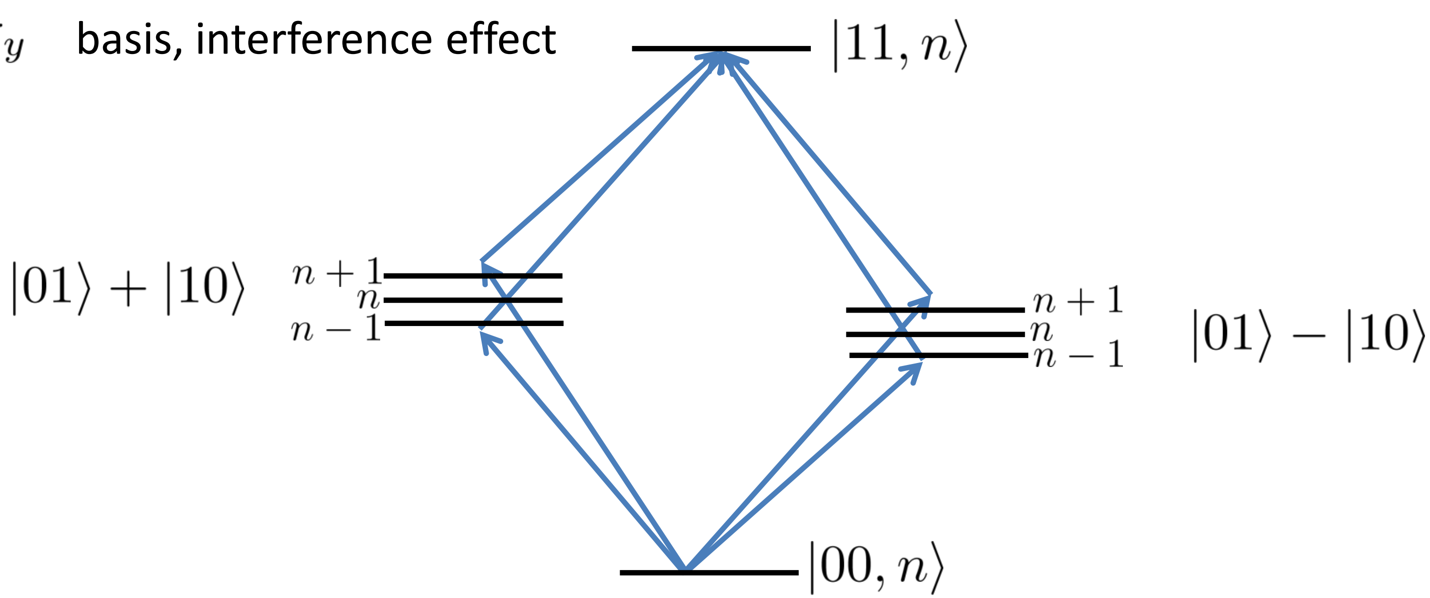
Realisations

σ_z basis, polarisation standing wave



(Leibfried et al. Nature 422 (2003))

σ_x, σ_y basis, interference effect



Examples: Quantum simulation

Go to limit of large motional detuning
(very little entanglement between spin and motion)

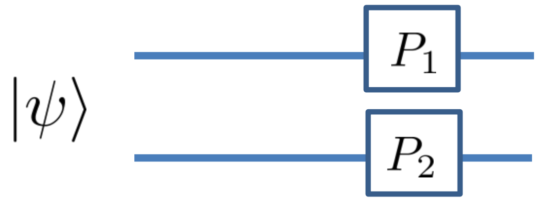
$$\Omega \ll \delta$$

$$\Phi_{10} = \Phi_{01} \simeq \frac{\Omega^2}{\delta} t$$

Allows creation of condensed-matter Hamiltonians
(Friedenauer et al. Nat. Phys 4, 757-761 (2008)
Kim et al. Nature 465, 7298 (2010))

$$H_{\text{eff}} \simeq \frac{\Omega^2}{\delta} s_1^z s_2^z \qquad H_{\text{eff}} \simeq \frac{\Omega^2}{\delta} \sum_{i \neq j}^{N} s_i^z s_j^z$$

State and entanglement characterisation



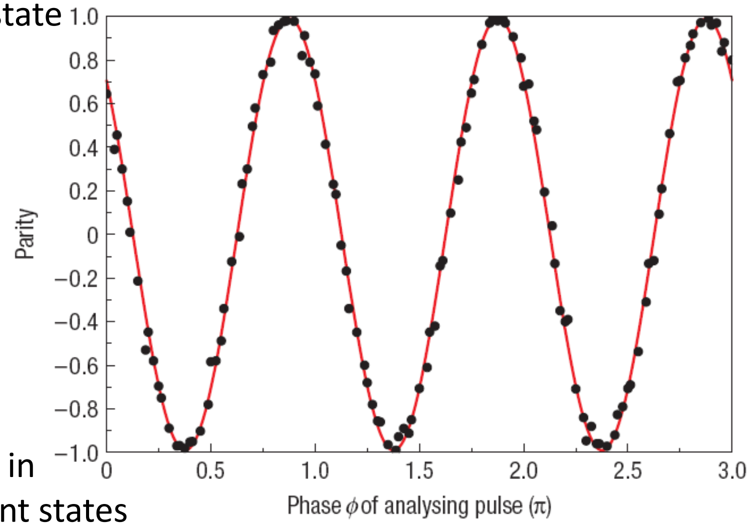
Detect

8, 6, 7, 4, 9, 0, 0, 1, 1, 6, 1, 9, 0, 0…
5, 4, 3,11, 4, 1, 0, 0, 1, 8, 0, 8, 1, 0…

Entanglement – correlations…

$$(|11\rangle + |00\rangle)\sqrt{2}$$
$$P_1 = P_2 = R(\theta = \pi/2, \phi)$$

Qubits in the same state



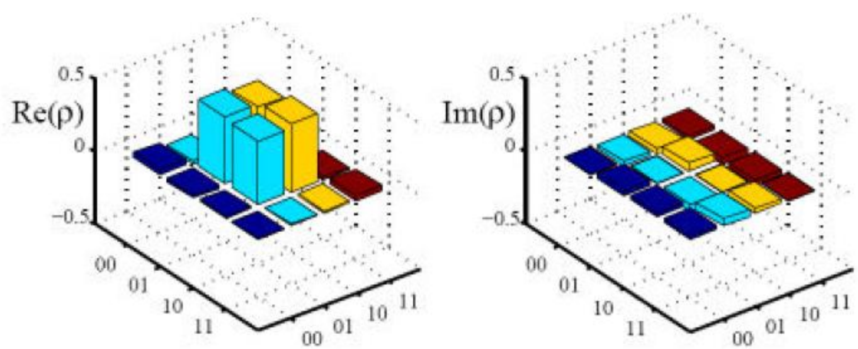
Qubits in different states

F = 0.993 (Innsbruck)

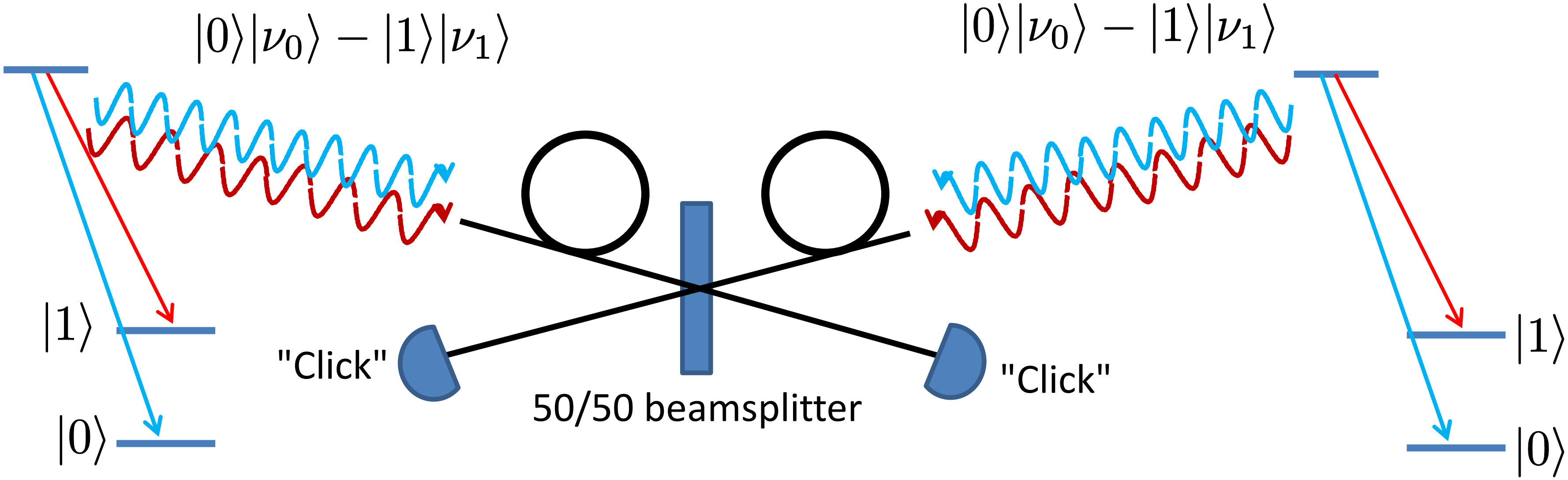
Benhelm et al. Nat. Phys 4, 463(2008)

Choose 12 different settings of P_1, P_2

Reconstruct density matrix

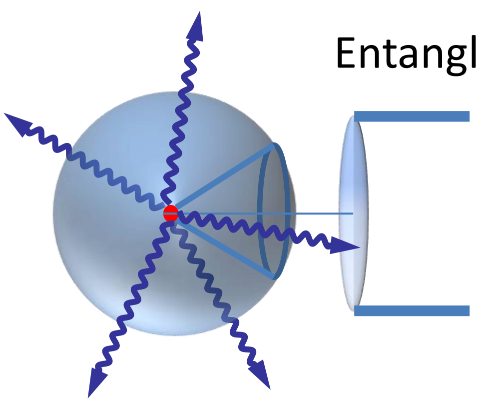


Remote entanglement: probabilistic



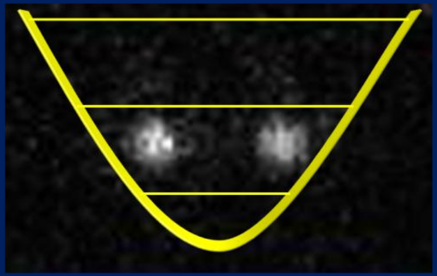
If("Click"&"Click") $(|01\rangle - |10\rangle)/\sqrt{2}$

Entangled ions separated by 1m (Moehring et al. Nature 449, 68 (2008))



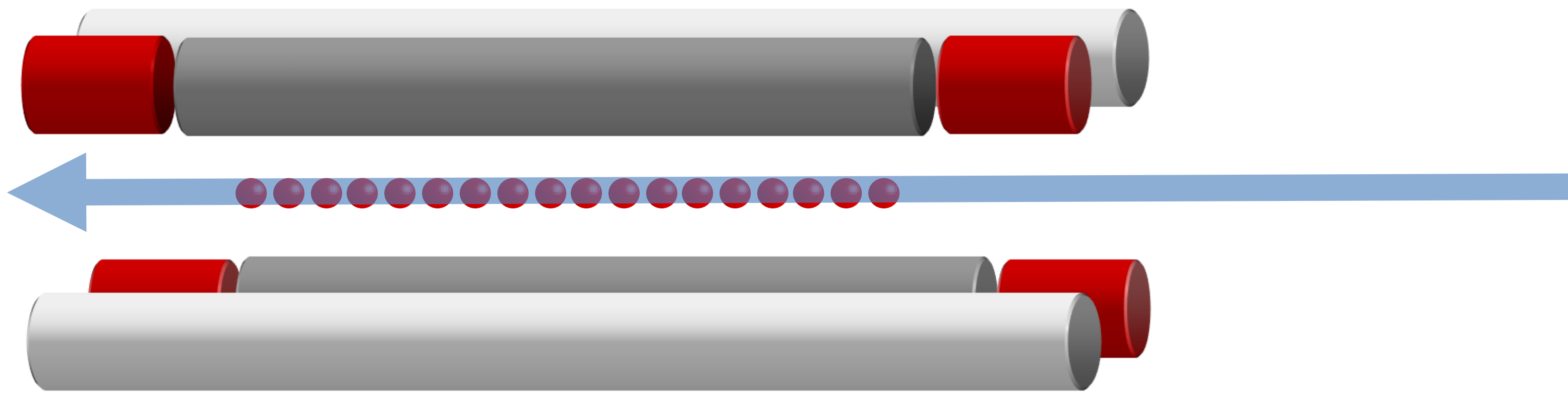
Currently P(both click) = 2e-8, 1 entangled pair per 8.5 minute

Towards large-scale entanglement



Dealing with large numbers of ions

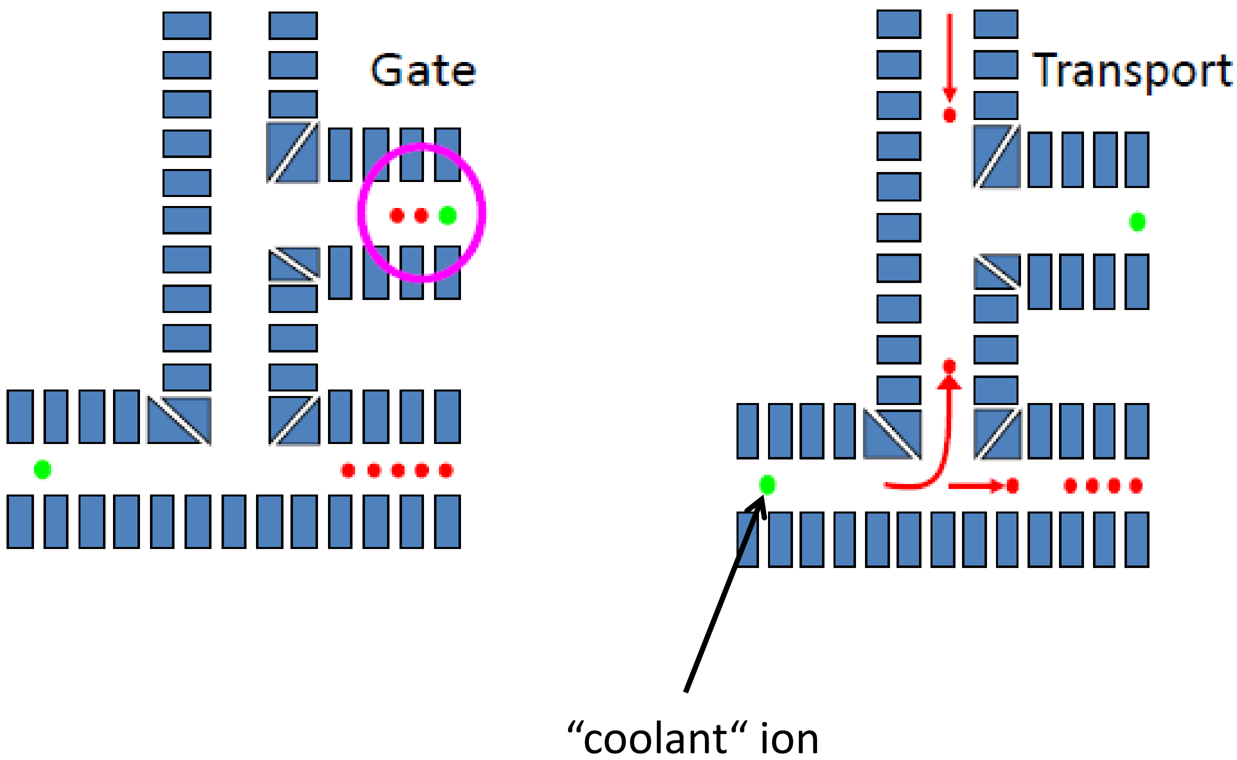
Load more ions



Technical requirement	Limitation
Spectral mode addressing	Mode density increases
Many ions	Heating rates proportional to N
Simultaneous laser addressing	Ions take up space (separation > 2 micron) Laser beams are finite-size

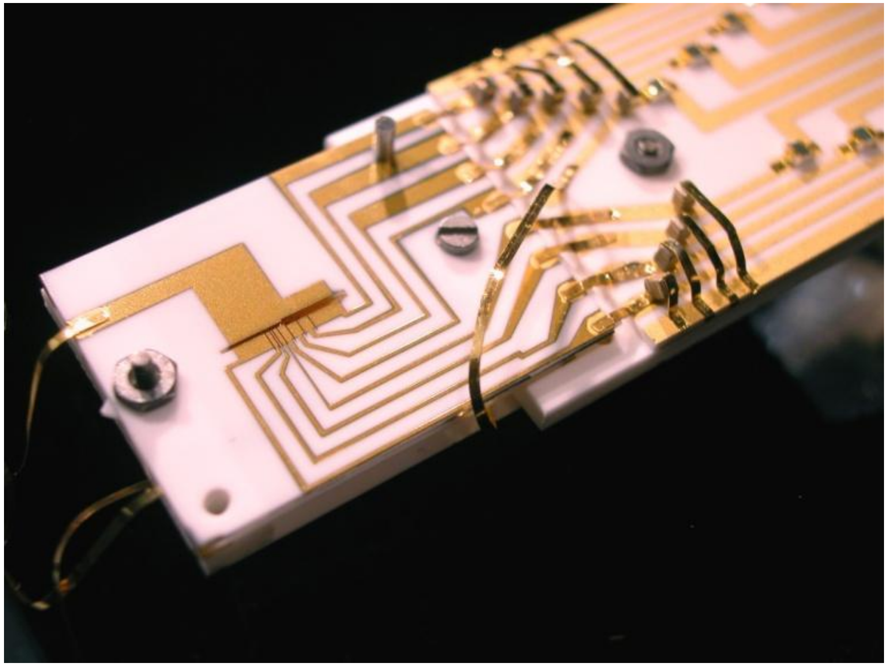
Isolate small numbers of ions

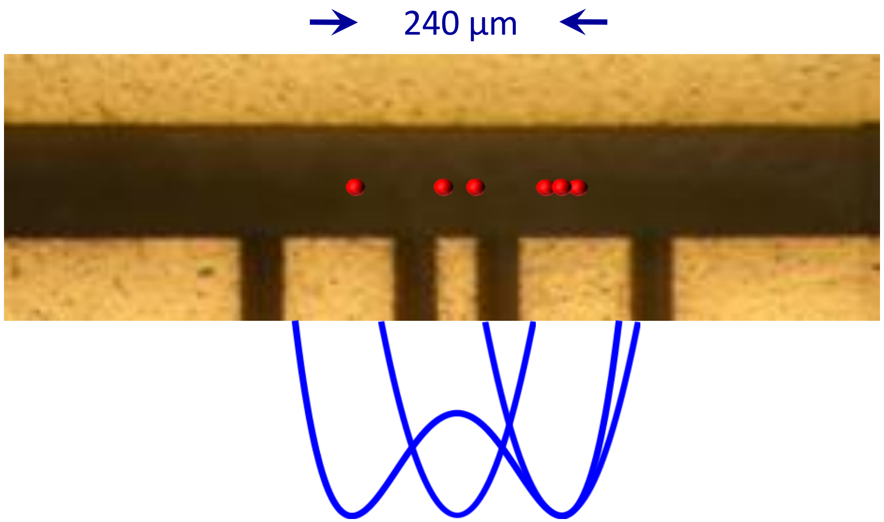
Wineland et al. J. Res. Nat. Inst. St. Tech, (1998)



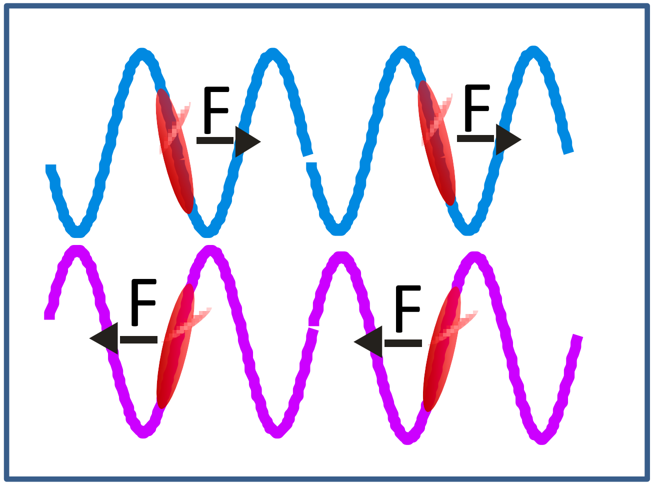
Technological challenge – large numbers of electrodes, many control regions

Qubit transport with ions





Move: 20 us, Separate 340 us, 0.5 quanta/separation

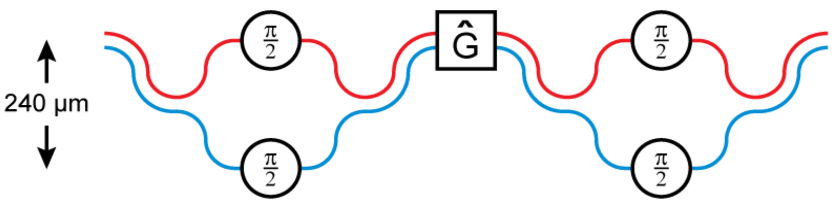


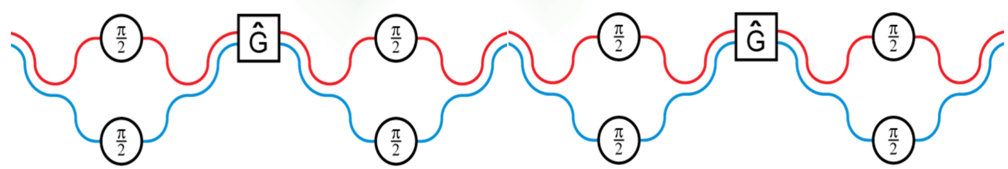
Internal quantum states of ions unaffected by transport
Motional states are affected (problem!)

Combining shuttling with all other tasks

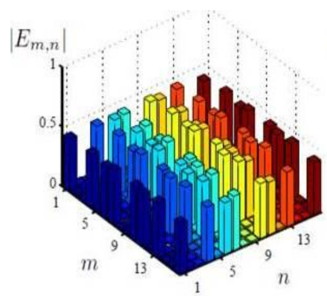
Home et al. Science 325, 1227 (2009)

Sympathetic cooling with additional ion species mitigates ion heating

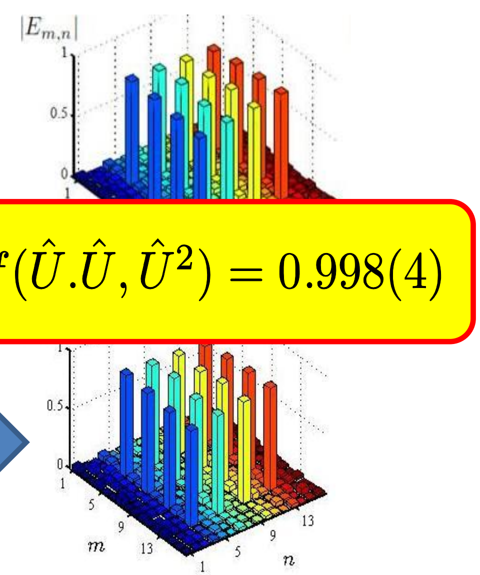




Absolute value of reconstructed process matrices



Mathematical Repetition



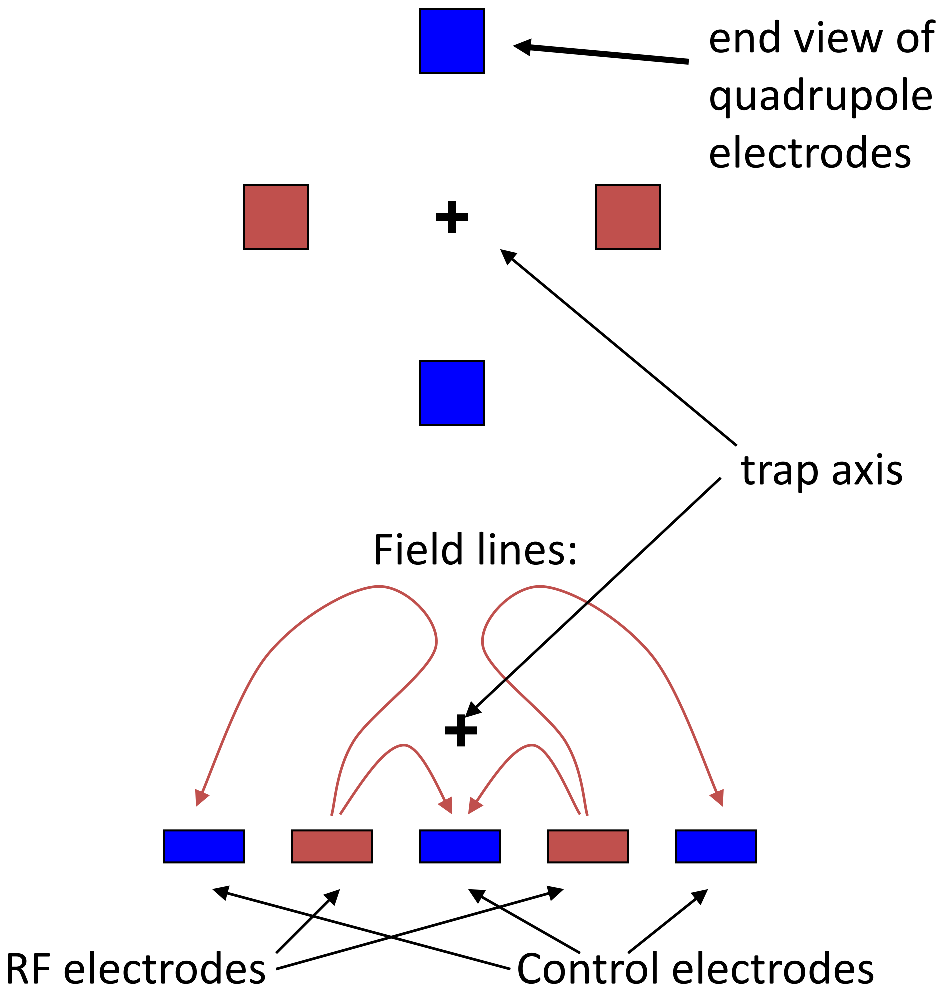
$$f(\hat{U}.\hat{U},\hat{U}^2) = 0.998(4)$$

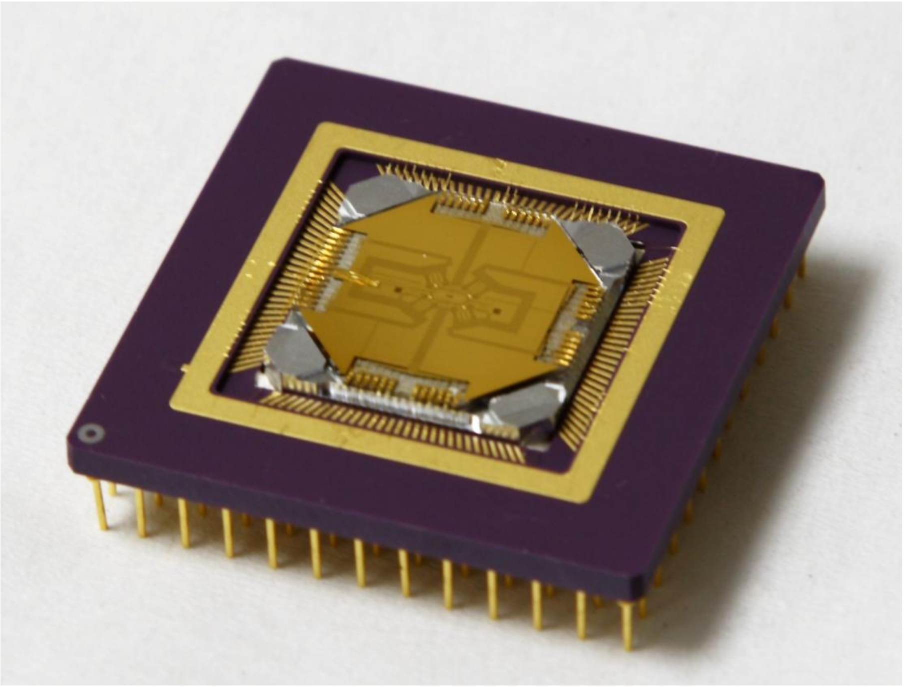
Gate performance and qubits maintained while transporting information

Trapping ions on a chip

For microfabrication purposes, desirable to deposit trap structures on a surface

(Chiaverini *et al.*, Quant. Inf. & Computation (2005), Seidelin et al. PRL 96, 253003 (2006))

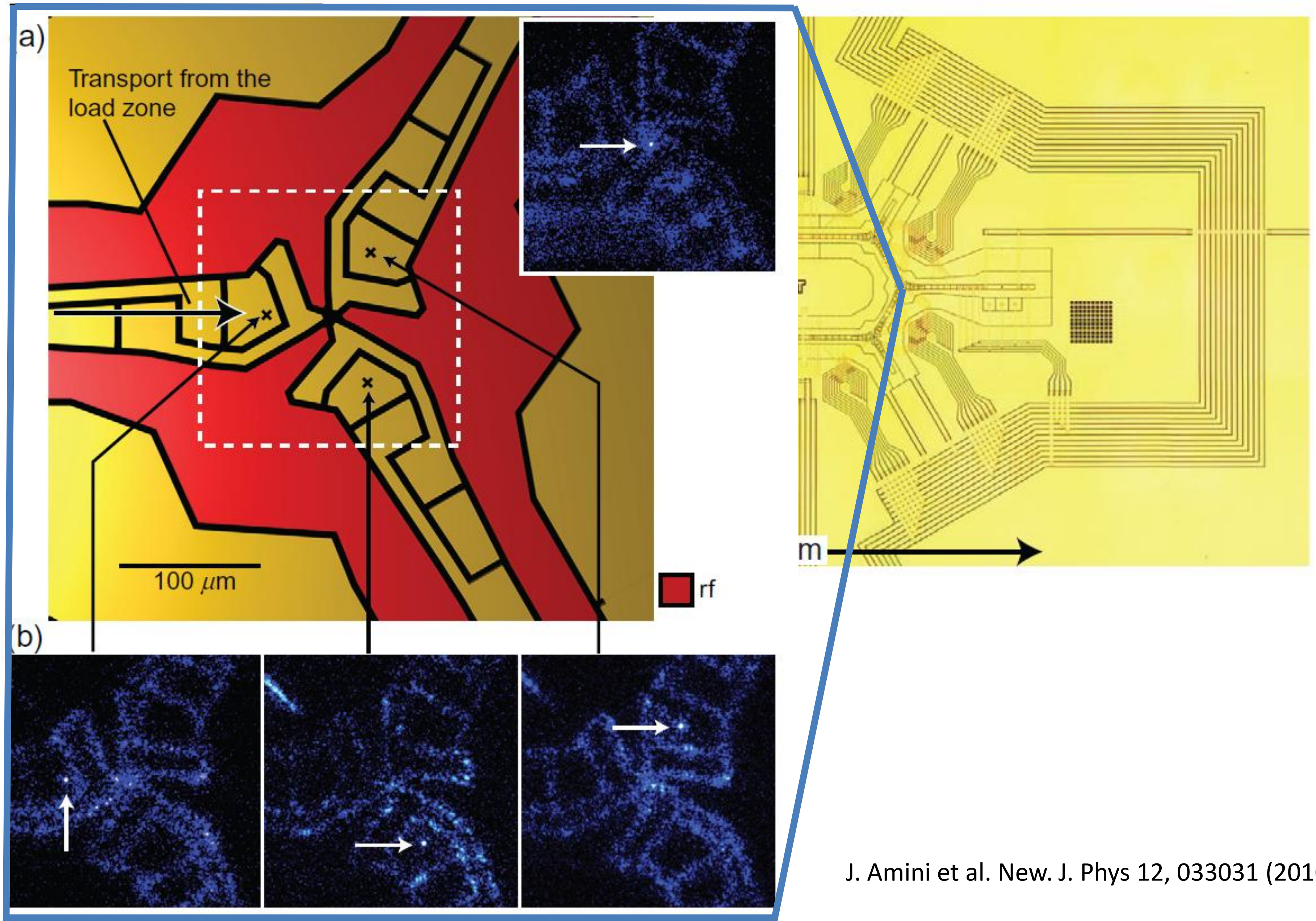




Challenges: shallow trap depth (100 meV)
charging of electrodes

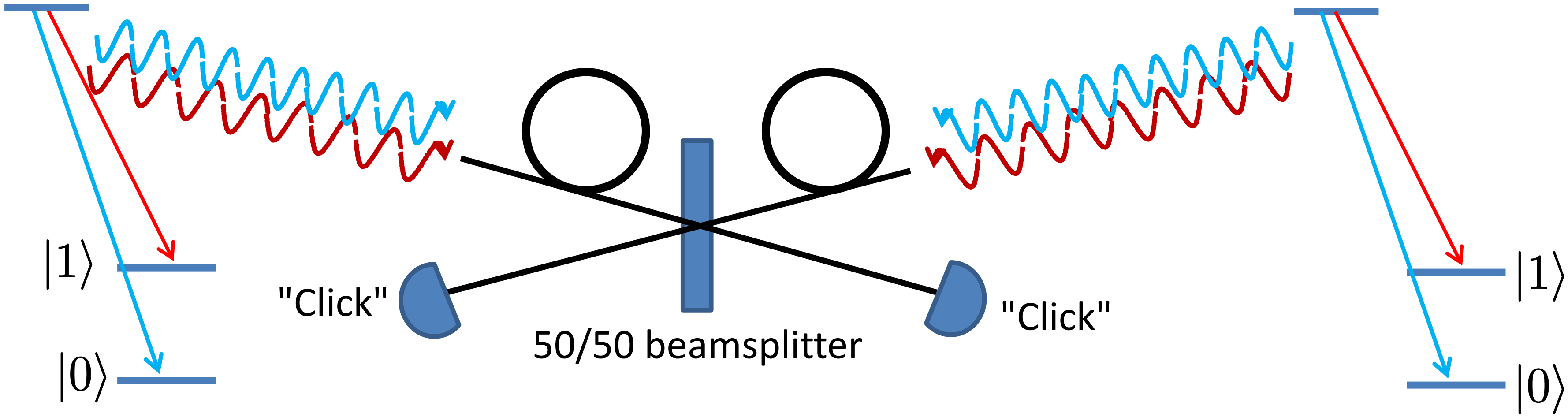
Opportunities: high gradients

Trapping ions on a (complicated) chip

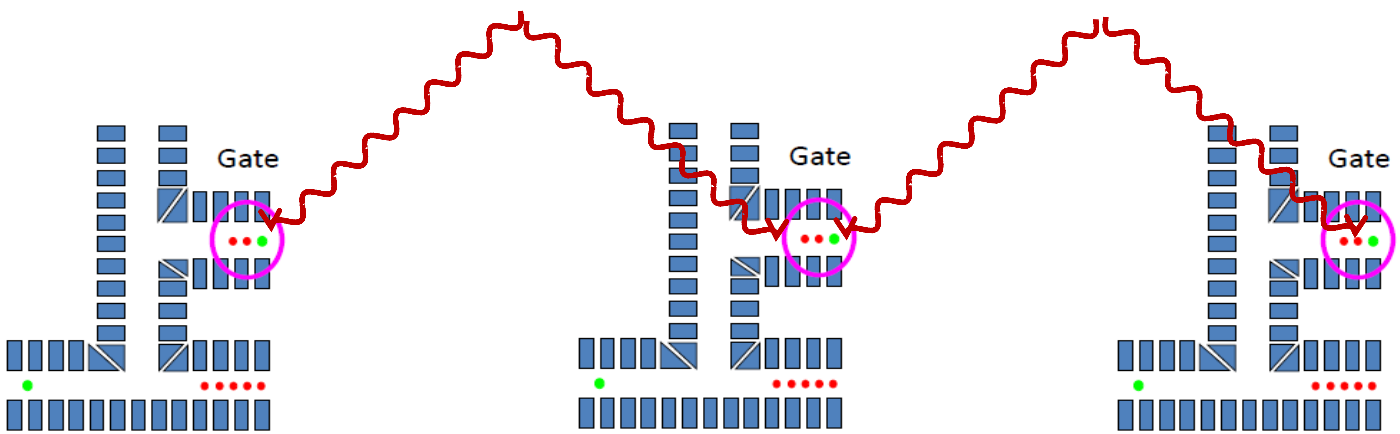


J. Amini et al. New. J. Phys 12, 033031 (2010)

Distributing entanglement: probabilistic



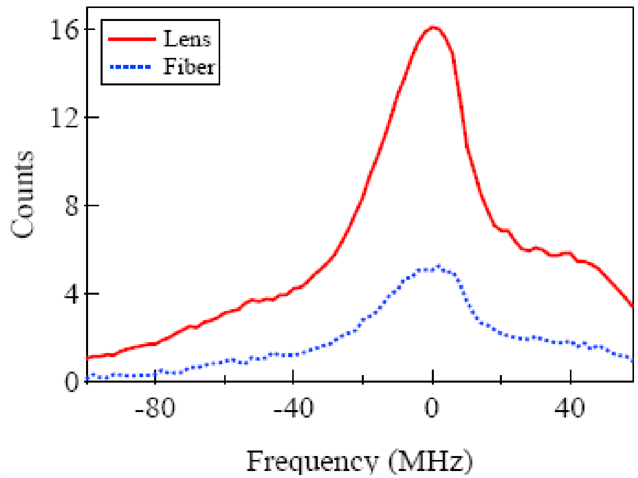
Entangled ions separated by **1m** (Moehring et al. Nature 449, 68 (2008))

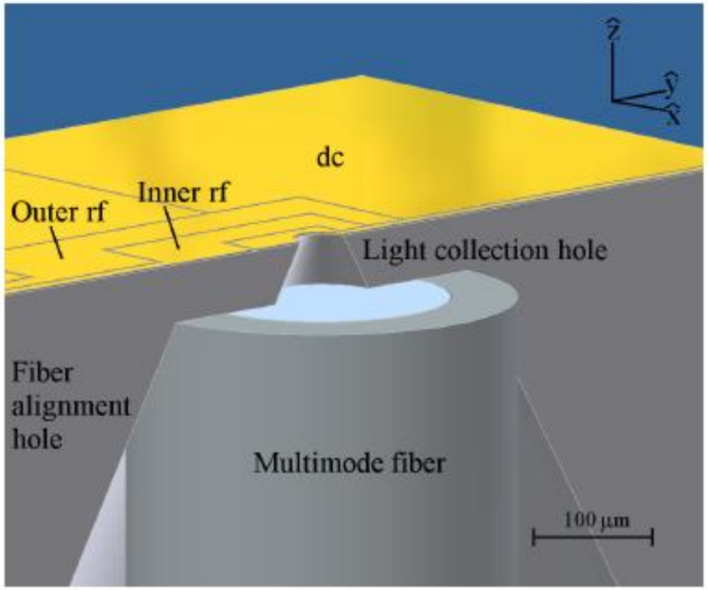


Entangled states are a resource for teleportation

Integrated components

Light collection

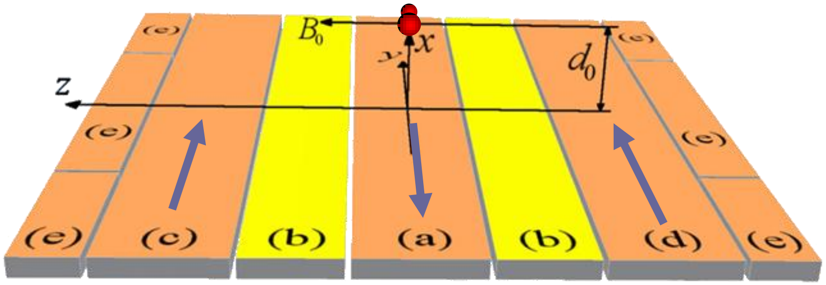




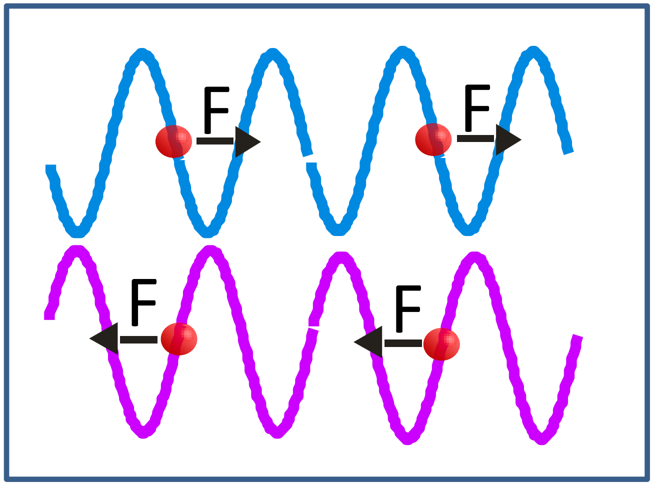
Vandevender et al. PRL 105, 023001 (2010)

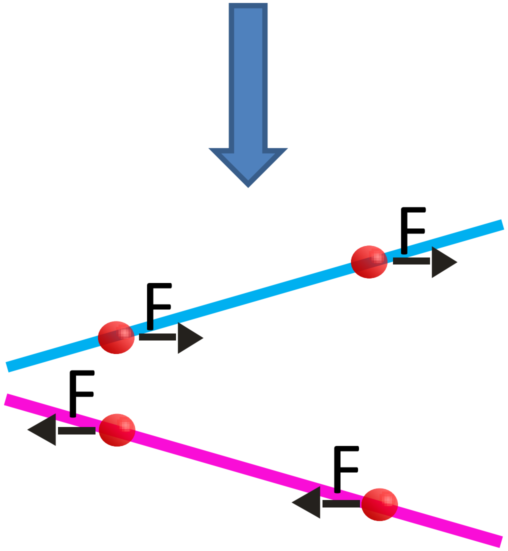
Integrated Components 2

Perform gates using r.f. magnetic fields derived from currents on trap surface



Ospelkaus et al. PRL 101, 090502 (2009)

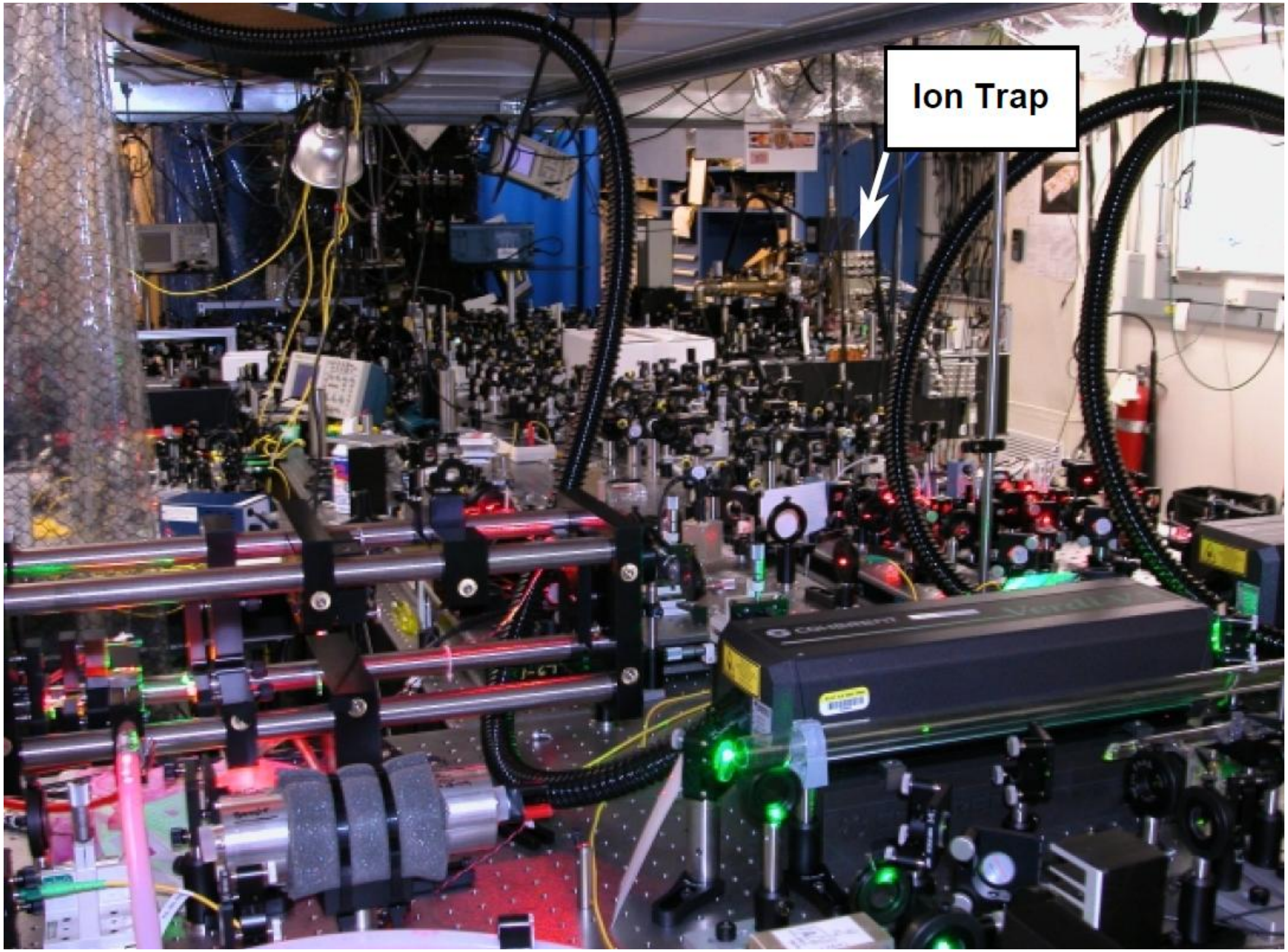




Magnetic field **gradients** offer alternative route to state-dependent potentials

- High-fidelity gates possible at higher ion temperatures
- r.f. easier to stabilize than laser beams

Apparatus - considerable



Example: NIST experiments now firing 1000s of laser pulses in an experimental sequence

Selected results

QIP protocols

Deterministic teleportation – Barrett et al., Haffner et al., Nature 429 (2004)

Entanglement purification – Reichle et al. Nature 443, 838-841 (2006)

Quantum error-correction - Chiaverini et al. Nature 432, 602-605 (2005)
(simple demonstration)

Arbitrary 2-qubit control - Hanneke et al. Nature Physics 6, 13-16 (2010)

Entangled states

GHZ states with up to 14 qubits - Schindler et al. arxiv:1009.6126 (2010)

$$|\psi\rangle = \frac{1}{\sqrt{2}}\left(|00000000000000\rangle + |11111111111111\rangle\right)$$

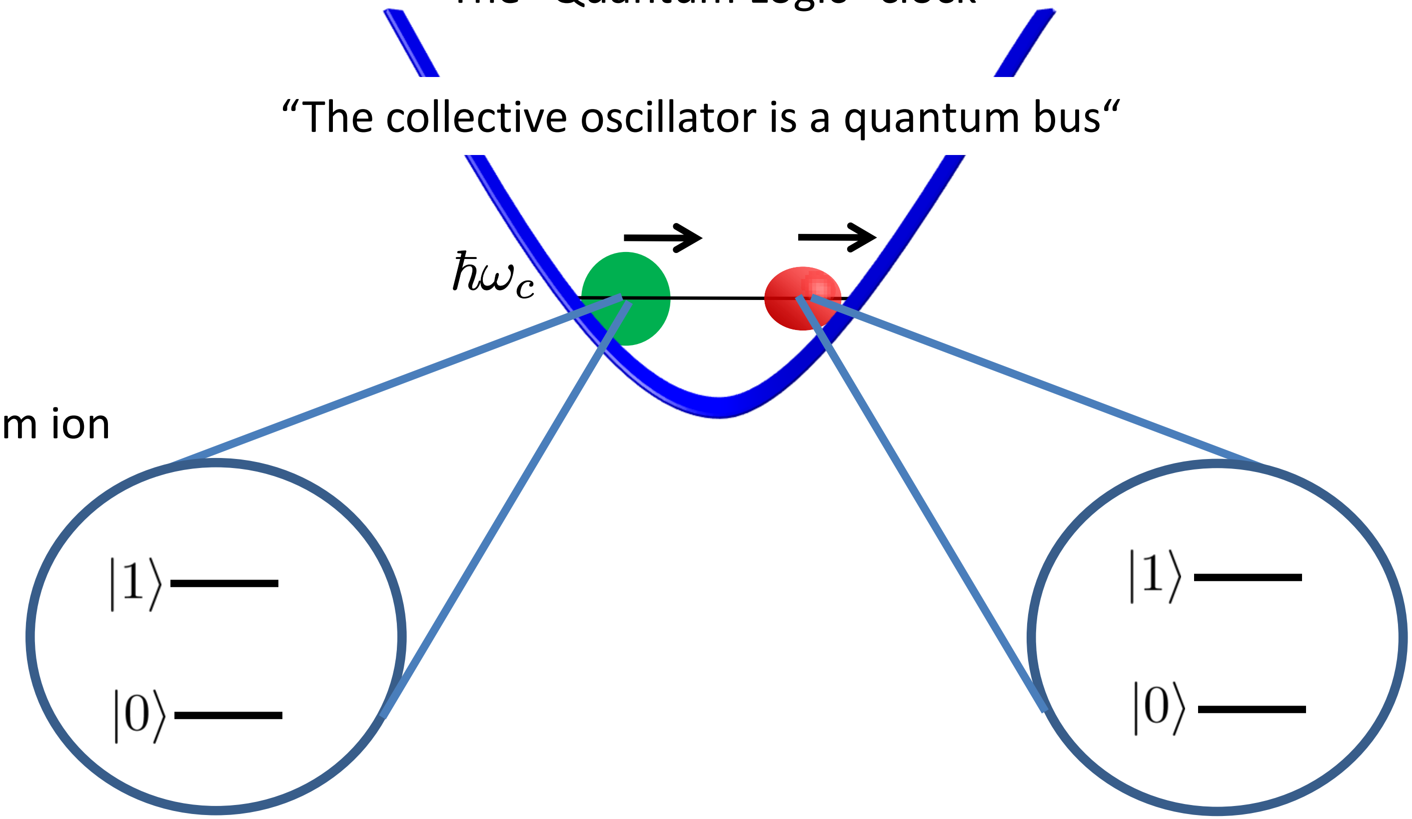
Entangled states of mechanical oscillators – Jost et al. Nature 459, 683 (2009)

Trapped-Ion applications

The “Quantum Logic“ clock

“The collective oscillator is a quantum bus“

Aluminium ion



Forbidden transition, lifetime 21 seconds

Frequency insensitive to light shifts from blackbody radiation

Cooling and readout transitions at 167 nm – vacuum UV!

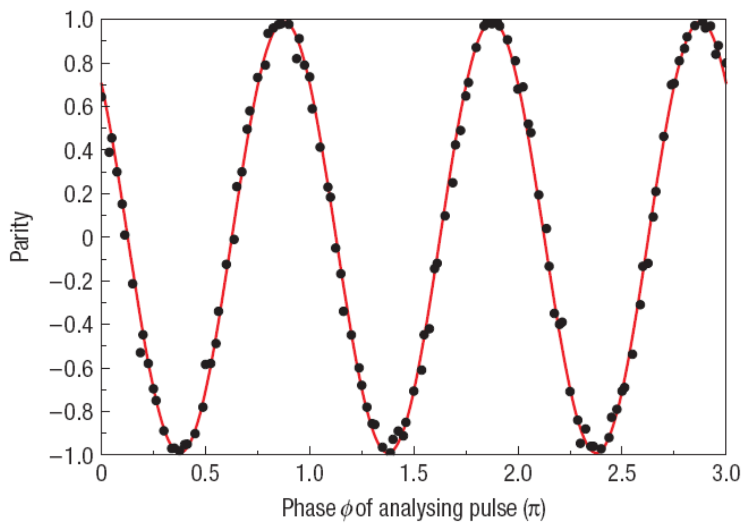
Beryllium/Magnesium

Cooling and readout

Frequency ratio of two clocks is stable at 8 parts in 10^{18}

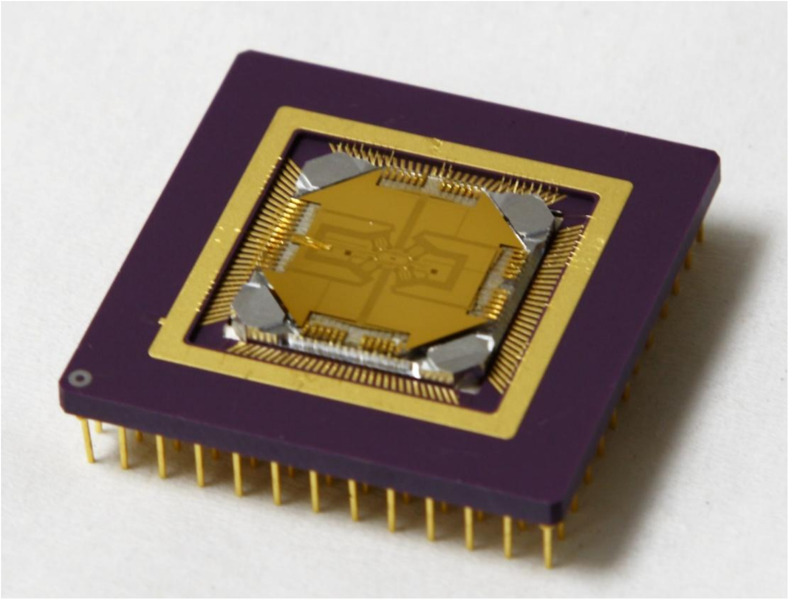
Measure difference in gravitation with height change of 20 cm

Trapped-ion summary



Have achieved quantum control
of up to N ions (latest N revealed next!)

Have demonstrated all basic components required
to create large scale entangled states



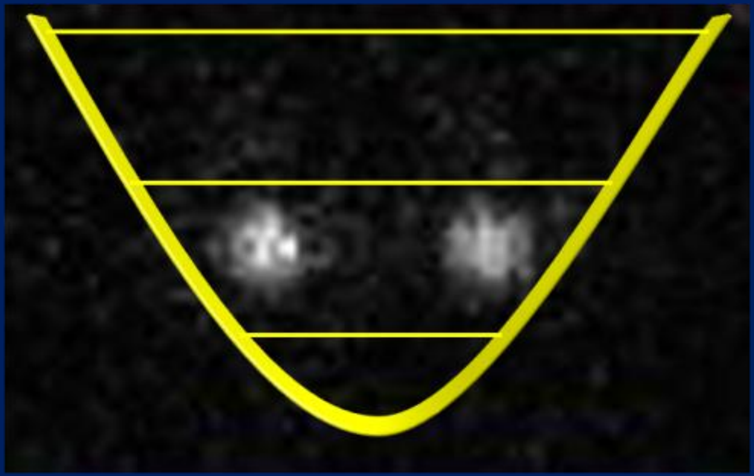
Working on:

Higher precision

New manipulation methods

Scaling to many ions

ETH
Eidgenössische Technische Hochschule Zürich
Swiss Federal Institute of Technology Zurich



www.tiqi.ethz.ch

Teleportation – a simple quantum information protocol.

Puzzle – can we transmit an <u>unknown</u> quantum state by only sending classical information?

We could try to measure the state, then reconstruct it at the other end
But if we don't know the state, what basis do we choose?

> On average, the best overlap we can get is 2/3, classically

Choose co-ords such that original state is $|0\rangle$

Rotate state into measurement basis, at unknown angle θ, ϕ, and measure

$$P(0) = \cos^2(\theta/2), P(1) = \sin^2(\theta/2)$$

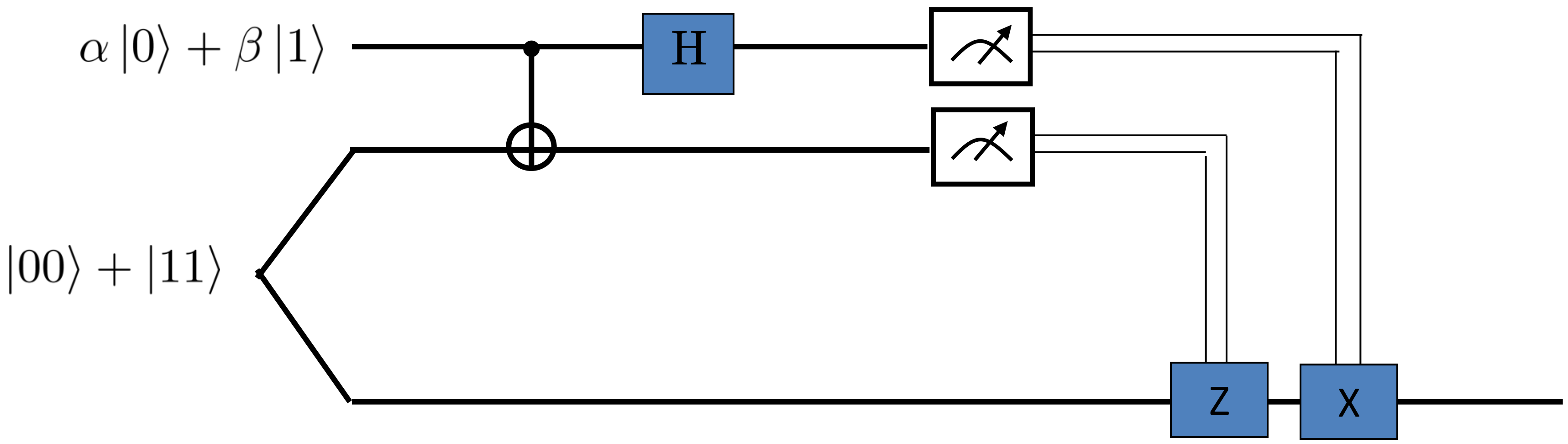
Now at the other end, reproduce the state you measured: the overlap with the initial state is

$$P(0)\cos^2(\theta/2) + P(1)\sin^2(\theta/2)$$

Integrate over surface of Bloch sphere, you get 2/3

Teleportation – a simple quantum information protocol.

What if we have half of an entangled pair at each of the source and destination?



Note again that despite the spatial separation, we can't describe the parts locally.

Initially: $(\alpha|0\rangle + \beta|1\rangle)(|00\rangle + |11\rangle)$

CNOT gate $(\alpha|000\rangle + \alpha|011\rangle + \beta|110\rangle + \beta|101\rangle)$

Teleportation – a simple quantum information protocol.

CNOT gate $(\alpha\,|000\rangle + \alpha\,|011\rangle + \beta\,|110\rangle + \beta\,|101\rangle)$

Hadamard (basis change) produces:

$$\begin{aligned} & \alpha(|0\rangle + |1\rangle)(|00\rangle + |11\rangle) \\ + \quad & \beta(|0\rangle - |1\rangle)(|10\rangle + |01\rangle) \end{aligned}$$

Regrouping terms:

Measure

$\|00\rangle(\alpha\|0\rangle + \beta\|1\rangle)$	measure 0,0, do nothing
$+\|01\rangle(\alpha\|1\rangle + \beta\|0\rangle)$	measure 0,1 apply X
$+\|10\rangle(\alpha\|0\rangle - \beta\|1\rangle)$	measure 1,0 apply Z
$+\|11\rangle(\alpha\|1\rangle - \beta\|0\rangle)$	measure 1,1, apply X then Z

NB: we can recover the qubit perfectly, by passing 2 bits of classical information…

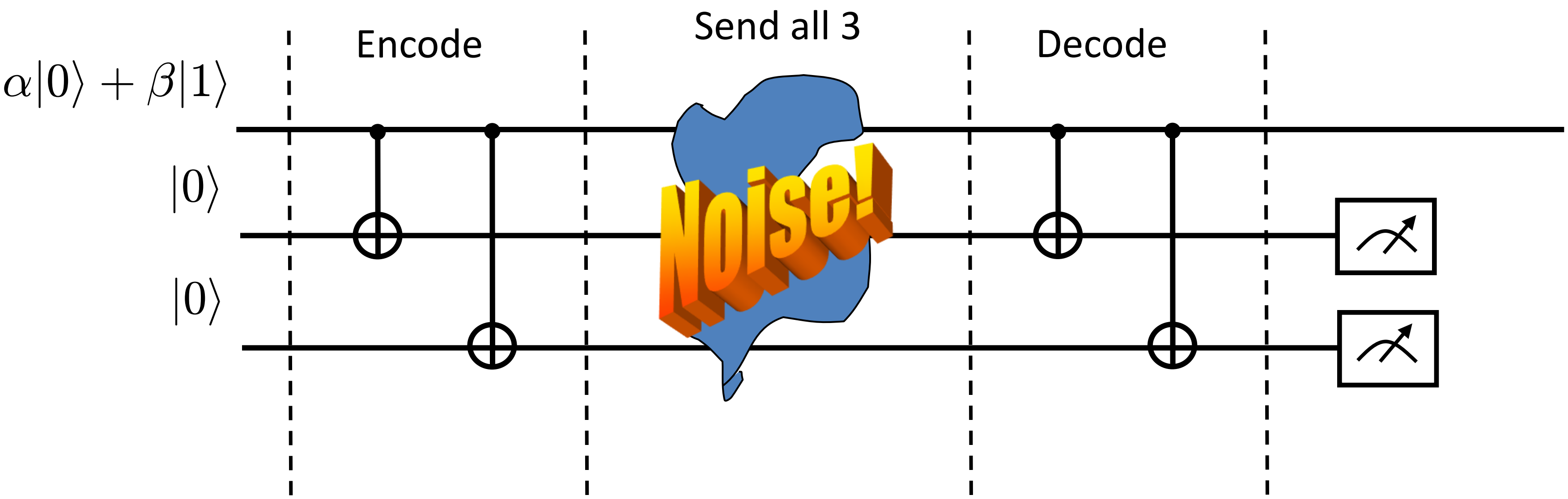
Trapped-ion realisation

A simple Error Correction Protocol

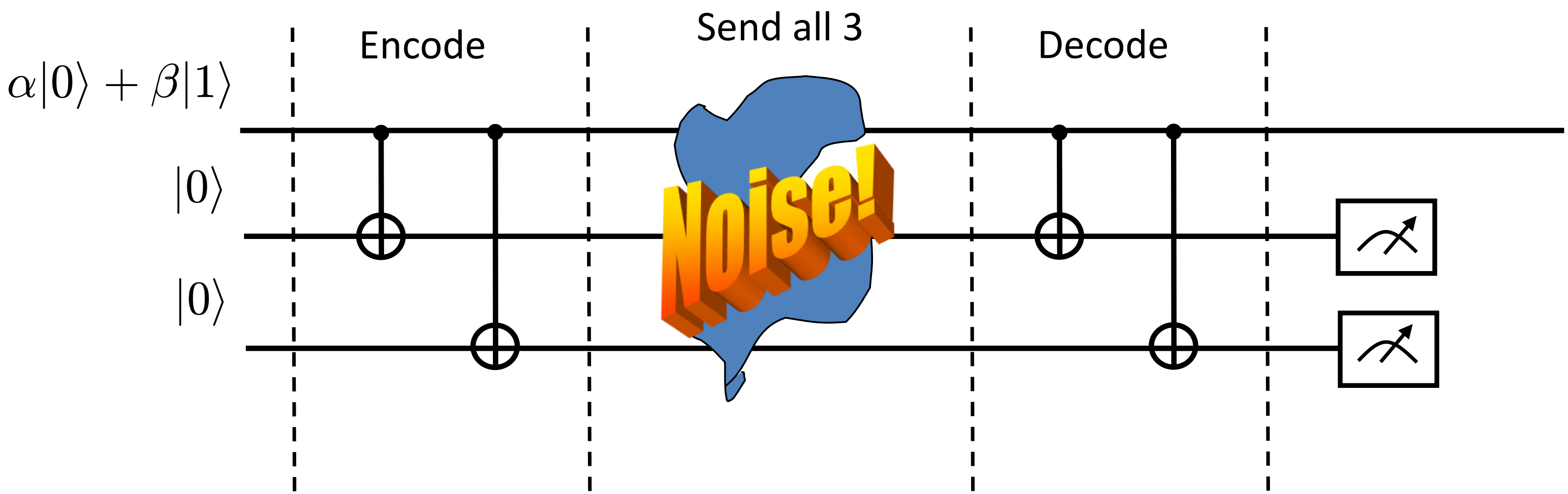
A problem: We want to send a qubit to a friend, but it risks having a X gate applied to it with probability *p* on the way, because of random noise.

Note: We can't measure the stored information, and send that, since we then only have a 2/3 probability of success. Can we win?

Consider the following circuit:



A simple Error Correction Protocol

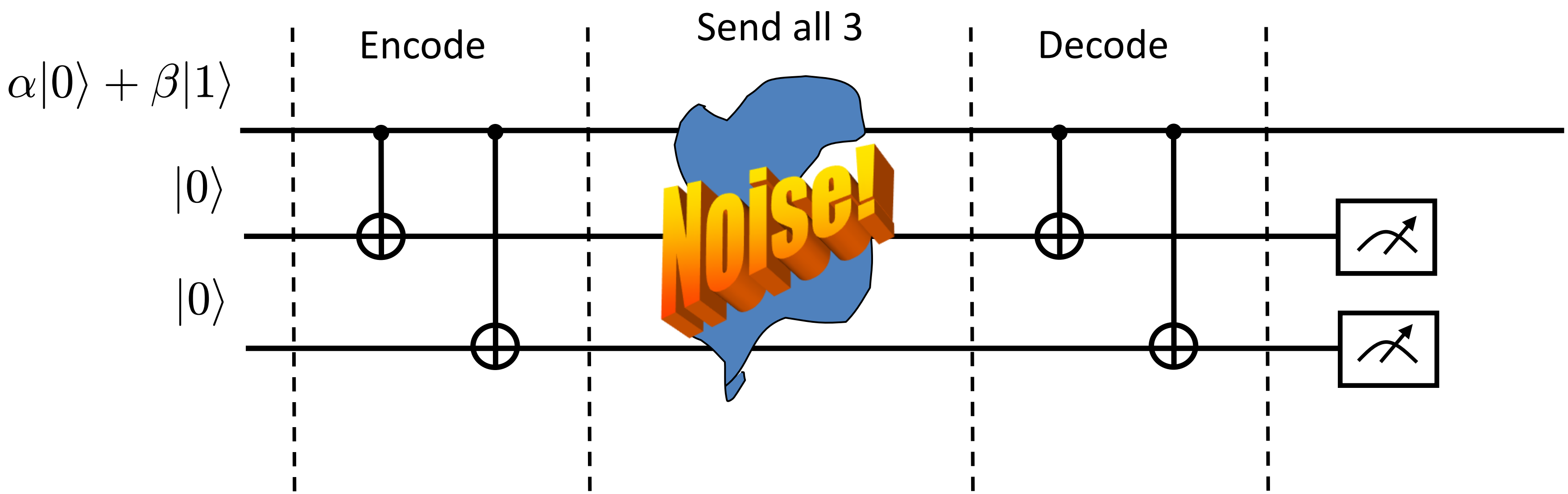


ENCODE:

Initially:	$(\alpha\|0\rangle + \beta\|1\rangle)\|00\rangle$
CNOT c1, t2	$(\alpha\|00\rangle + \beta\|11\rangle)\|0\rangle$
CNOT c1, t3	$\alpha\|000\rangle + \beta\|111\rangle$

NB: The quantum information belongs to no one qubit – it is shared between all three…

A simple Error Correction Protocol



Noise!

Flips qubit 1 with probability p
Flips qubit 2 with probability p
Flips qubit 3 with probability p

Therefore flips none, with probability $(1-p)^3$
flips one, with probability $p(1-p)^2$
flips two, with probability $p^2(1-p)$
flips three, with probability p^3

A simple Error Correction Protocol

Probability	Noise Aftermath	After decoding	Action
$(1-p)^3$	$\alpha\|000\rangle + \beta\|111\rangle$	$(\alpha\|0\rangle + \beta\|1\rangle)\|00\rangle$	Nothing
$p(1-p)^2$	$\alpha\|100\rangle + \beta\|011\rangle$	$(\alpha\|1\rangle + \beta\|0\rangle)\|11\rangle$	Apply X_1
	$\alpha\|010\rangle + \beta\|101\rangle$	$(\alpha\|0\rangle + \beta\|1\rangle)\|01\rangle$	Nothing
	$\alpha\|001\rangle + \beta\|110\rangle$	$(\alpha\|0\rangle + \beta\|1\rangle)\|10\rangle$	Nothing
$p^2(1-p)$	$\alpha\|110\rangle + \beta\|001\rangle$	$(\alpha\|1\rangle + \beta\|0\rangle)\|01\rangle$	
	$\alpha\|101\rangle + \beta\|010\rangle$	$(\alpha\|1\rangle + \beta\|0\rangle)\|10\rangle$	
	$\alpha\|011\rangle + \beta\|100\rangle$	$(\alpha\|0\rangle + \beta\|1\rangle)\|11\rangle$	
p^3	$\alpha\|111\rangle + \beta\|000\rangle$	$(\alpha\|1\rangle + \beta\|0\rangle)\|00\rangle$	

Measure

So we only fail with probability $3p^2(1-p) + p^3$

Improves things if p< ½; a lot if p << 1/2