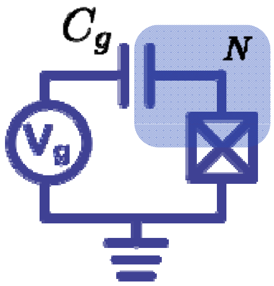
A Charge Qubit: The Cooper Pair Box

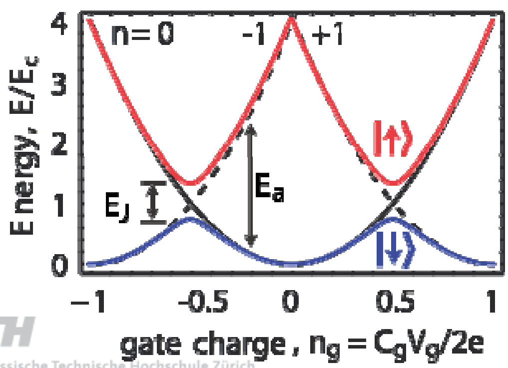


$$H_{el} = E_C N^2$$

$$H = E_C (N - N_g)^2 - E_J \cos\delta$$

$$[\delta, N] = i \quad \rightarrow \quad e^{\pm i\delta}|n\rangle = |n \pm 1\rangle$$

$$H = \sum_N \left[E_C (N - N_g)^2 |N\rangle\langle N| - \frac{E_J}{2} (|N\rangle\langle N+1| + |N+1\rangle\langle N|) \right]$$



Charging energy: $E_C = \frac{(2e)^2}{2C_\Sigma}$

Gate charge: $N_g = \frac{C_g V_g}{2e}$

Josephson energy: $E_J = \frac{I_0 \Phi_0}{2\pi} = \frac{h\Delta}{8e^2 R_J}$

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Bouchiat et al. Physica Scripta 176, 165 (1998)

Cooper pair box Hamiltonian:

Hamiltonian: $\hat{H} = E_c(\hat{N} - N_g)^2 - E_J \cos\hat{\delta}$ $= \frac{E_J}{2}(e^{i\hat{\delta}} + e^{-i\hat{\delta}})$

gate charge $N_g = \frac{C_g V_g}{2e}$

electrostatic charging energy — magnetic energy Josephson coupling Energy

$$E_c = \frac{(2e)^2}{2C_\Sigma} \qquad E_J = \frac{\phi I_c}{2\pi}$$

Hamiltonian in charge representation:

$$\hat{H} = E_c (N - N_g)^2 |N\rangle\langle N| - \frac{E_J}{2} \sum_N (|N+1\rangle\langle N| + |N\rangle\langle N+1|)$$

easy to diagonalize numerically

$$\hat{H} = \begin{pmatrix} \cdots & \cdots & \cdots & \cdots & \cdots \\ \cdots & E_c(-1-N_g)^2 & -E_J/2 & 0 & \cdots \\ \cdots & -E_J/2 & E_c(0-N_g)^2 & -E_J/2 & \cdots \\ \cdots & 0 & -E_J/2 & E_c(1-N_g)^2 & \cdots \\ \cdots & \cdots & \cdots & \cdots & \cdots \end{pmatrix}$$

relation between phase and number basis:

$$|\delta\rangle = \frac{1}{\sqrt{2\pi}} \sum_N e^{i\hat{N}\delta} |N\rangle \quad \text{with} \quad e^{i\hat{\delta}}|N\rangle = |N+1\rangle$$

Phase representation of Cooper pair box Hamiltonian:

$$\hat{H} = E_C(\hat{N} - N_g)^2 - E_J\cos\hat{\delta} \qquad \text{with} \qquad \hat{N} = \frac{\hat{Q}}{2e} = -i\hbar\frac{1}{2e}\frac{\partial}{\partial\phi}$$

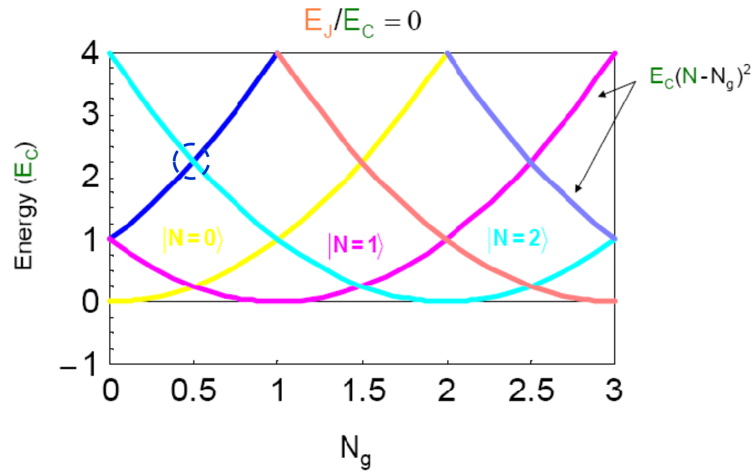
$$= E_C\left(-i\frac{\partial}{\partial\delta} - N_g\right)^2 - E_J\cos\hat{\delta} \qquad\qquad = -i\frac{\partial}{\partial\delta}$$

Equivalent solution to the Hamiltonian can be found in both representations, e.g. by numerically solving the Schrödinger equation for the charge (N) representation or analytically solving the Schrödinger equation for the phase (δ) representation.

$$\hat{H}|\psi\rangle = E|\psi\rangle$$

solutions for $E_J = 0$:

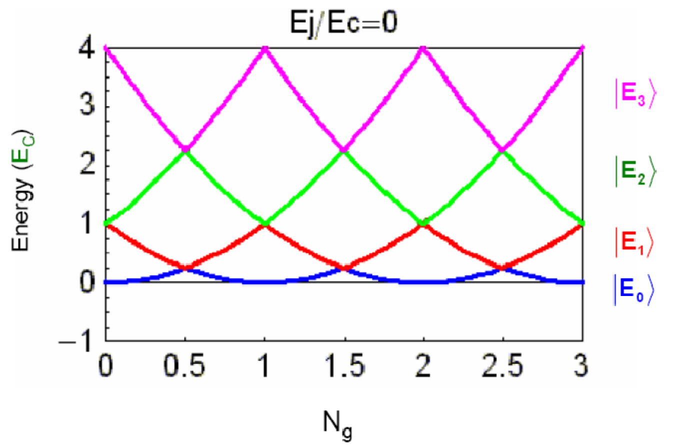
- crossing points are charge degeneracy points



Energy Levels

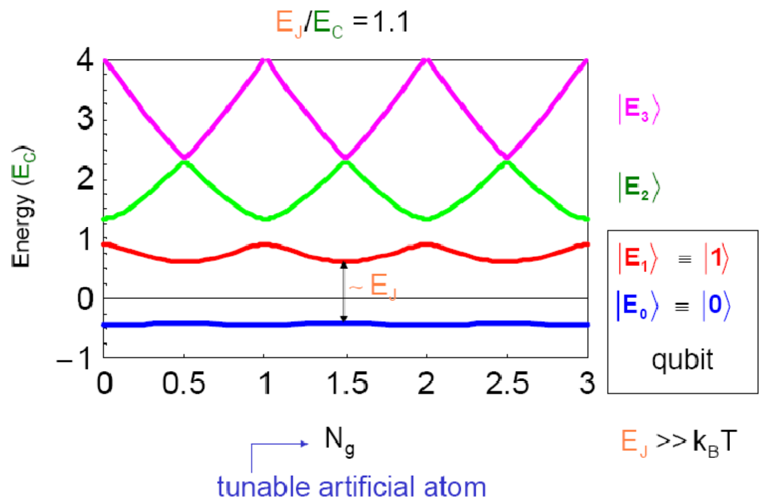
energy level diagram for $E_J=0$:

- energy bands are formed
- bands are periodic in N_g

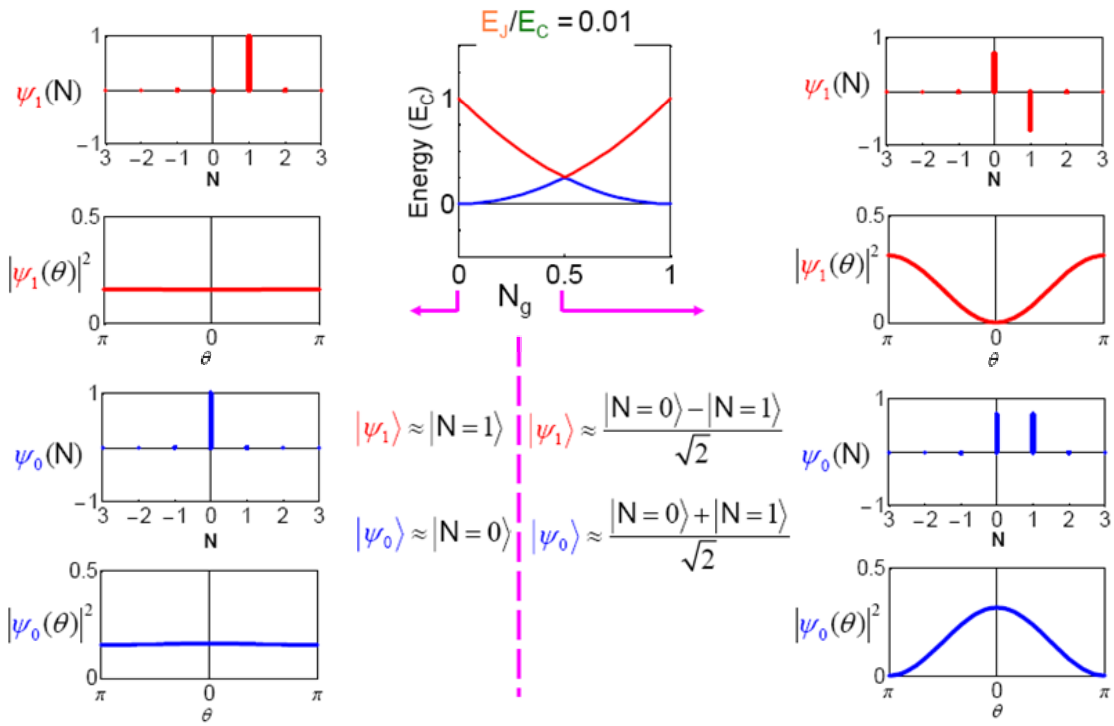


energy bands for finite E_J

- Josephson coupling lifts degeneracy
- E_J scales level separation at charge degeneracy

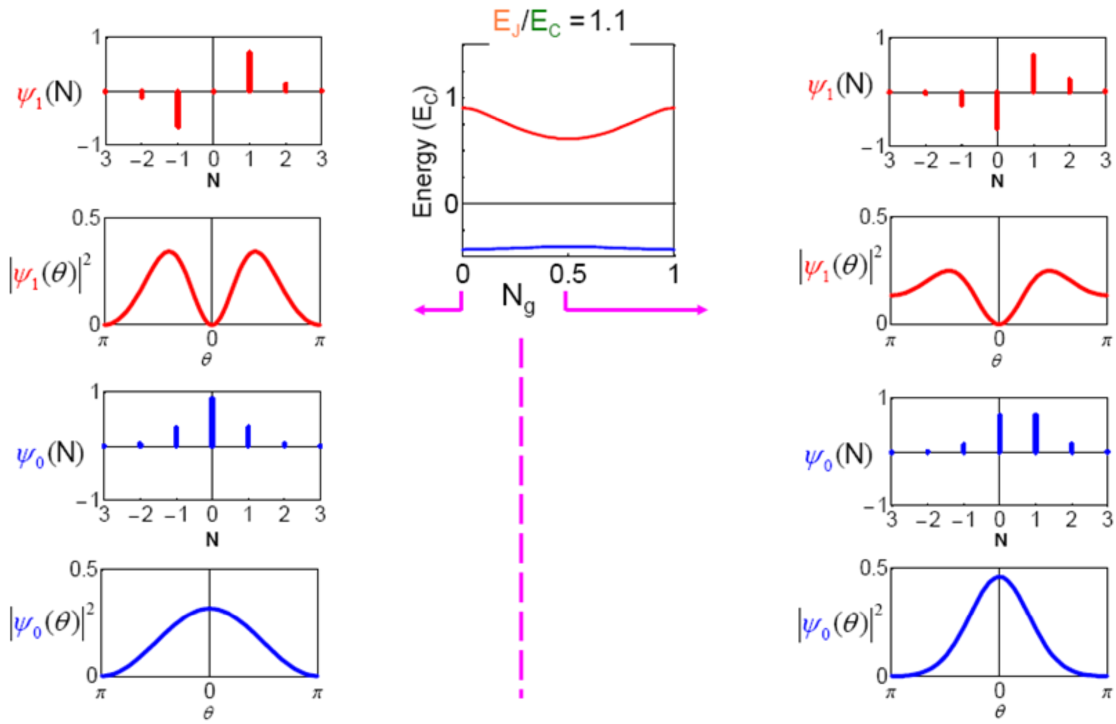


Charge and Phase Wave Functions ($E_j << E_c$)



courtesy Saclay

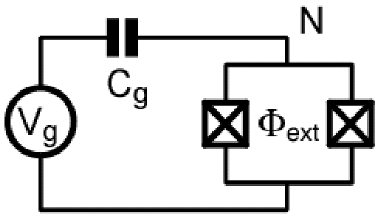
Charge and Phase Wave Functions (E_j ~ E_c)



courtesy Saclay

Tuning the Josephson Energy

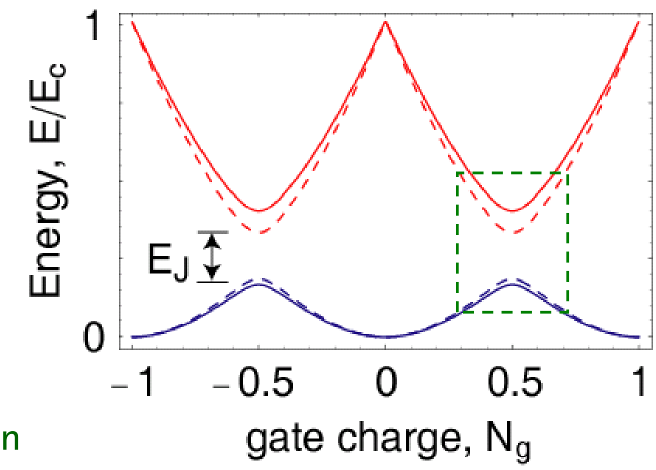
split Cooper pair box in perpendicular field



$$H = E_C\left(N - N_g\right)^2 - E_{J,\max}\cos\left(\pi\frac{\Phi_{\text{ext}}}{\Phi_0}\right)$$

SQUID modulation of Josephson energy

$$E_J = E_{J,\max}\cos\left(\pi\frac{\Phi_{\text{ext}}}{\Phi_0}\right)$$



consider two state approximation

J. Clarke, *Proc. IEEE* **77**, 1208 (1989)

Two State Approximation

$$\mathbf{H}_{\text{CPB}} = \mathbf{H}_{\text{el}} + \mathbf{H}_{\text{J}} = E_C(N - N_g)^2 - E_J\cos\delta$$

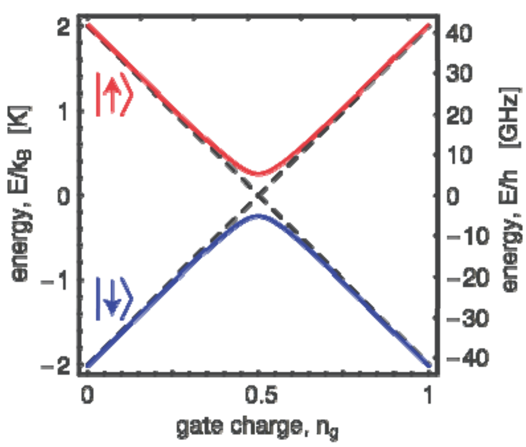
$$\mathbf{H}_{\text{CPB}} = \sum_N\left[E_C(N-N_g)^2\left|N\right\rangle\left\langle N\right| - \frac{E_J}{2}\left(\left|N\right\rangle\left\langle N+1\right| + \left|N+1\right\rangle\left\langle N\right|\right)\right]$$

Restricting to a two-charge Hilbert space:

$$N = \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix} = \frac{1-\sigma_z}{2}$$

$$\cos\delta = \frac{\sigma_x}{2}$$

$$\begin{aligned} \mathbf{H}_{\text{CPB}} &= -\frac{E_C}{2}(1-2N_g)\sigma_z - \frac{E_J}{2}\sigma_x \\ &= -\frac{1}{2}(E_{\text{el}}\sigma_z + E_J\sigma_x) \end{aligned}$$



Shnirman *et al., Phys. Rev. Lett.* **79**, 2371 (1997)

Cavity QED with Electronic Circuits

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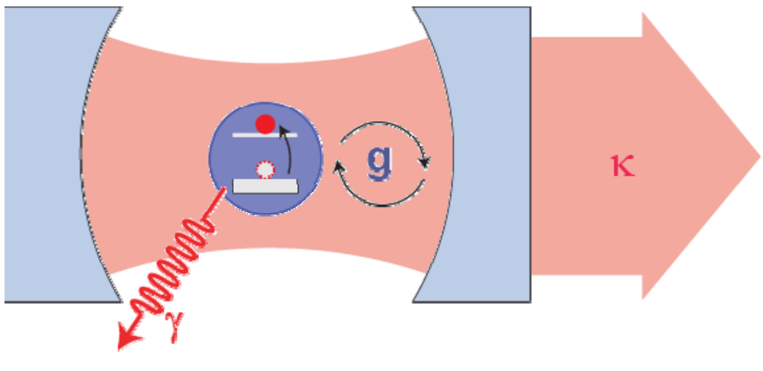
Cavity Quantum Electrodynamics

coupling photons to qubits:



Jaynes-Cummings Hamiltonian

$$H = \hbar\omega_r\left(a^\dagger a + \frac{1}{2}\right) + \frac{\hbar\omega_a}{2}\sigma^z + \hbar g(a^\dagger\sigma^- + a\sigma^+) + H_\kappa + H_\gamma$$

strong coupling limit $(g = dE_0/\hbar > \gamma, \kappa, 1/t_{\text{transit}})$

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D. Walls, G. Milburn, Quantum Optics (Spinger-Verlag, Berlin, 1994)

Dressed States Energy Level Diagram

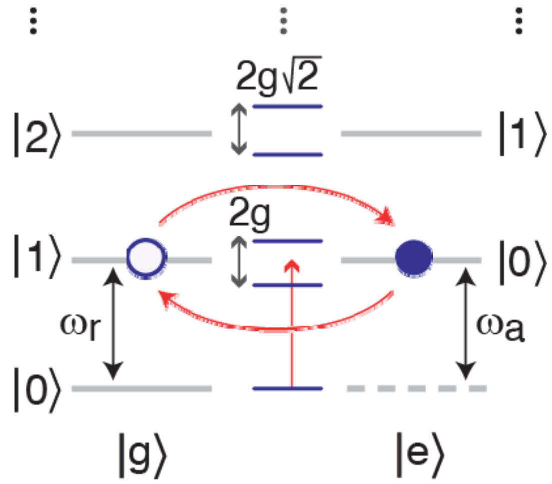
$$H = \hbar\omega_r\left(a^\dagger a + \frac{1}{2}\right) + \frac{\hbar\omega_a}{2}\sigma^z + \hbar g(a^\dagger\sigma^- + a\sigma^+)$$

in resonance:

$$\omega_a - \omega_r = \Delta = 0$$

strong coupling limit:

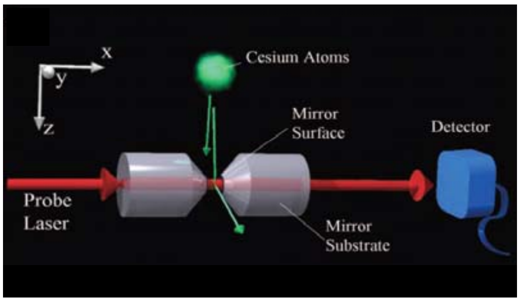
$$g = \frac{dE_0}{\hbar} > \gamma,\, \kappa$$



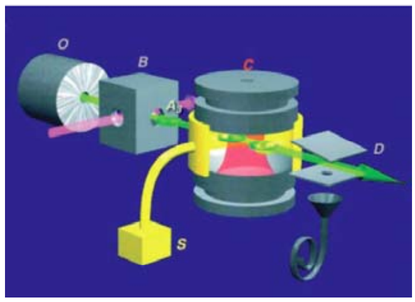
Jaynes-Cummings Ladder

Atomic cavity quantum electrodynamics reviews:
J. Ye., H. J. Kimble, H. Katori, *Science* **320**, 1734 (2008)
S. Haroche & J. Raimond, Exploring the Quantum, *OUP Oxford* (2006)

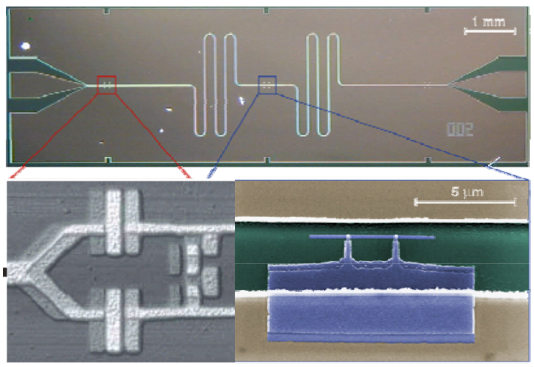
Cavity Quantum Electrodynamics (QED)



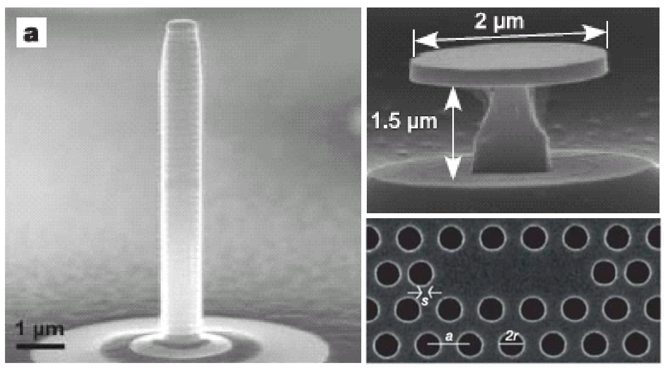
alkali atoms
MPQ, Caltech, ...



Rydberg atoms
ENS, MPQ, ...



superconductor circuits
Yale, Delft, NTT, ETHZ, NIST, ...



semiconductor quantum dots
Wurzburg, ETHZ, Stanford ...

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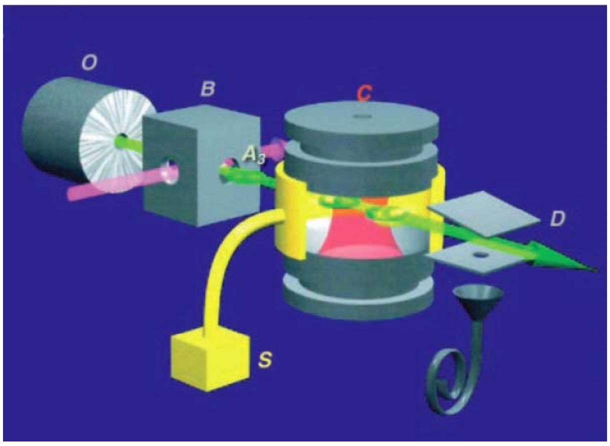
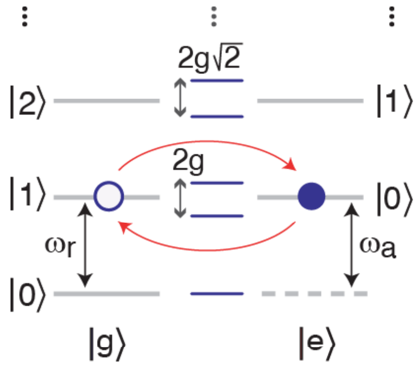
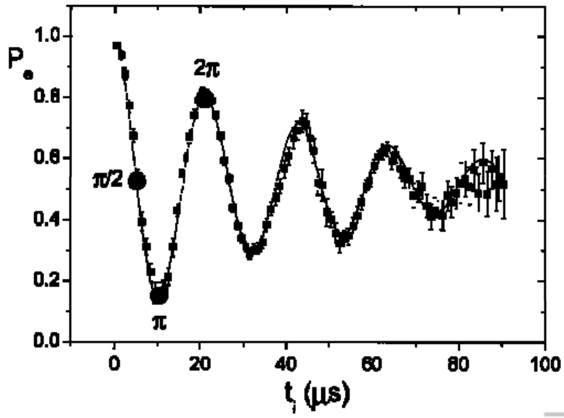
Vacuum Rabi Oscillations with Rydberg Atoms





Review: J. M. Raimond, M. Brune, and S. Haroche
Rev. Mod. Phys. **73**, 565 (2001)
P. Hyafil, ..., J. M. Raimond, and S. Haroche,
Phys. Rev. Lett. **93**, 103001 (2004)



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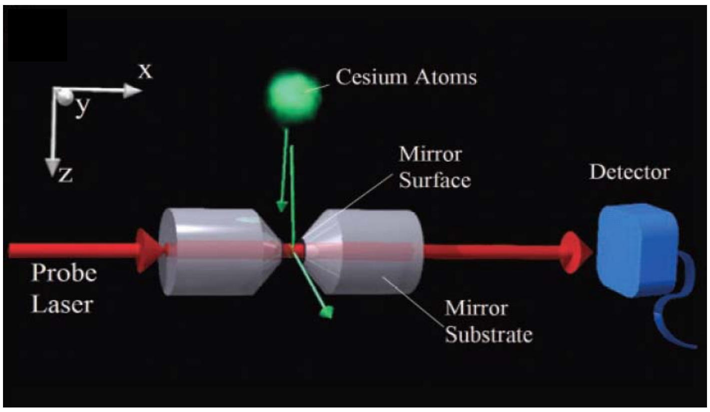
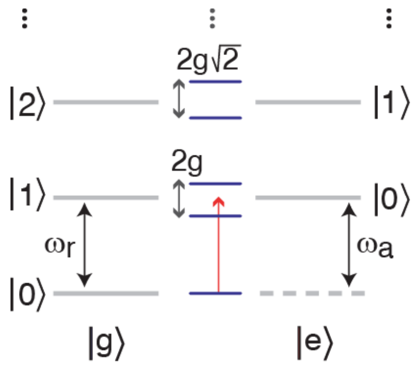
Vacuum Rabi Mode Splitting with Alkali Atoms





R. J. Thompson, G. Rempe, & H. J. Kimble,
Phys. Rev. Lett. **68** 1132 (1992)
A. Boca, ... , J. McKeever, & H. J. Kimble
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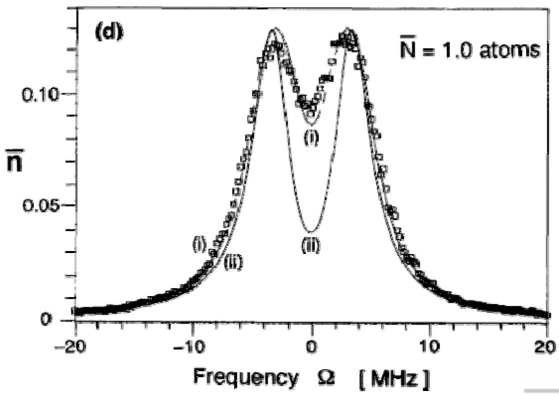

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