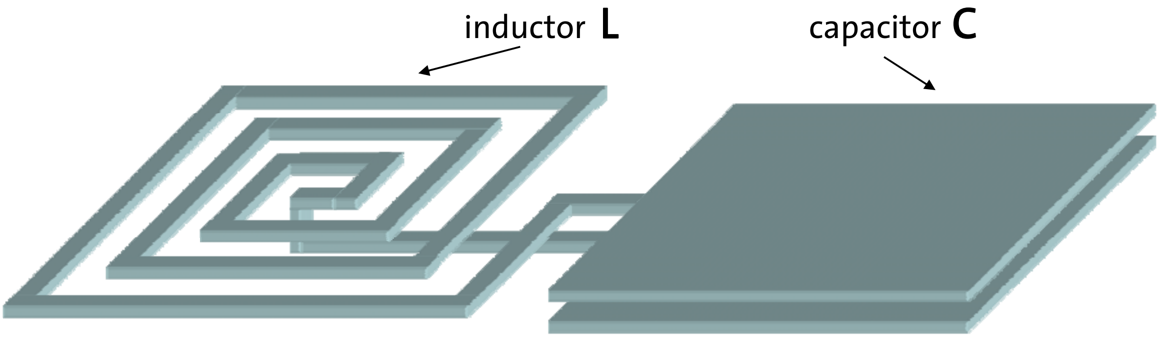
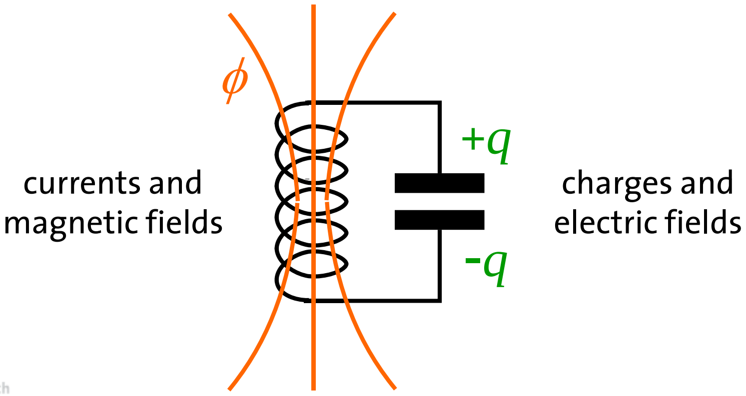
Realization of H.O.: Lumped Element Resonator

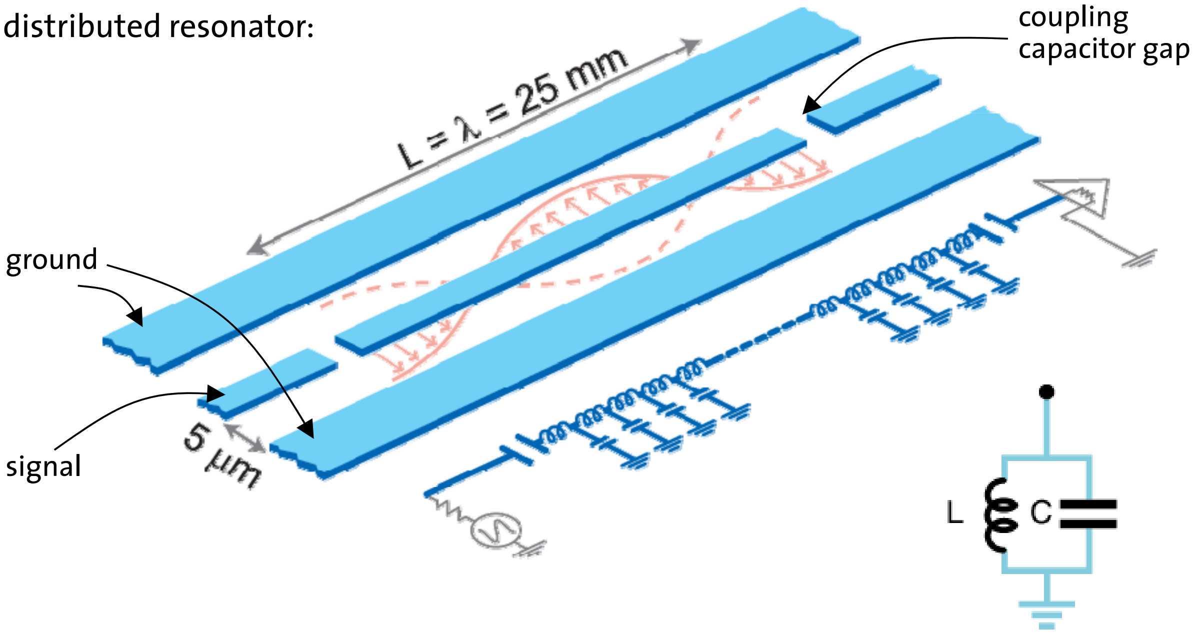


a harmonic oscillator



Realization of H.O.: Transmission Line Resonator

distributed resonator:

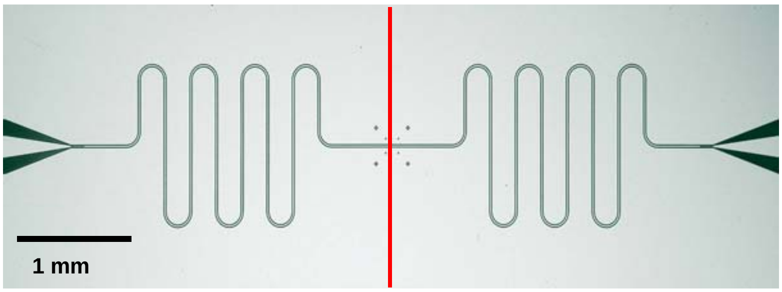


- coplanar waveguide resonator
- close to resonance: equivalent to lumped element LC resonator

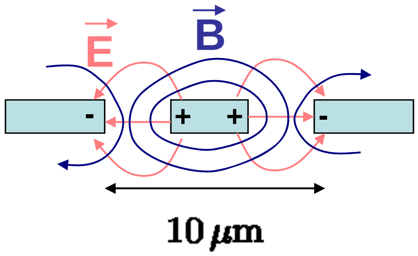
M. Goeppl *et al.*, Coplanar Waveguide Resonators for Circuit QED, *Journal of Applied Physics* **104**, 113904 (2008)

Realization of Transmission Line Resonator

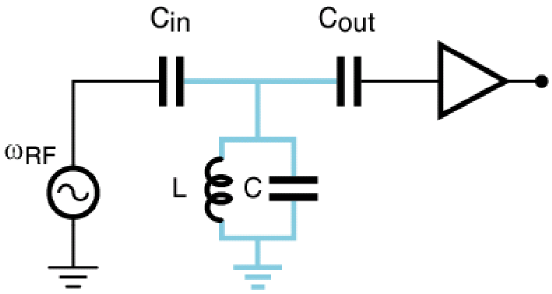
coplanar waveguide:



cross-section of transm. line (TEM mode):

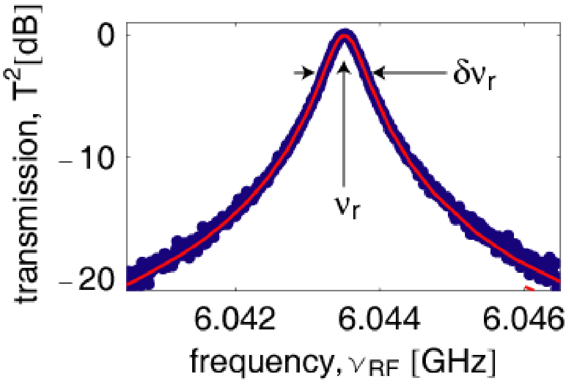


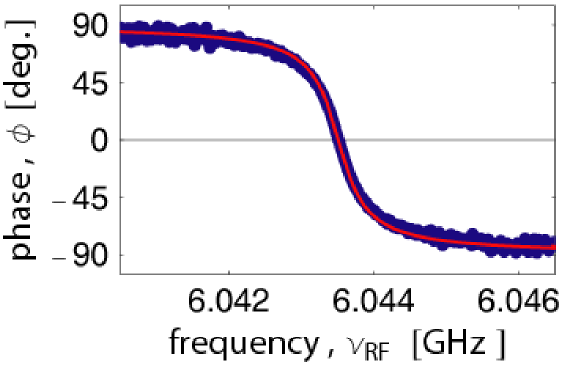
measuring the resonator:



photon lifetime (quality factor) controlled by coupling capacitors $C_{in/out}$

Resonator Quality Factor and Photon Lifetime





resonance frequency:

$$\nu_r = 6.04\,\mathrm{GHz}$$

quality factor:

$$Q = \frac{\nu_r}{\delta\nu_r} \approx 10^4$$

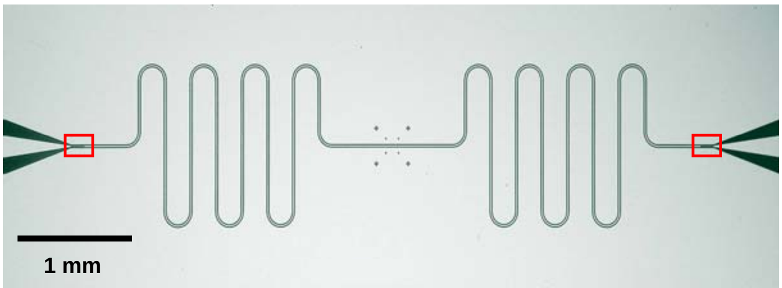
photon decay rate:

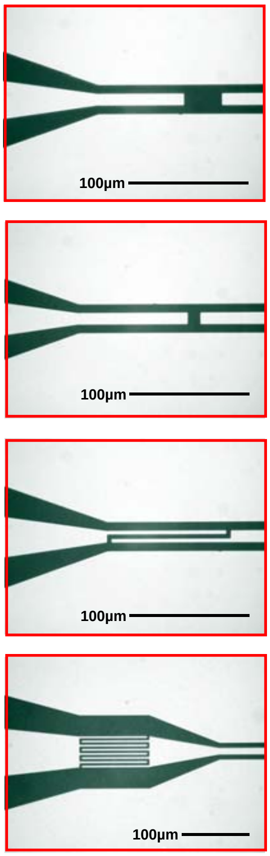
$$\frac{\kappa}{2\pi} = \frac{\nu_r}{Q} \approx 0.8\,\mathrm{MHz}$$

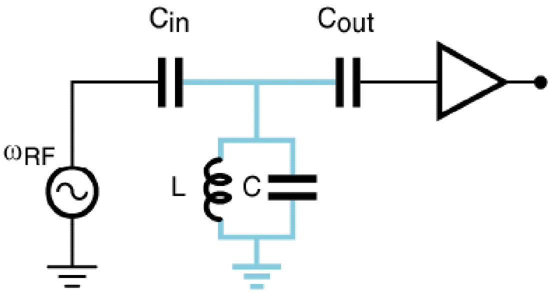
photon lifetime:

$$T_\kappa = 1/\kappa \approx 200\,\mathrm{ns}$$

Controlling the Photon Life Time

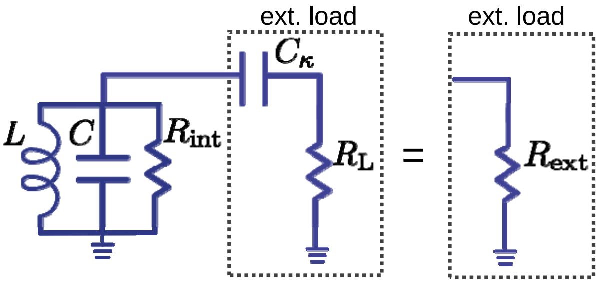


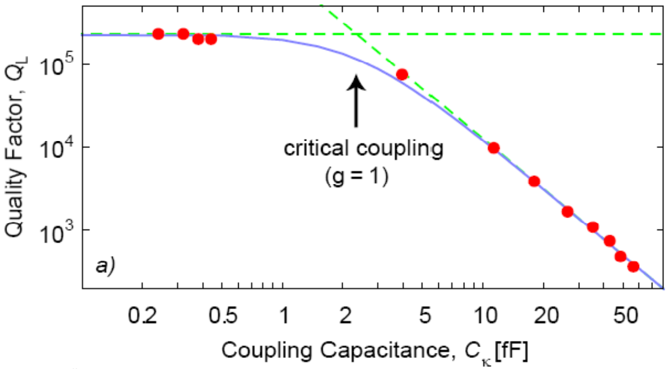


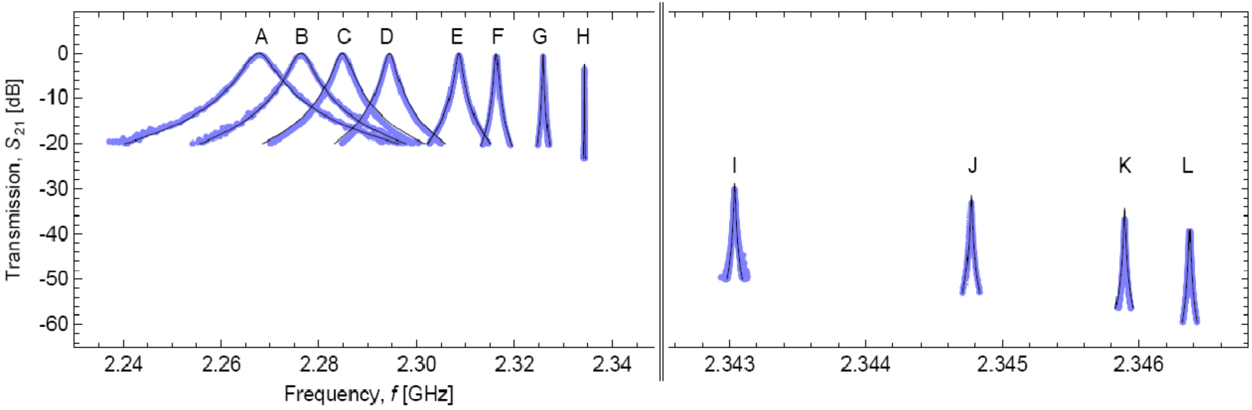


photon lifetime (quality factor)
controlled by coupling capacitor $C_{in/out}$

Quality Factor Measurement

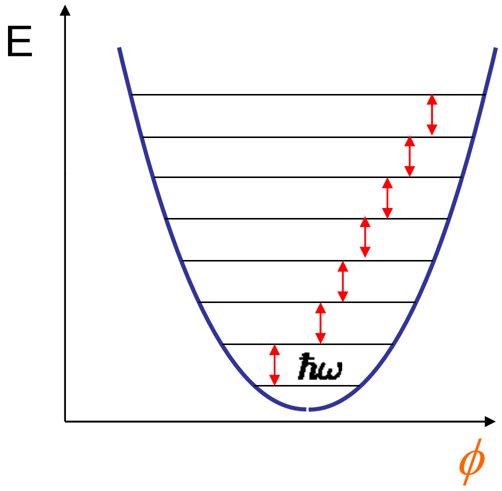






J. Appl. Phys. **104**, 113904 (2008)

Quantum Harmonic Oscillator at Finite Temperature



thermal occupation:

$$\langle n_{\mathrm{th}} \rangle = \frac{1}{\exp\left(h\nu/k_B T\right) - 1}$$

low temperature required:

$$\hbar\omega \gg k_B T$$

10 GHz ~ 500 mK ↗ 20 mK ↖

$$\langle n_{\mathrm{th}} \rangle \sim 10^{-11}$$

How to Prove that a Harmonic Oscillator is Quantum?

:

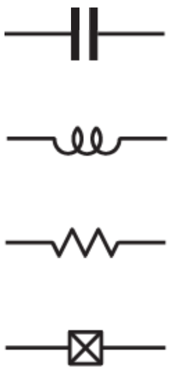
- resonance frequency
- average charge (momentum)
- average flux (position)

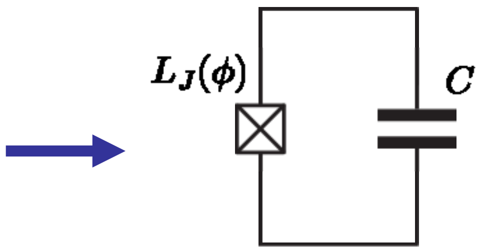
all averaged quantities are identical for a purely harmonic oscillator in the classical or quantum regime

solution:

- make oscillator non-linear in a controllable way

Constructing Non-Linear Quantum Electronic Circuits





Josesphson junction:
a non-dissipative nonlinear element (inductor)

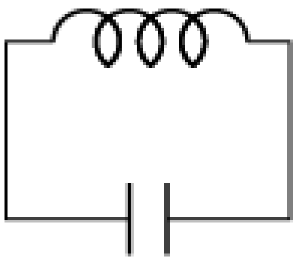
$$L_J(\phi) = \left(\frac{\partial I}{\partial \phi}\right)^{-1} = \frac{\phi_0}{2\pi I_c}\frac{1}{\cos(2\pi\phi/\phi_0)}$$

electronic
artificial atom

Review: M. H. Devoret, A. Wallraff and J. M. Martinis, *condmat/0411172* (2004)

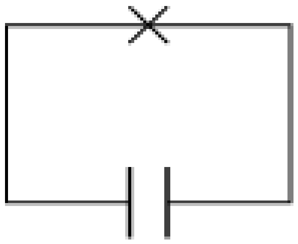
Linear vs. Nonlinear Superconducting Oscillators

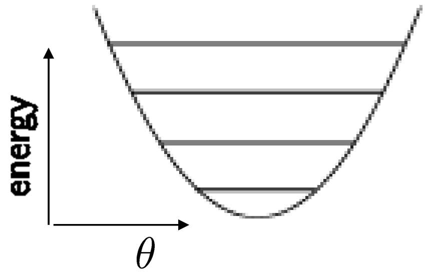
LC resonator



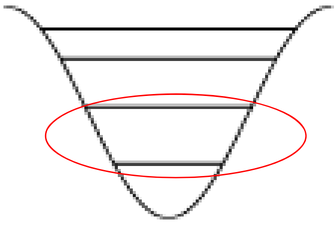
Josephson junction resonator

Josephson junction = nonlinear inductor



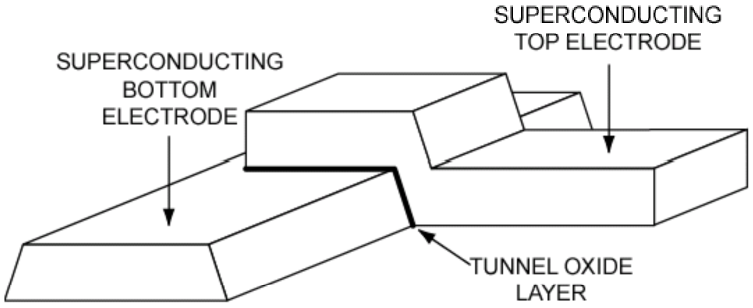


anharmonicity → effective two-level system



A Low-Loss Nonlinear Element

a (superconducting) Josephson junction

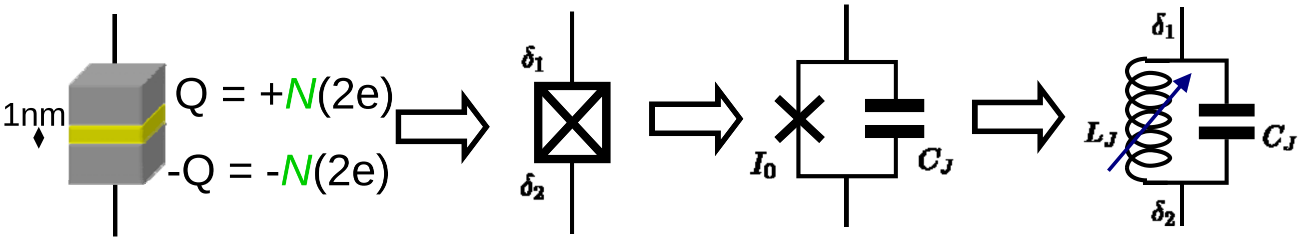


- superconductors: Nb, Al
- tunnel barrier: AlO_x

M. Tinkham, *Introduction to Superconductivity* (Krieger, Malabar, 1985).

Josephson Tunnel Junction

the only non-linear LC resonator with no dissipation (BCS, $k_B T \ll \Delta$)



tunnel junction parameters:

- critical current I_0
- junction capacitance C_J
- high internal resistance R_J

Josephson relations:

$$I = I_0 \sin \delta$$

$$V = \phi_0 \frac{\partial \delta}{\partial t}$$

flux quantum: $\phi_0 = \frac{h}{2e}$

phase difference: $\delta = \delta_2 - \delta_1$

derivation of Josephson effect, see e.g.: chap. 21 in R. A. Feynman: Quantum mechanics, The Feynman Lectures on Physics. Vol. 3 (Addison-Wesley, 1965)

The Josephson junction as a non-linear inductor

induction law: $V = -L \frac{\partial I}{\partial t}$

Josephson effect: dc-Josephson equation $I = I_c \sin\delta$

$$\frac{\partial I}{\partial t} = I_c \cos\delta \frac{\partial \delta}{\partial t}$$

ac-Josephson equation $V = \frac{\phi_0}{2\pi}\frac{\partial \delta}{\partial t} = \underbrace{\frac{\phi_0}{2\pi I_c}\frac{1}{\cos\delta}}_{L_J}\frac{\partial I}{\partial t}$

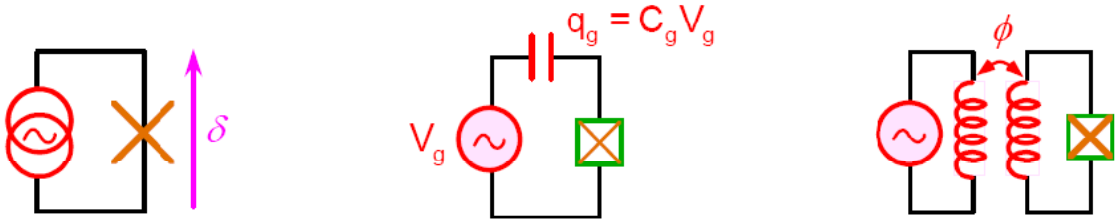
Josephson inductance

$$L_J = \underbrace{\frac{\phi_0}{2\pi I_c}}_{\text{specific Josephson inductance}} \underset{\uparrow\ \text{nonlinearity}}{\frac{1}{\cos\delta}}$$

A typical characteristic Josephson inductance for a tunnel junction with $I_c = 100$ nA is $L_{J0} \sim 3$ nH.

review: M. H. Devoret *et al.*,
Quantum tunneling in condensed media, *North-Holland,* (1992)

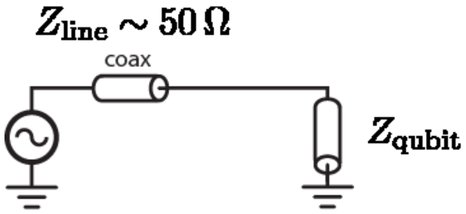
How to Make Use of the Josephson Junction in Qubits?



How is the control circuit important?

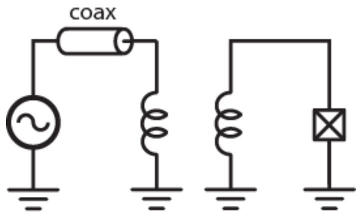
Controlling Coupling to the E.M. Environment

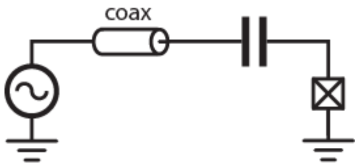
coupling to environment (bias wires):



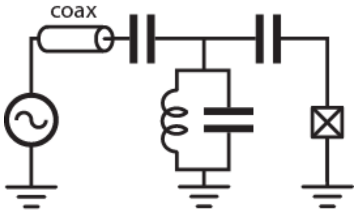
decoherence
from energy relaxation
(spontaneous emission)

decoupling using non-resonant impedance transformers:





using resonant impedance transformers

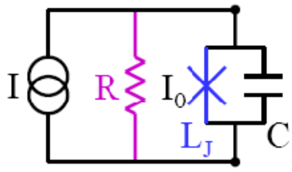


control spontaneous emission
by circuit design

The Current Biased Phase Qubit...

... supplementary information on a different type of
superconducting qubit.

Current Biased Phase Qubit



The bias current I distributes into a Josephson current through an ideal Josephson junction with critical current I_c, through a resistor R and into a displacement current over the capacitor C.

Kirchhoff's law:

$$I_b = I_s + I_R + I_c$$
$$= I_c \sin\delta + \frac{V}{R} + C\dot{V}$$

$$I_c = \dot{Q}_c = C\dot{V}$$
$$I_R = V/R$$
$$I_s = I_c \sin\delta$$

use Josephson equations:

$$I_b = I_c \sin\delta + \frac{\Phi_0}{2\pi R}\dot{\delta} + \frac{\Phi_0 C}{2\pi}\ddot{\delta}$$

W.C. Stewart, Appl. Phys. Lett. **2**, 277, (1968)
D.E. McCumber, J. Appl. Phys. **39**, 3 113 (1968)

looks like equation of motion for a particle with mass m and coordinate δ in an external potential U:

$$m\ddot{\delta} + m\frac{1}{RC}\dot{\delta} + \frac{\partial U(\delta)}{\partial\delta} = 0$$

particle mass: $m = C\,(\Phi_0/2\pi)^2$

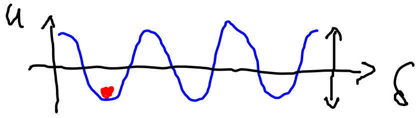
external potential: $U(\delta) = \frac{I_c\Phi_0}{2\pi}\left(-\frac{I_b}{I_c} - \cos\delta\right)$

Phase particle in a potential well

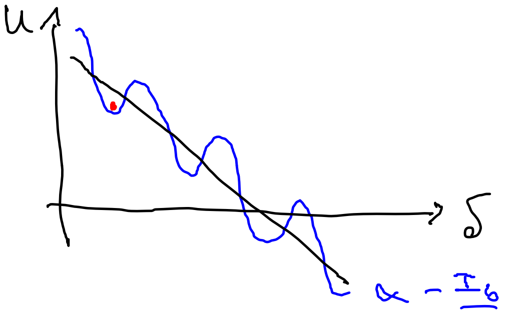
$$U(\delta) = \frac{I_c\Phi_0}{2\pi}\left(-\frac{I_b}{I_c}\delta - \cos\delta\right)$$

$$E_J = \frac{I_c\Phi_0}{2\pi}$$

cosine potential for $I_b = 0$:



'tilted washboard' potential for $I_b \neq 0$:



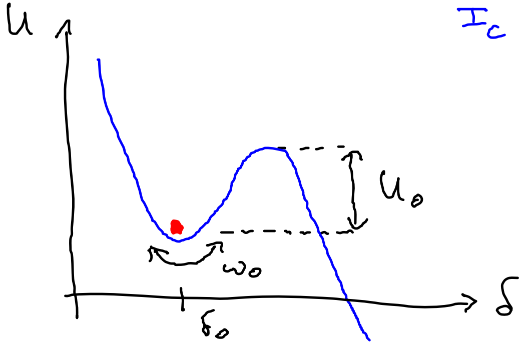
potential barrier:

$$U_0 = 2E_J\left[\sqrt{1-\gamma^2} - \gamma \arccos\gamma\right]$$

oscillation frequency:

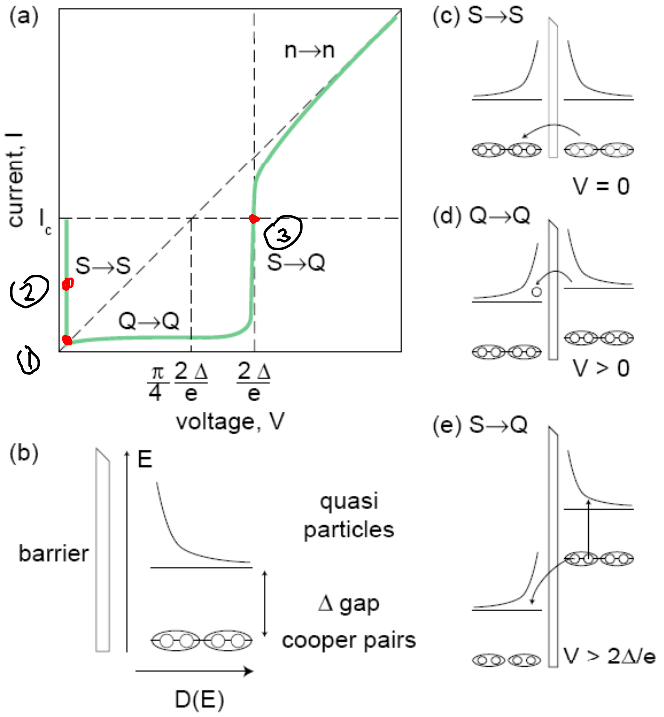
$$\omega_0 = \omega_p(1-\gamma^2)^{1/4} = \sqrt{\frac{U''(\delta_0)}{m}}$$

with: $\gamma = I_b/I_c$; $\omega_p = \sqrt{\frac{2\pi I_c}{\Phi_0 C}}$

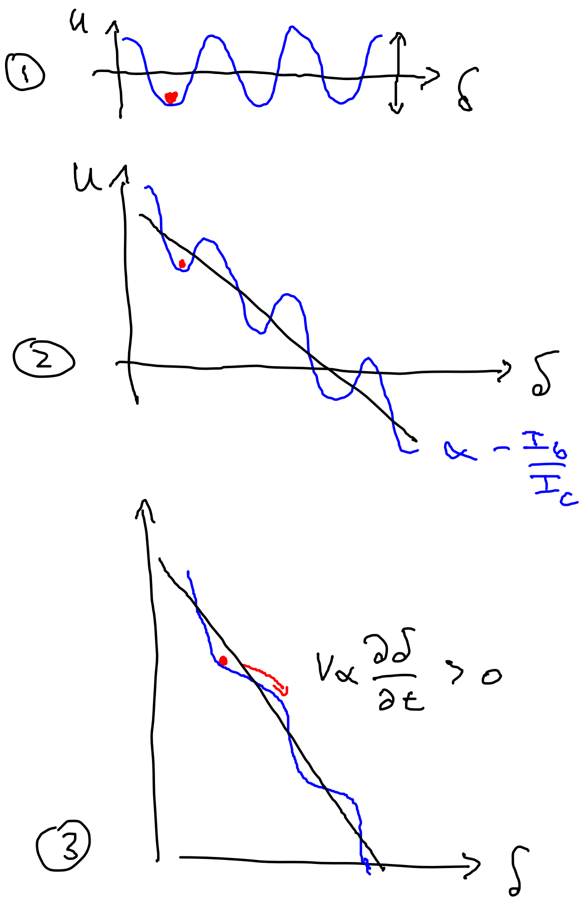


Current-voltage characteristics

typical I-V curve of underdamped Josephson junctions:



band diagram



Thermal Activation and Quantum Tunneling:

bias current dependence

$\omega_0(I)$; $U_0(I)$

thermal activation rate:

$$\Gamma_{th} \approx a_t \frac{\omega_0}{2\pi} \exp\left(-\frac{U_0}{k_B T}\right)$$

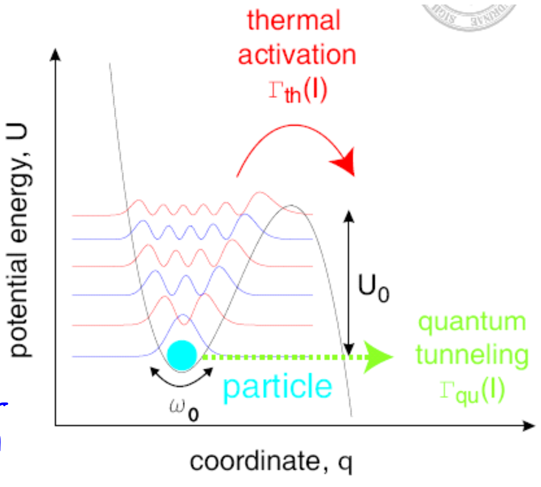
damping dependent prefactor

quantum tunneling rate:

$$\Gamma_{qu} = a_q \frac{\omega_0}{2\pi} \exp\left(-\frac{36}{5}\frac{U_0}{\hbar\omega_0}\right)$$

calculated using WKB method (exercise)

$$\Gamma_q = a_q \, \omega_0 \exp - \int_{\delta_1}^{\delta_2} \frac{1}{\hbar} \sqrt{2m(U(\delta) - E_0)}$$



energy level quantization:

$$E_n \approx \hbar\omega_0 (n + \tfrac{1}{2})$$

neglecting non-linearity

Quantum Mechanics of a Macroscopic Variable: The Phase Difference of a Josephson Junction
JOHN CLARKE, ANDREW N. CLELAND, MICHEL H. DEVORET, DANIEL ESTEVE, and JOHN M. MARTINIS
Science 26 February 1988 239: 992-997 [DOI: 10.1126/science.239.4843.992] (in Articles) Abstract » References » PDF »

Macroscopic quantum effects in the current-biased Josephson junction
M. H. Devoret, D. Esteve, C. Urbina, J. Martinis, A. Cleland, J. Clarke
in Quantum tunneling in condensed media, North-Holland (1992)

Early Results (1980's)

search for macroscopic quantum effects in superconducting circuits

theoretical predictions:

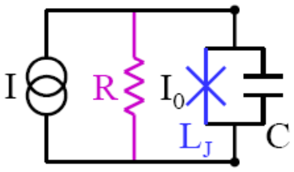
- tunneling ✓
- energy level quantization ✓
- coherence ✗

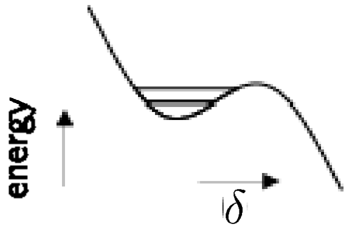
A.J. Leggett *et al.*,
Prog. Theor. Phys. Suppl. **69**, 80 (1980),
Phys. Scr. **T102**, 69 (2002).

short coherence times due to
strong coupling to em environment

experimental verification:

current biased JJ = phase qubit

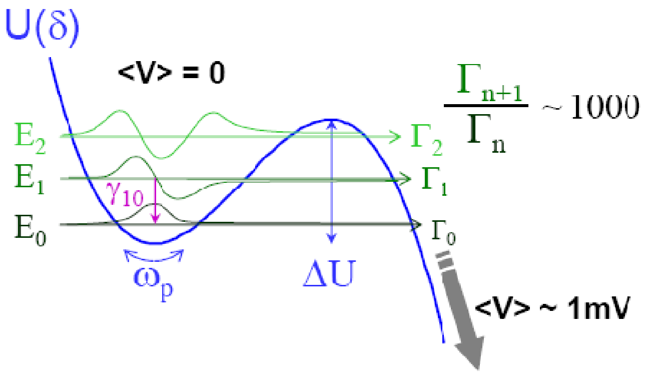


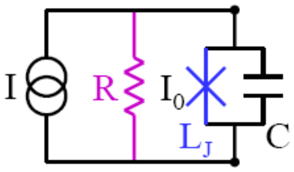


J. Clarke, J. Martinis, M. Devoret *et al.*, *Science* **239**, 992 (1988).

The Current Biased Phase Qubit

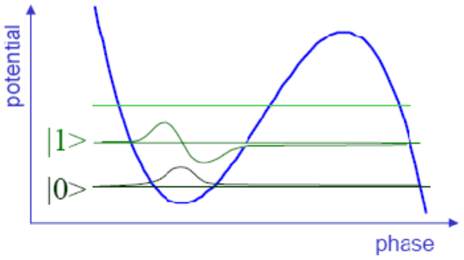
operating a current biased Josephson junction as a superconducting qubit:





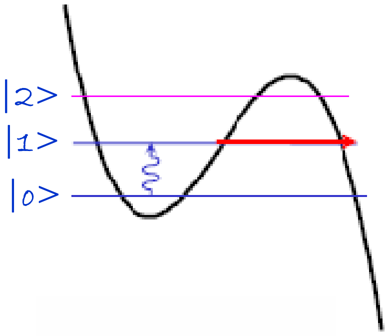
<u>initialization:</u>

wait for |1> to decay to |0>, e.g. by
spontaneous emission at rate γ_{10}



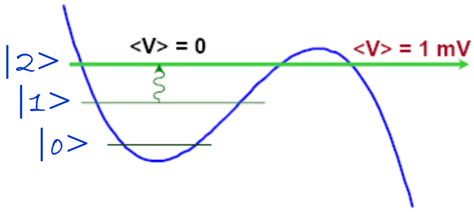
Read-Out Ideas

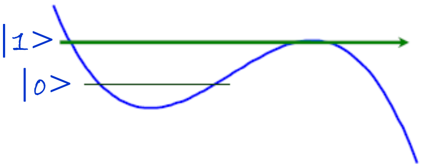
measuring the state of a current biased phase qubit



tunneling:

- prepare state |1> (pump)
- wait ($\Gamma_1 \sim 10^3\,\Gamma_0$)
- detect voltage
- |1> = voltage, |0> = no voltage





pump and probe pulses:

- prepare state |1> (pump)
- drive ω_{21} transition (probe)
- observe tunneling out of |2>

tipping pulse:

- prepare state |1>
- apply current pulse to suppress U_0
- observe tunneling out of |1>

The Cooper Pair Box ...

... a charge qubit.