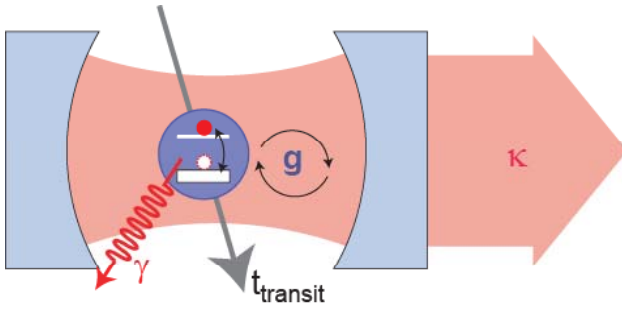


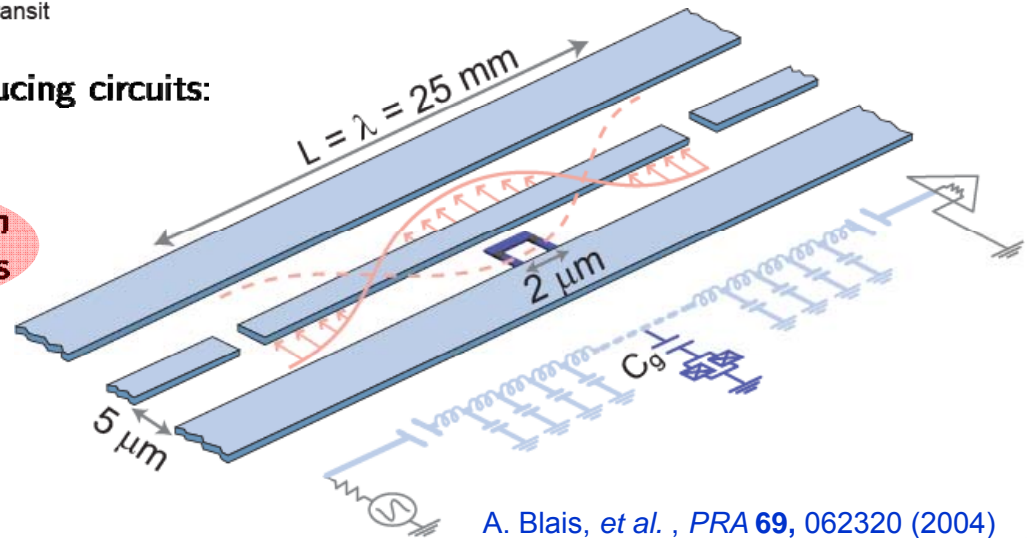
# Cavity QED with Superconducting Circuits



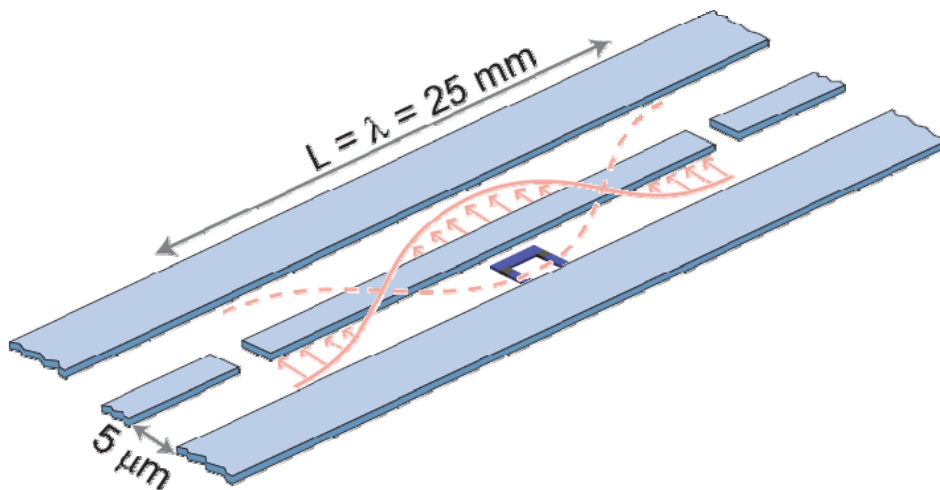
coherent quantum mechanics  
with individual photons and qubits ...

... in superconducting circuits:

circuit quantum  
electrodynamics



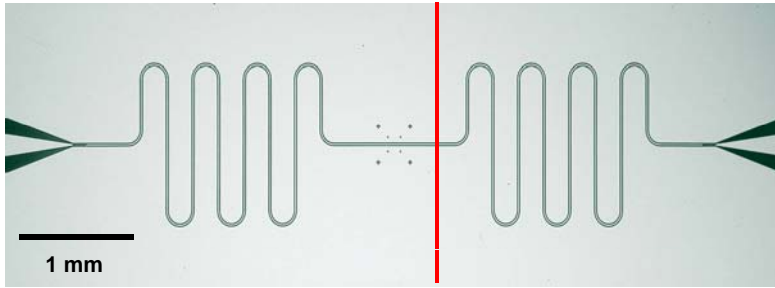
# Circuit Quantum Electrodynamics



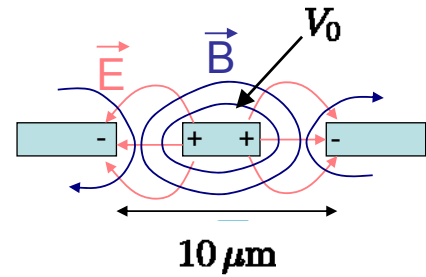
elements

- the cavity: a superconducting 1D transmission line resonator with **large vacuum field**  $E_0$  and **long photon life time**  $1/\kappa$
- the artificial atom: a Cooper pair box with large  $E_J/E_C$  with **large dipole moment**  $d$  and **long coherence time**  $1/\gamma$

# Vacuum Field in 1D Cavity



cross-section of transm. line (TEM mode):



voltage across resonator in vacuum state ( $n = 0$ )

harmonic oscillator

$$V_{0,rms} = \sqrt{\frac{\hbar\omega_r}{2C}} \approx 1 \mu V$$

$$H_r = \hbar\omega_r \left( a^\dagger a + \frac{1}{2} \right)$$

$$E_0 = \frac{V_{0,rms}}{b} \approx 0.2 \text{ V/m}$$

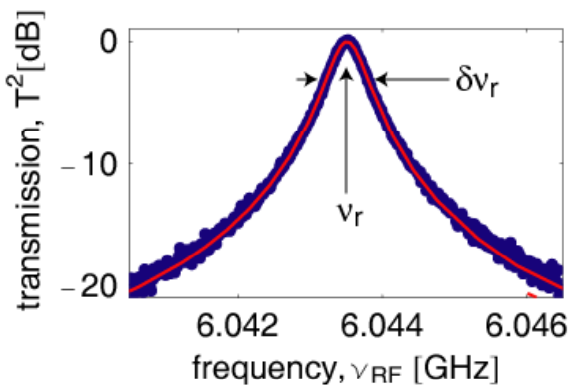
$\times 10^6$  larger than  $E_0$  in 3D microwave cavity

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for  $\omega_r/2\pi \approx 6 \text{ GHz}$  ( $C \sim 1 \text{ pF}$ ),  $b \approx 5 \mu\text{m}$

# Resonator Quality Factor and Photon Lifetime

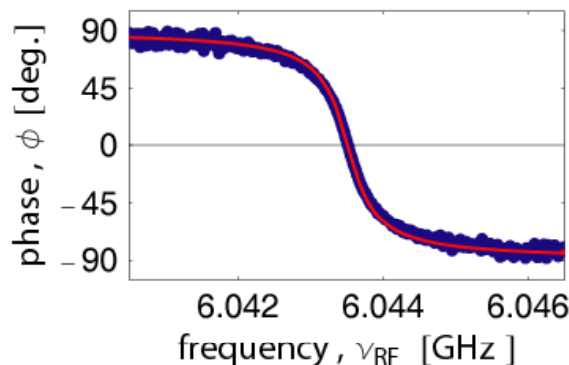


resonance frequency:

$$\nu_r = 6.04 \text{ GHz}$$

quality factor:

$$Q = \frac{\nu_r}{\delta\nu_r} \approx 10^4$$



photon decay rate:

$$\frac{\kappa}{2\pi} = \frac{\nu_r}{Q} \approx 0.8 \text{ MHz}$$

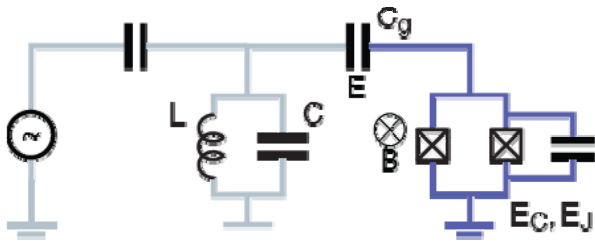
photon lifetime:

$$T_\kappa = 1/\kappa \approx 200 \text{ ns}$$

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# Qubit/Photon Coupling in a Circuit



qubit coupled to resonator

coupling strength:

$$\hbar g = eV_{0,rms} \frac{C_g}{C_\Sigma}$$

$$\Rightarrow \nu_{vac} = \frac{g}{\pi} \approx 1 \dots 300 \text{ MHz}$$

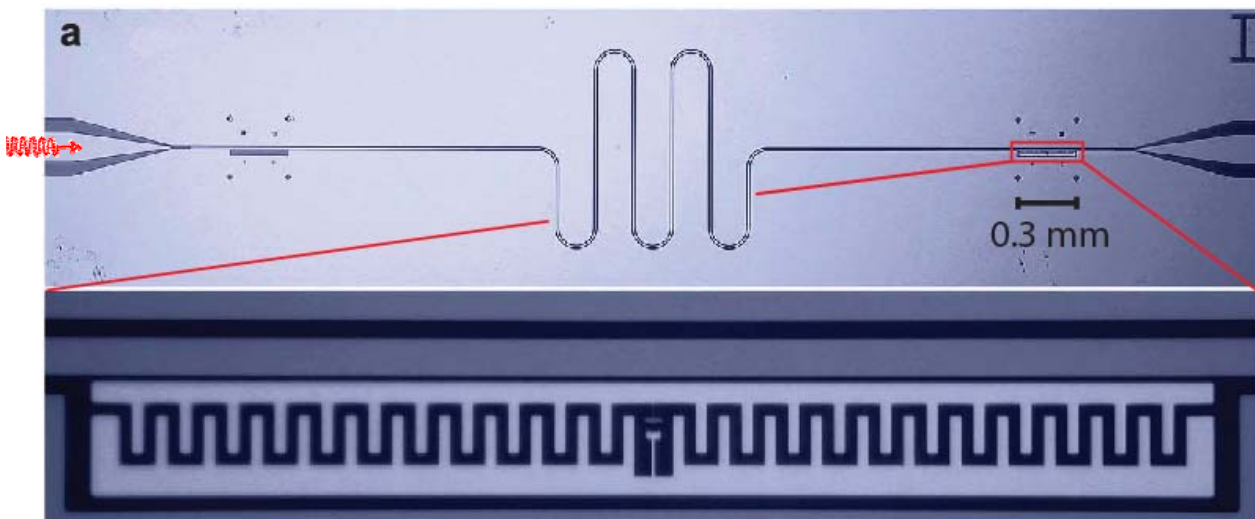
$g \gg [\kappa, \gamma]$  possible!



large effective dipole moment

$$d = \frac{\hbar g}{E_0} \sim 10^2 \dots 10^4 e a_0$$

# Circuit QED with One Photon

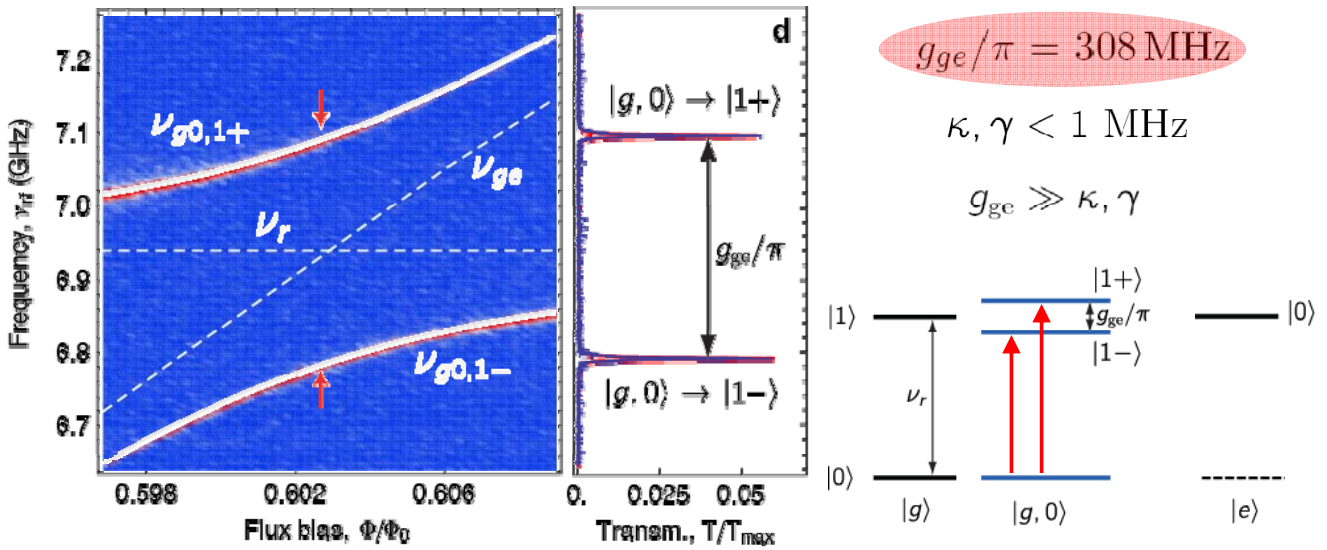


superconducting cavity QED circuit

# Resonant Vacuum Rabi Mode Splitting ...

... with one photon ( $n = 1$ ):

very strong coupling:



forming a 'molecule' of a qubit and a photon

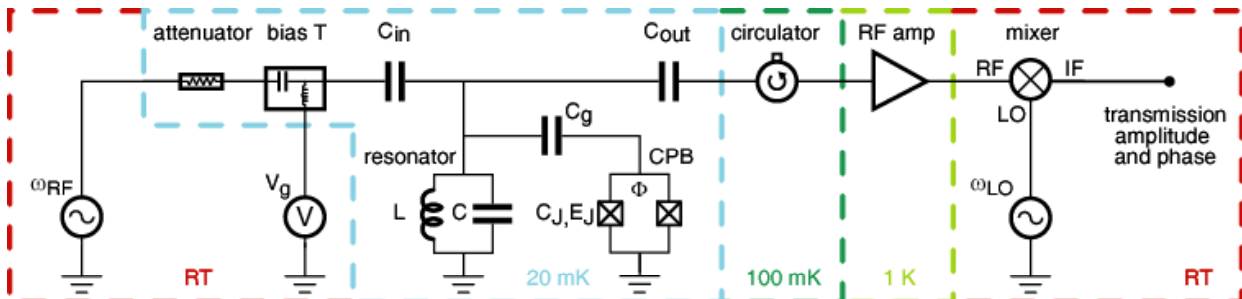
$$|1\pm\rangle = (|g, 1\rangle \pm |e, 0\rangle) / \sqrt{2}$$

ETH first demonstration: A. Wallraff, ... and R. J. Schoelkopf, *Nature (London)* **431**, 162 (2004)  
 Eidgenössische Technische Hochschule Zürich  
 Swiss Federal Institute of Technology Zurich  
 this data: J. Fink et al., *Nature (London)* **454**, 315 (2008)

# How to Measure Single Microwave Photons

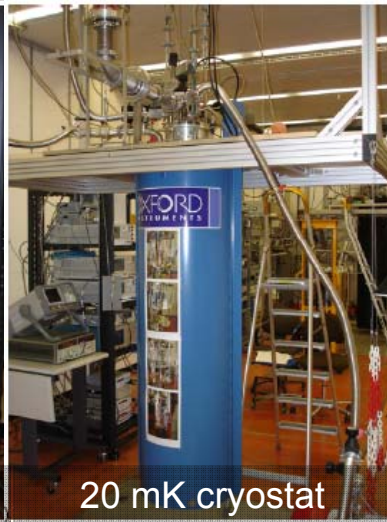
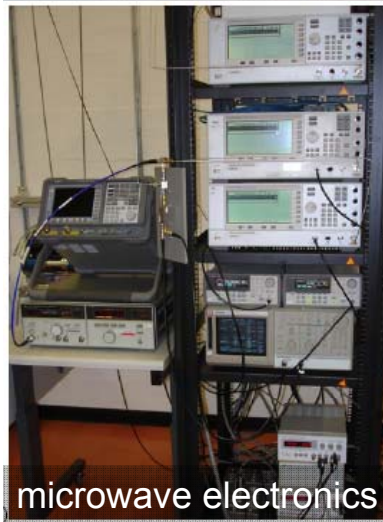
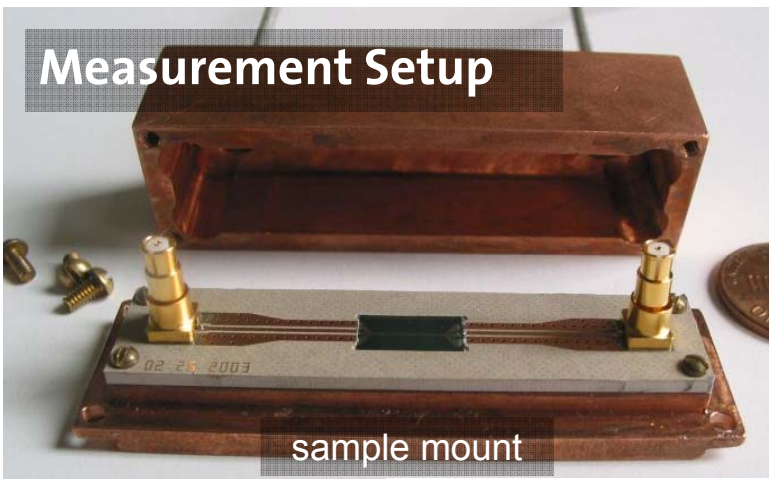
- average power to be detected

$$\rightarrow \langle n = 1 \rangle \hbar \omega_r \kappa / 2 \approx P_{RF} = -140 \text{ dBm} = 10^{-17} \text{ W}$$



- efficient with cryogenic low noise HEMT amplifier ( $T_N = 6$  K)
- prevent leakage of thermal photons (cold attenuators and circulators)

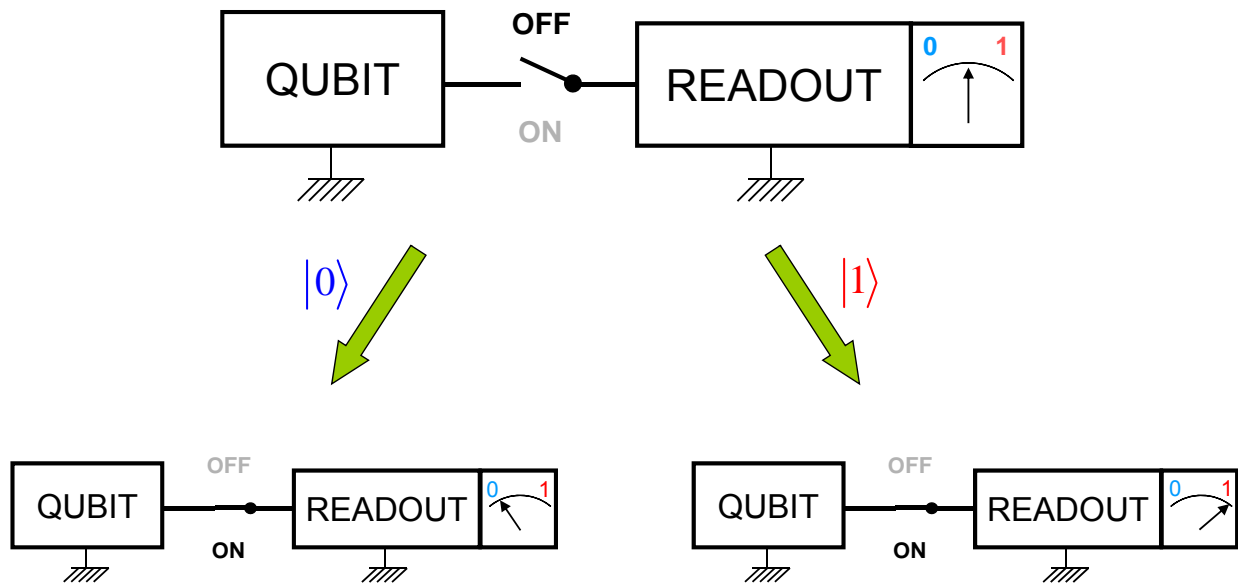




## Read-Out ...

... of a superconducting charge qubit

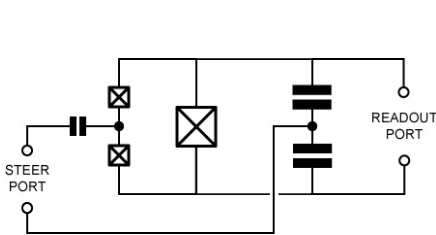
# Qubit Read Out



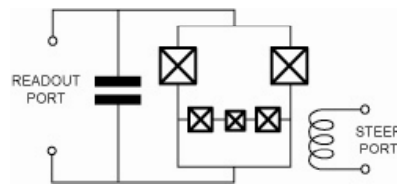
desired: good on/off ratio  
no relaxation in on state (QND)

# Read Out Strategies

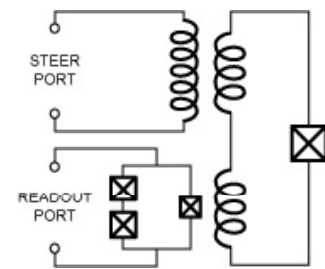
demolition measurements (switching/latching measurements)



Quantonium (Saclay, Yale)

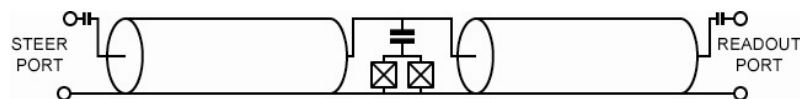


Flux Qubit (TU Delft, NEC)



Phase Qubit (NIST, UCSB)

quantum non-demolition (QND) measurements



Yale (circuit QED)

also: Chalmers, Delft, Yale (JBA)

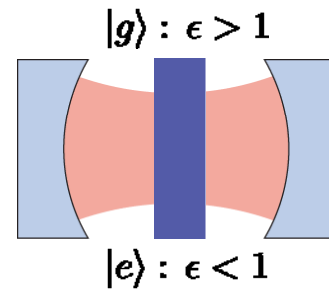
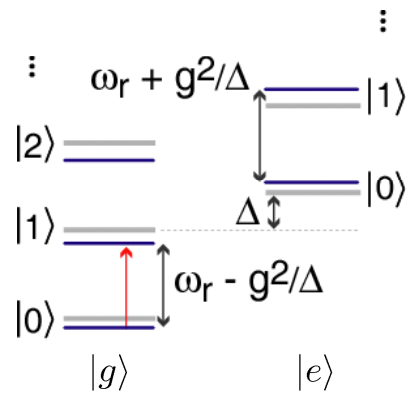
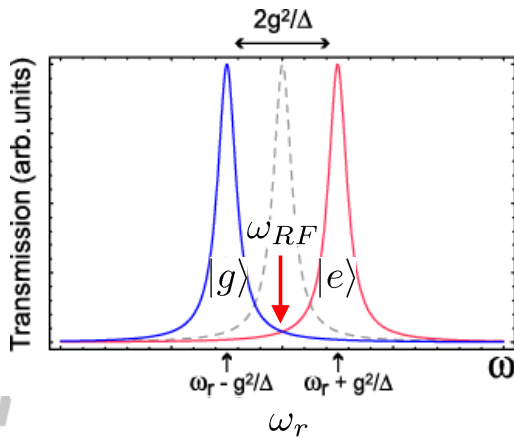
# Non-Resonant Qubit-Photon Interaction

approximate diagonalization in the dispersive limit  $|\Delta| = |\omega_a - \omega_r| \gg g$

$$H \approx \hbar \left( \omega_r + \frac{g^2}{\Delta} \sigma_z \right) a^\dagger a + \frac{1}{2} \hbar \left( \omega_a + \frac{g^2}{\Delta} \right) \sigma_z$$

//    //

**cavity frequency shift  
and qubit ac-Stark shift**    **Lamb shift**



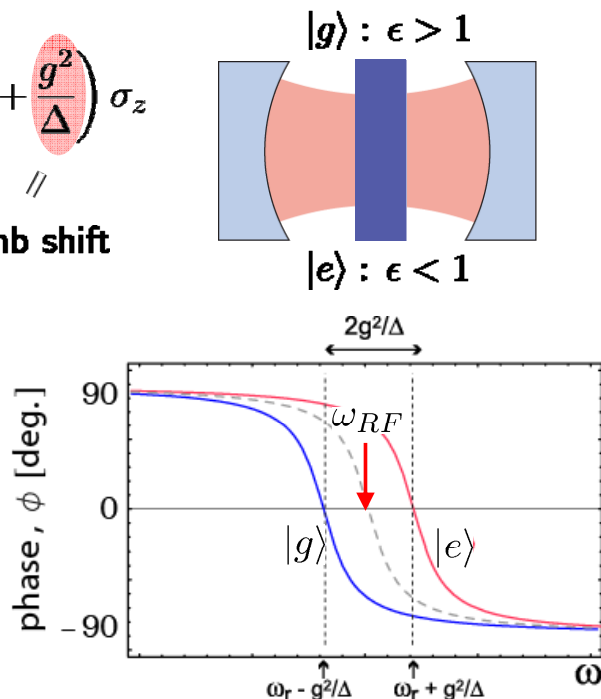
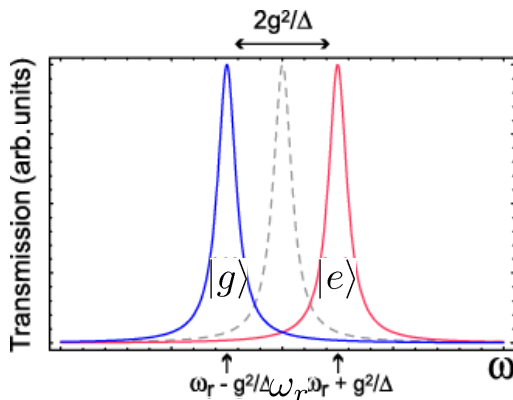
# Non-Resonant Qubit-Photon Interaction

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//    //

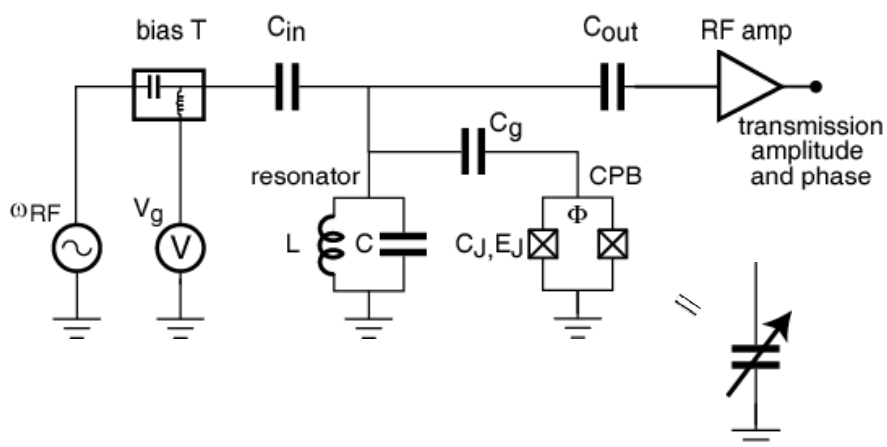
**cavity frequency shift  
and qubit ac-Stark shift**    **Lamb shift**



# Qubit Spectroscopy with Dispersive Read-Out ...

... additional material

## Measurement Technique



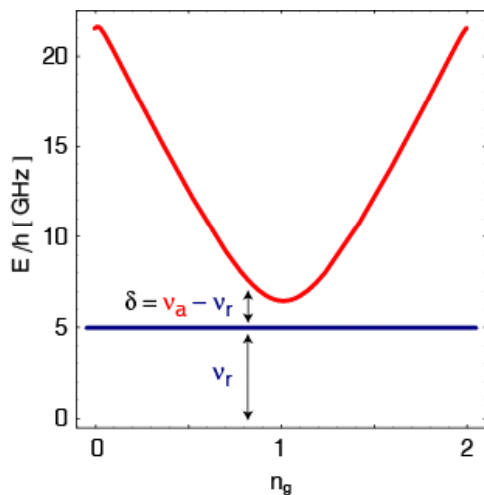
- measurement of microwave transmission amplitude  $T$  and phase  $\phi$
- intra-cavity photon number controllable from  $n \sim 10^3$  to  $n \ll 1$



# Dispersive Shift of Resonance Frequency

sketch of qubit level separation:

$$\Delta = 2\pi\delta > g$$

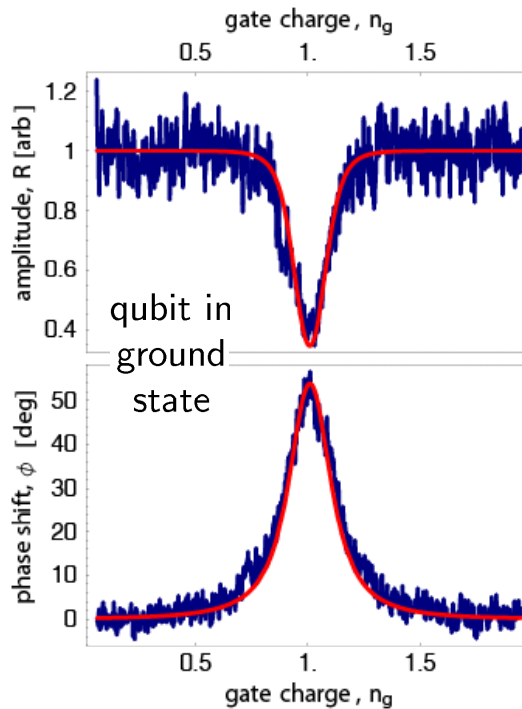


$$g/\pi = \nu_{\text{vac}} = 11 \text{ MHz}$$

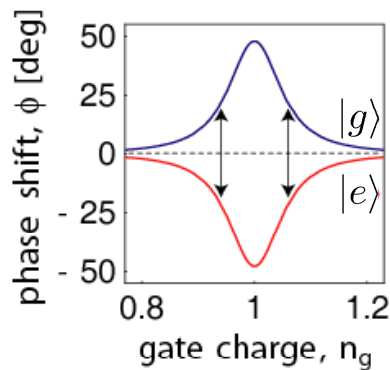
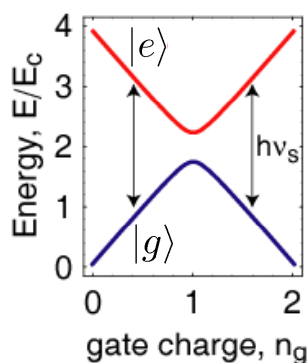
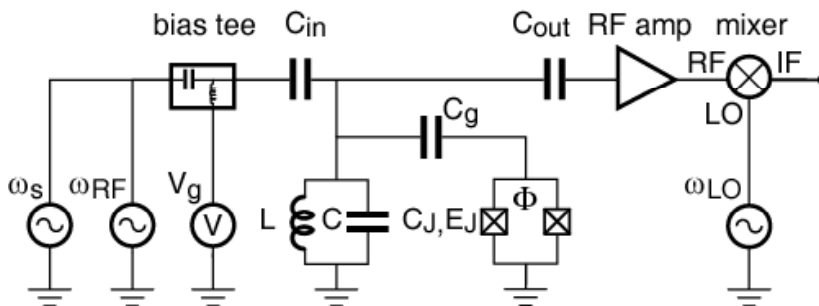
$$\Delta(n_g = 1)/2\pi = 66 \text{ MHz}$$

$$n = 10$$

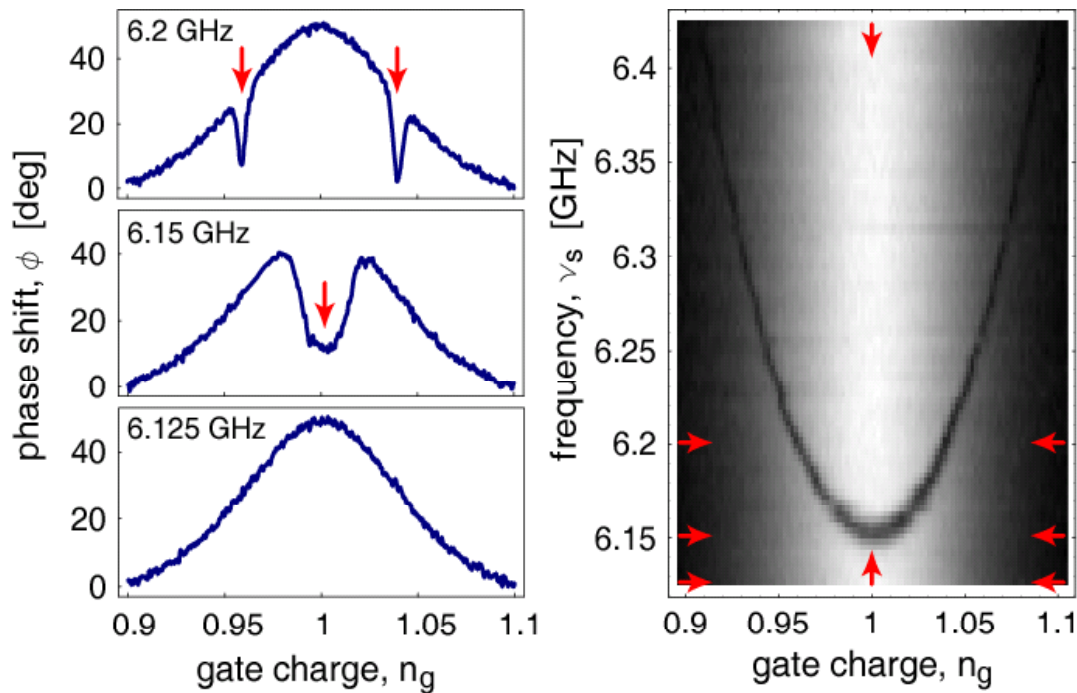
measured resonator transmission amplitude and phase:



# Qubit Spectroscopy with Dispersive Read-Out



# CW Spectroscopy of Cooper Pair Box

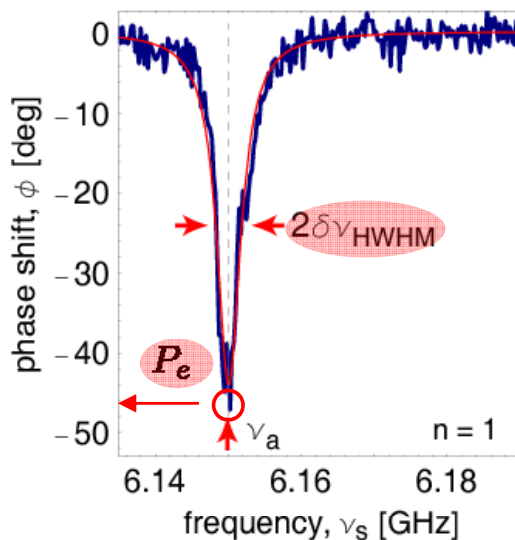


detuning  $\Delta_{r,a}/2\pi \sim 100$  MHz      extracted:  $E_J = 6.2$  GHz,  $E_C = 4.8$  GHz

## Line Shape

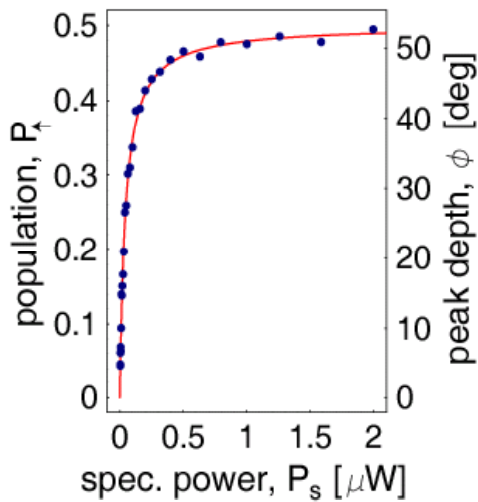
excited state population (steady-state Bloch equations):

$$P_e = 1 - P_g = \frac{1}{2} \frac{\Omega_R^2 T_1 T_2}{1 + (T_2 \Delta_{s,a})^2 + \Omega_R^2 T_1 T_2}$$



- fixed drive  $P_s \propto \Omega_R^2 = n_s \omega_{\text{vac}}^2$
- varying  $\Delta_{s,a} = \omega_s - \tilde{\omega}_a$
- weak continuous measurement ( $n \sim 1$ )
- at charge degeneracy ( $n_g = 1$ )

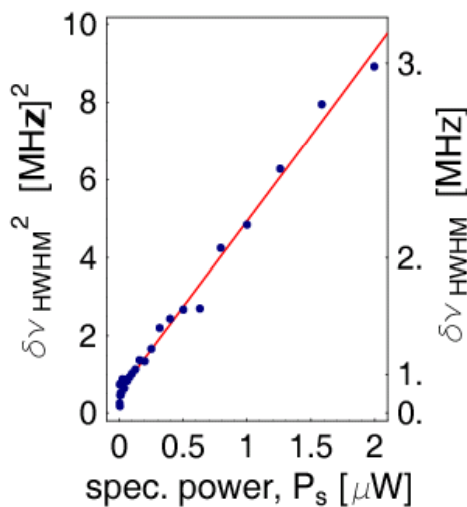
# Excited State Population



peak depth  $\rightarrow$  population (saturation):

$$P_e = 1 - P_g = \frac{1}{2} \frac{\Omega_R^2 T_1 T_2}{1 + \Omega_R^2 T_1 T_2}$$

# Line Width

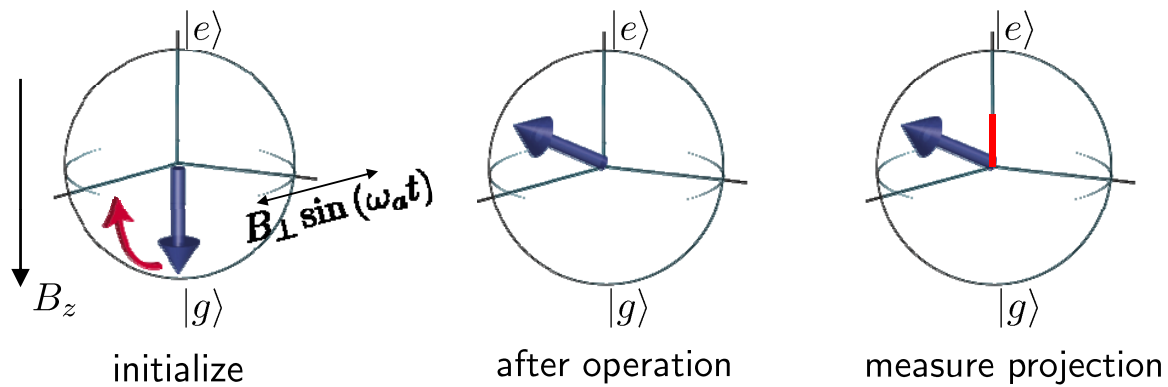


line width  $\rightarrow$  coherence time:

$$2\pi\delta\nu_{\text{HWHM}} = \frac{1}{T_2'} = \sqrt{\frac{1}{T_2^2} + \Omega_R^2 \frac{T_1}{T_2}}$$

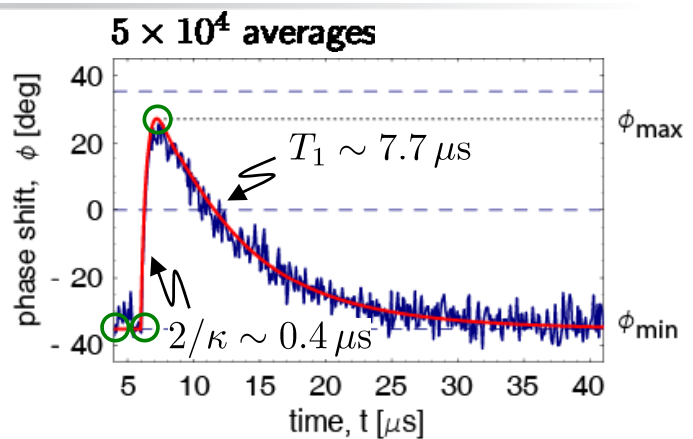
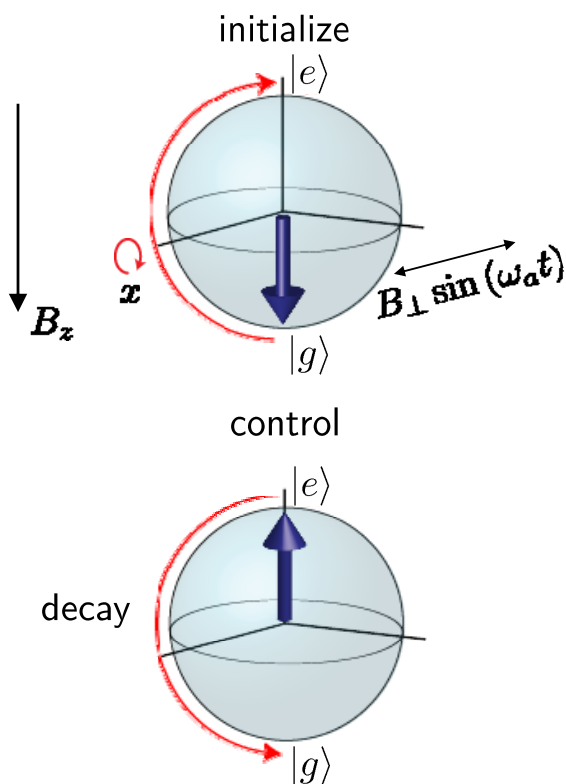
$\text{Min}(\delta\nu_{\text{HWHM}}) \sim 750 \text{ kHz} \rightarrow T_2 > 200 \text{ ns}$

# Coherent Control of a Qubit in a Cavity



- qubit state represented on a Bloch sphere
- vary length, amplitude and phase of microwave pulse to control qubit state

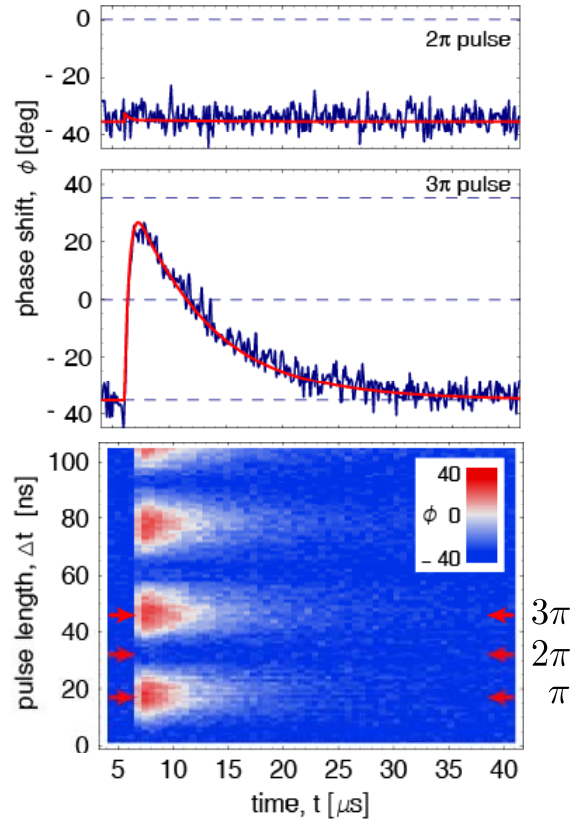
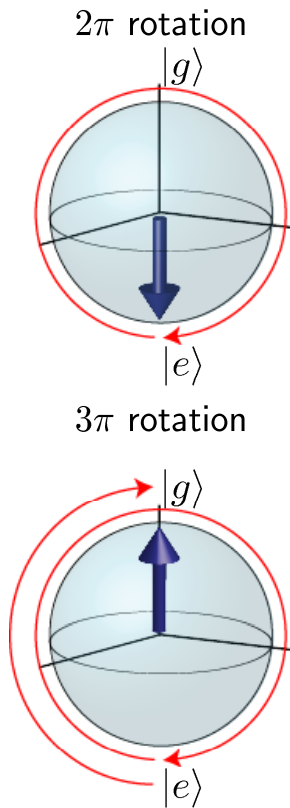
# Qubit Control and Readout



measurement properties:

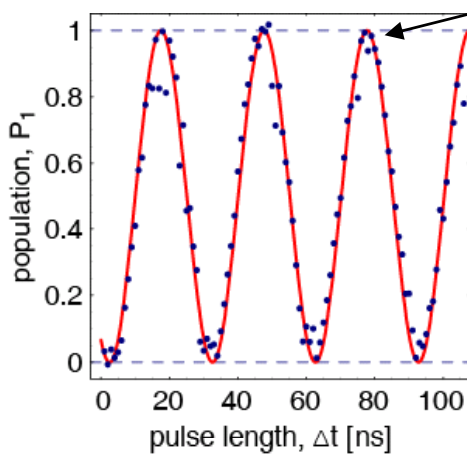
- continuous
- dispersive
- quantum non-demolition
- in good agreement with predictions

# Varying the Control Pulse Length



# High Visibility Rabi Oscillations

Rabi oscillations:



visibility  $95 \pm 5\%$

for superconducting qubits:

- high visibility
- well characterized and understood measurement
- good control accuracy

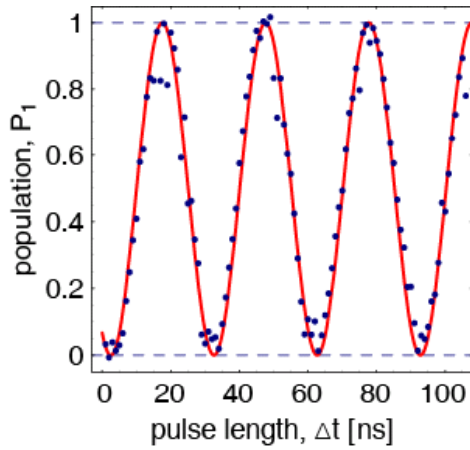


# Rabi Frequency

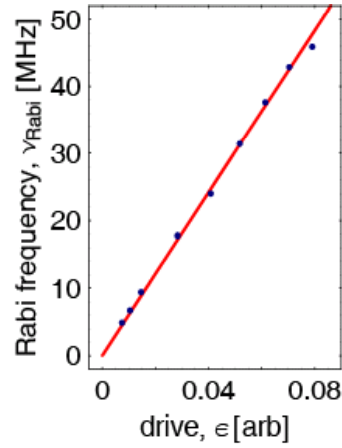
pulse scheme:



Rabi oscillations:



Rabi frequency:

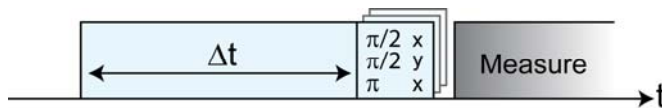


- linear dependence of Rabi frequency on microwave amplitude

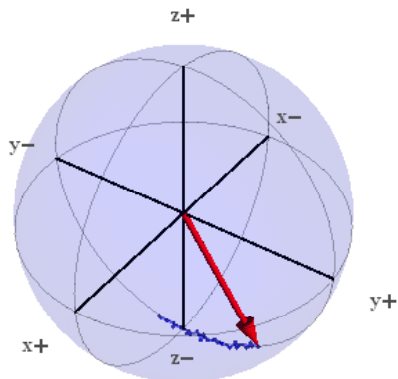
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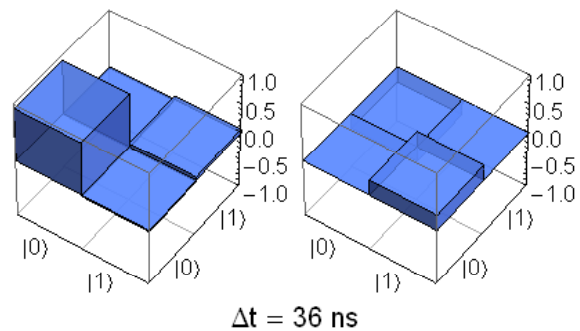
Rabi rotation pulse sequence:



experimental Bloch vector:



experimental density matrix:



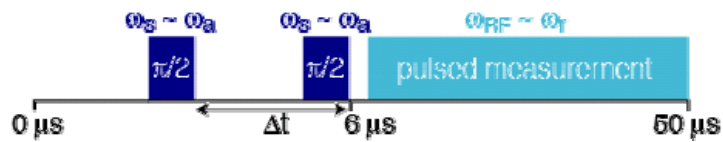
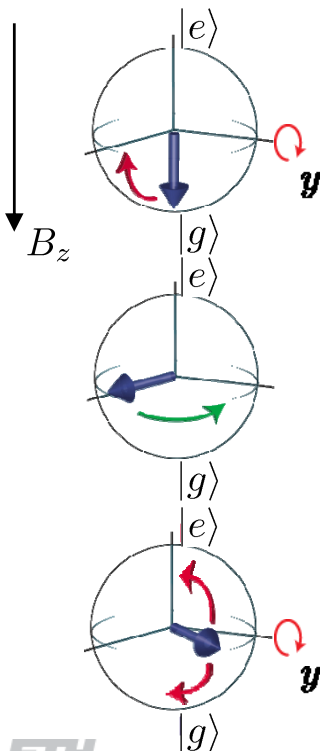
ETH

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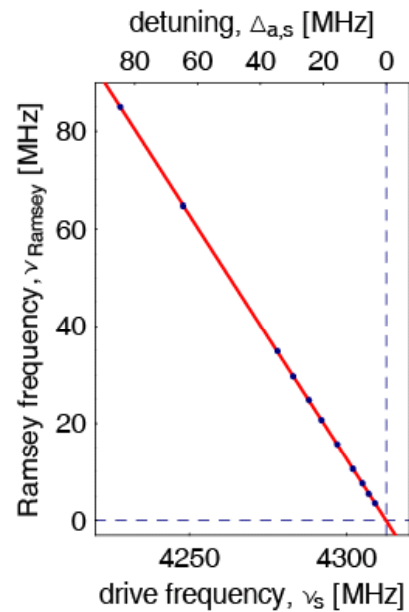
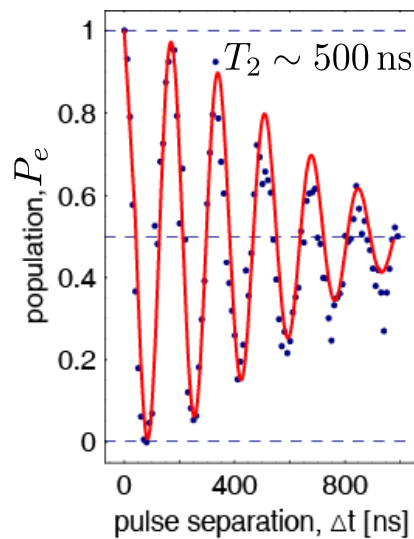
# Measurements of Coherence Time

## Coherence Time Measurement: Ramsey Fringes

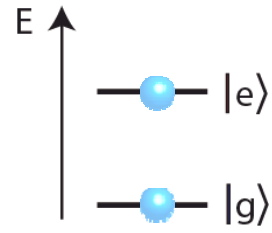
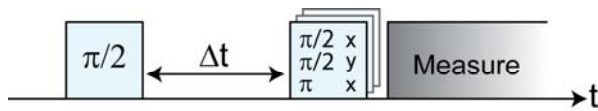
pulse scheme:



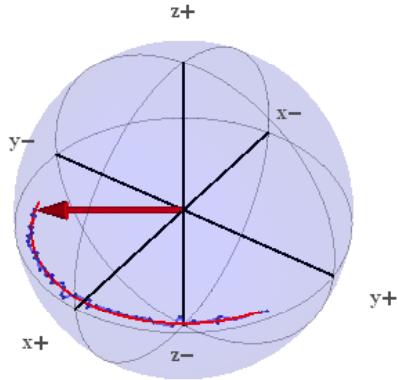
Ramsey fringes:



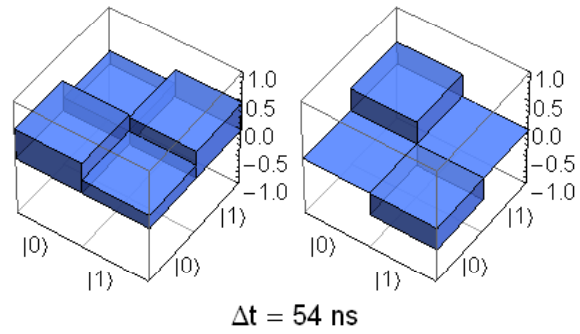
pulse sequence:



experimental Bloch vector:

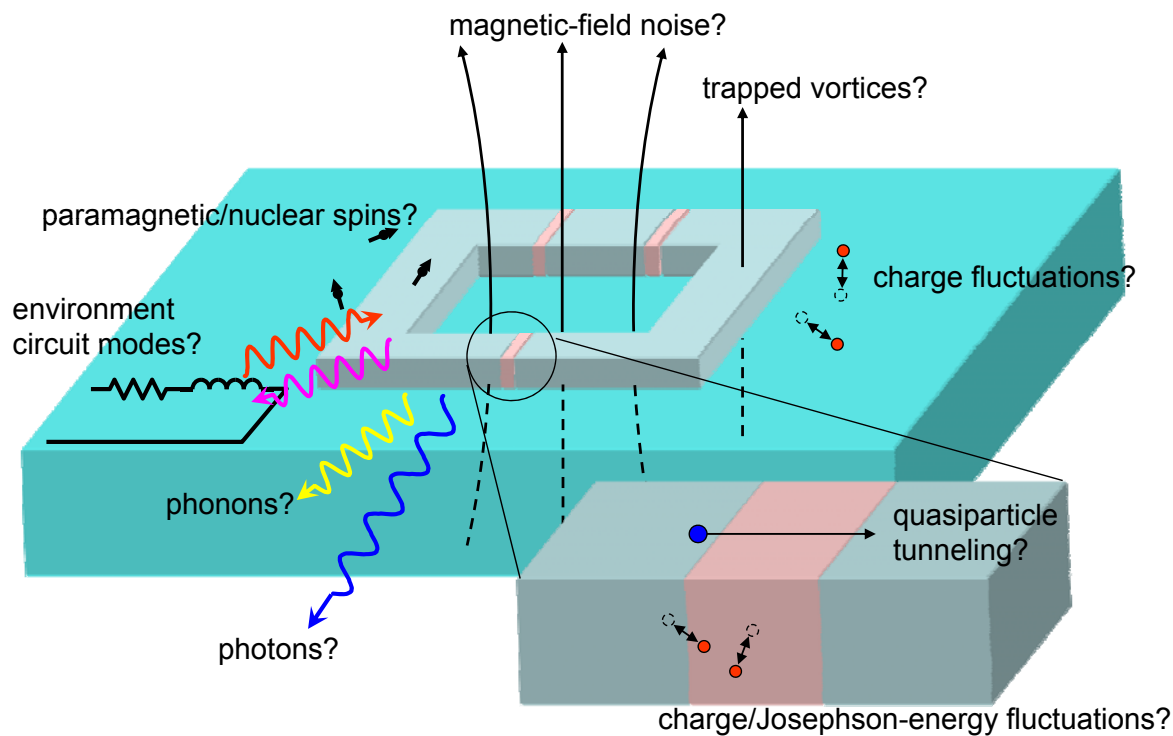


experimental density matrix:



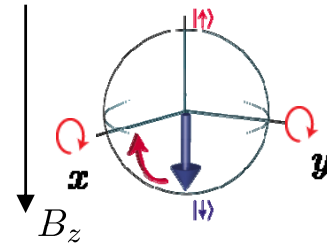
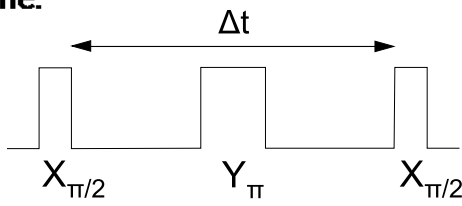
**Decoherence ...**  
**... additional material**

# Sources of Decoherence

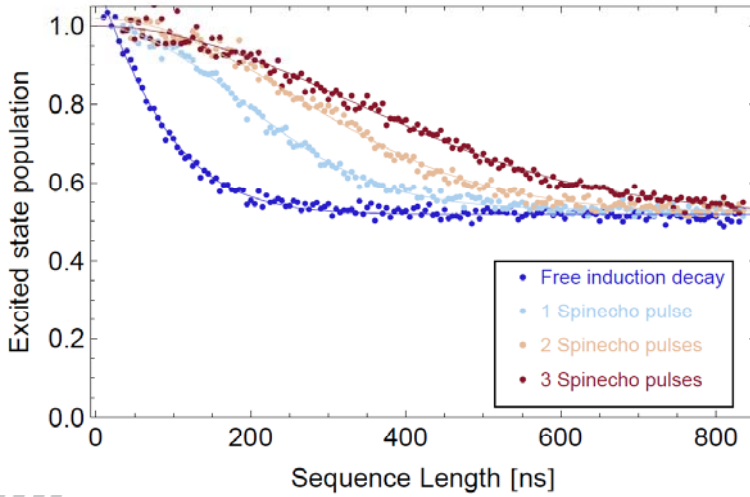


- remove sources of decoherence
  - improve materials
- use dynamic methods to counteract specific sources of decoherence
  - spin echo
  - geometric manipulations
- reduce sensitivity of quantum systems to specific sources of decoherence
  - make use of symmetries in design and operation

**pulse scheme:**



**result:**

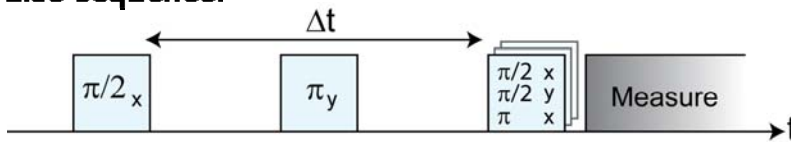


- refocusing
- elimination of low frequency fluctuations
- increased effective coherence time

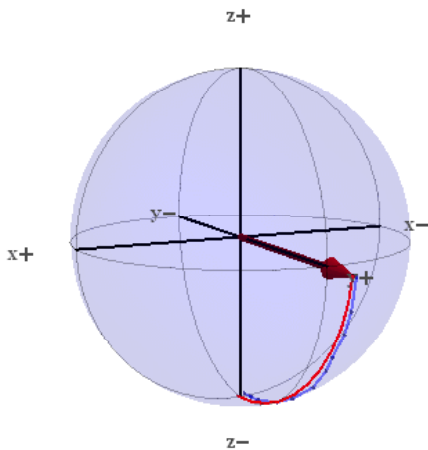


## Tomography of a Spin Echo

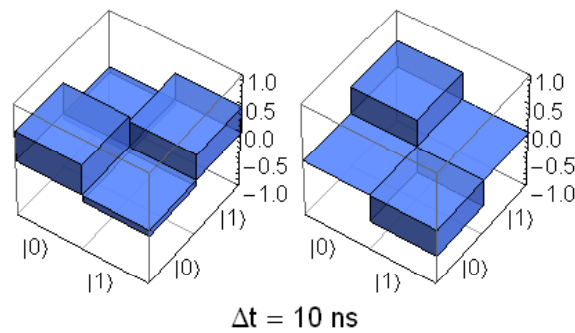
**pulse sequence:**



**experimental Bloch vector:**



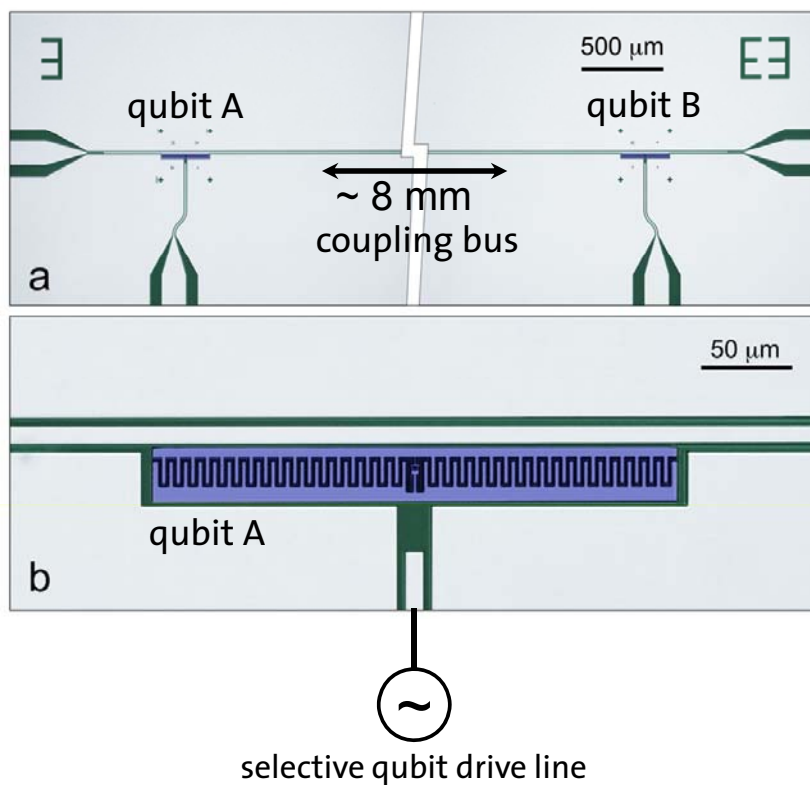
**experimental density matrix:**





# Coupling Superconducting Qubits and Generating Entanglement using Sideband Transitions

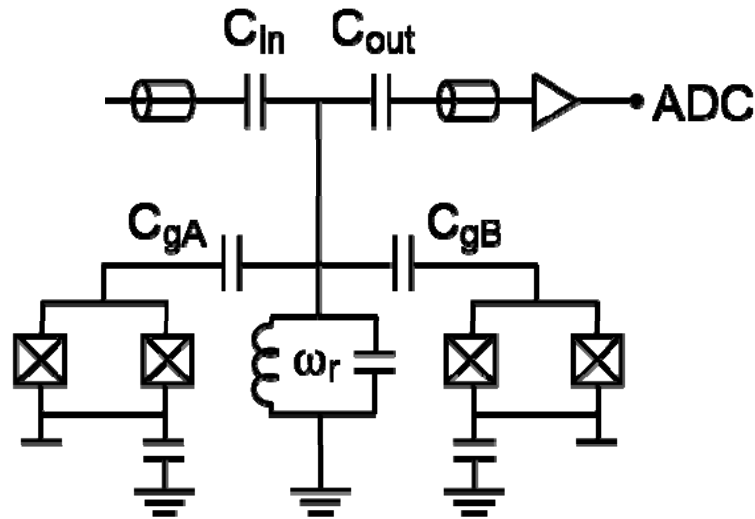
## 2-Qubit Chip



- Two near identical superconducting qubits
- Local control of magnetic flux allows independent selection of qubit transition frequencies
- Local drive lines allow selective excitation of individual qubits

## 2-Qubit Circuit with Selective Control

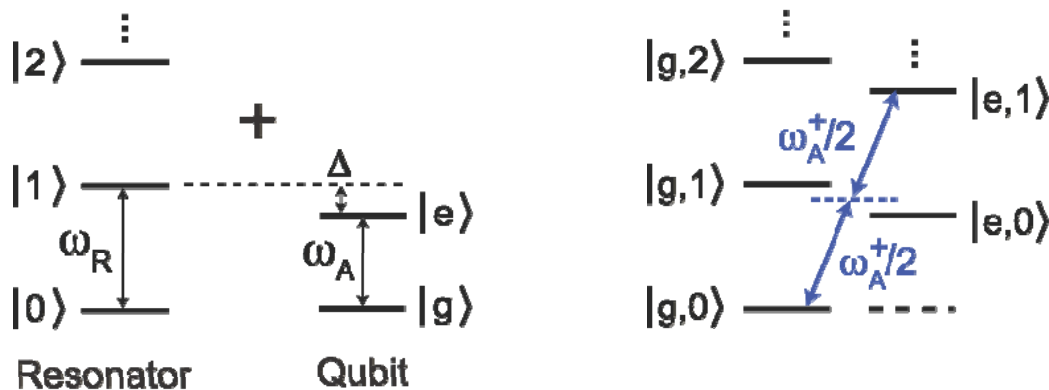
joint dispersive  
read-out



Local magnetic  
fields created  
using small  
inductively  
coupled coils

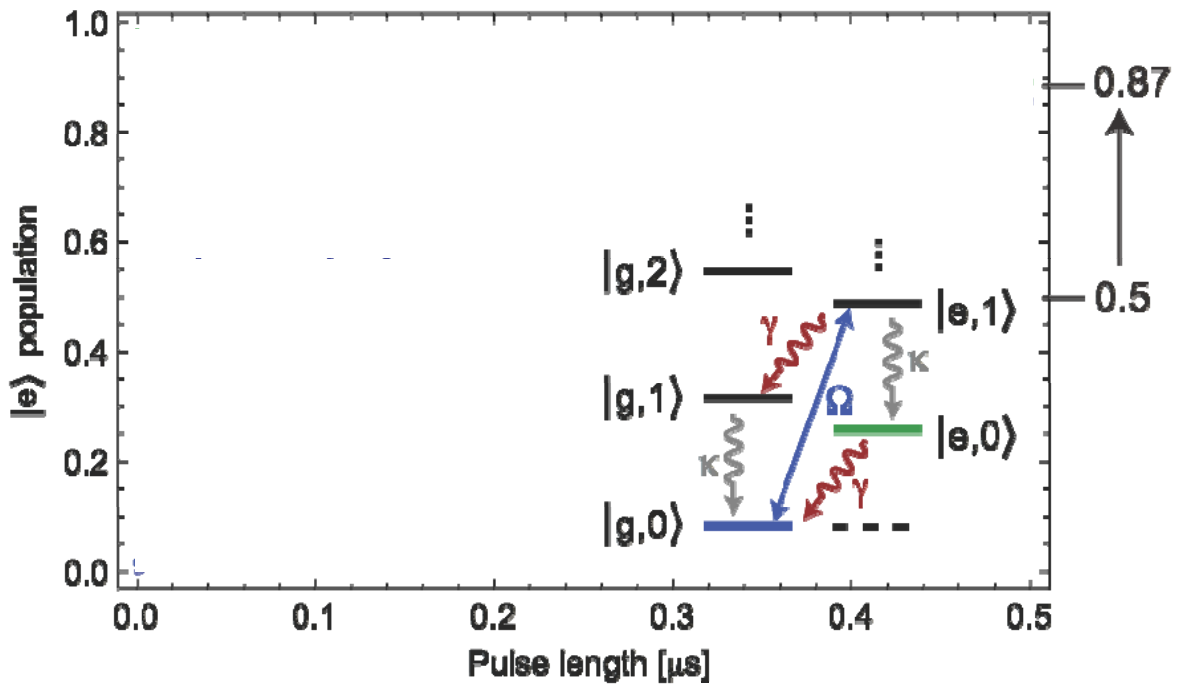
Selective qubit excitation  
using locally capacitively  
coupled drive lines

## Sideband Transitions in Circuit QED



$$\omega_A^+/2 = (\omega_R + \omega_A)/2$$

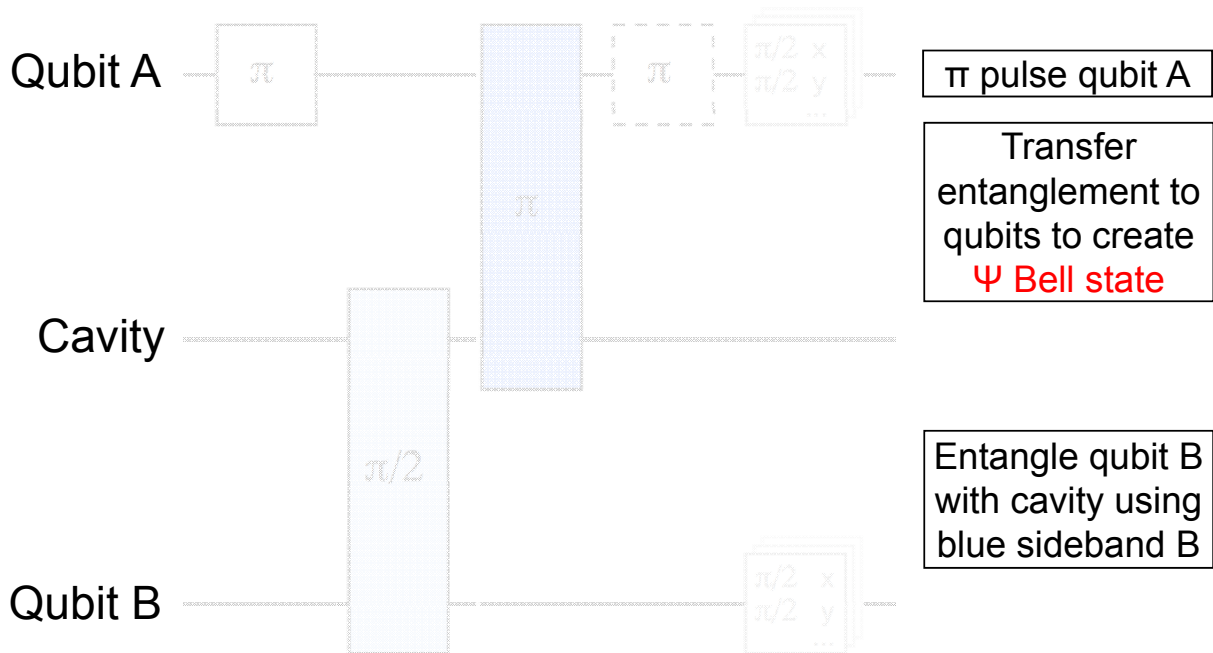
- › dispersive coupling allows joint excitations to be driven
- › sideband transitions forbidden to first order: use two photon transition



simultaneous excitation of qubit and resonator:  $|g,0\rangle \rightarrow |e,1\rangle$

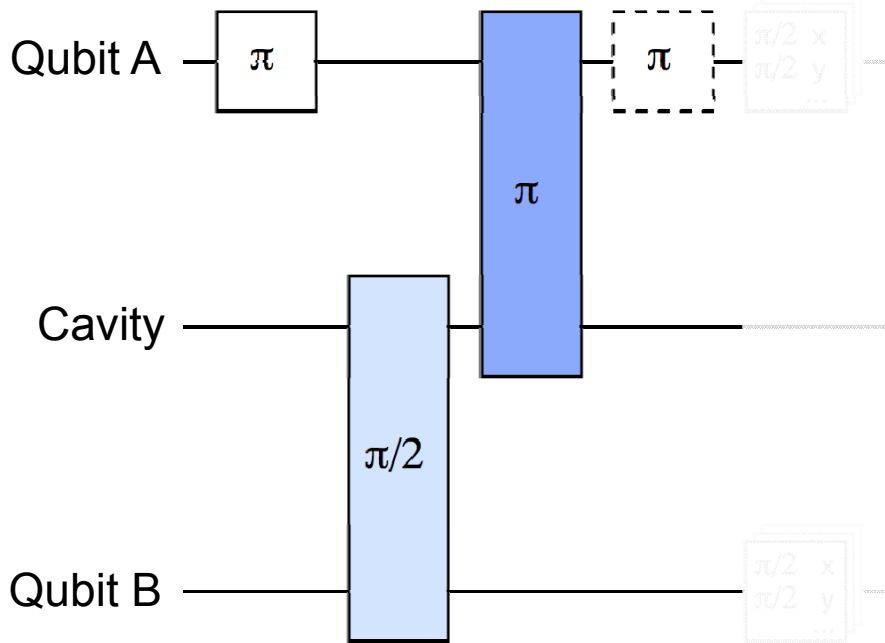
entangle a qubit with a photon on the bus:  $|g,0\rangle \rightarrow |g,0\rangle + |e,1\rangle$

## Bell State Preparation



$$|gg0\rangle \rightarrow |eg0\rangle \rightarrow \frac{1}{\sqrt{2}}(|eg0\rangle + |ee1\rangle) \rightarrow \frac{1}{\sqrt{2}}(|eg\rangle + |ge\rangle) \otimes |0\rangle$$

# Bell State Preparation



$\pi$  pulse qubit A to convert to  $\Phi$  Bell state

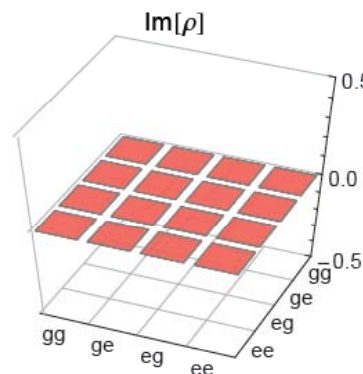
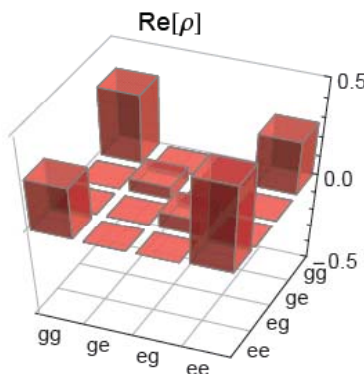
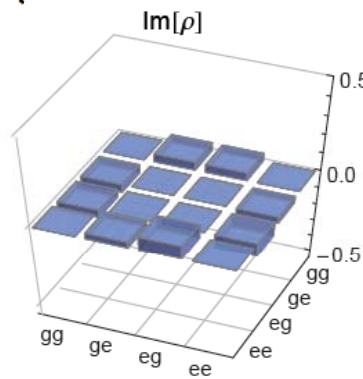
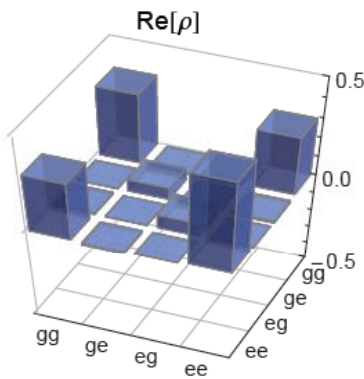
$\pi$  pulse qubit A

Transfer entanglement to qubit A to create  $\Psi$  Bell state

Characterise the entanglement with cavity state tomography and joint msmnt

$$\dots \rightarrow \frac{1}{\sqrt{2}}(|eg\rangle + |ge\rangle) \otimes |0\rangle \rightarrow \frac{1}{\sqrt{2}}(|gg\rangle + |ee\rangle) \otimes |0\rangle$$

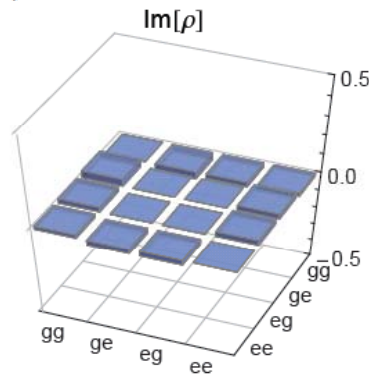
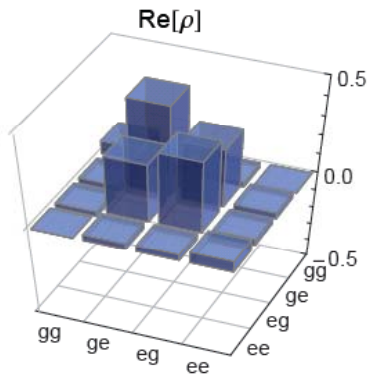
$$|\Phi_+\rangle = \frac{1}{\sqrt{2}}(|gg\rangle + |ee\rangle)$$



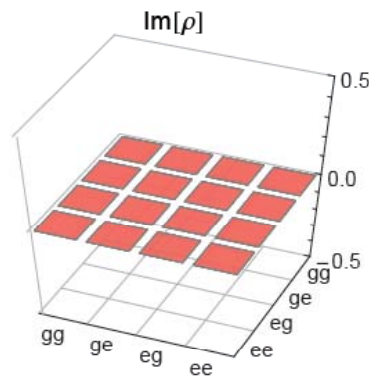
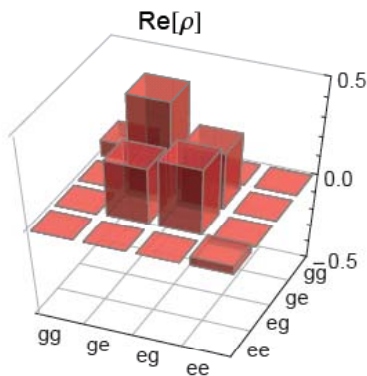
experimental state fidelity:  
F = 86%  
concurrence:  
0.541  
entanglement of formation:  
0.371

overlap with calculation  
F = 99%

$$|\Psi_+\rangle = \frac{1}{\sqrt{2}}(|ge\rangle + |eg\rangle)$$



experimental  
state fidelity:  
F = 86%  
concurrence:  
0.518  
entanglement  
of formation :  
0.374



overlap with  
calculation  
F = 99%



## DiVincenzo Criteria fulfilled for Superconducting Qubits

for Implementing a Quantum Computer in the standard (circuit approach) to quantum information processing (QIP):

- #1. A scalable physical system with well-characterized qubits. ✓
- #2. The ability to initialize the state of the qubits. ✓
- #3. Long (relative) decoherence times, much longer than the gate-operation time. ✓
- #4. A universal set of quantum gates. ✓
- #5. A qubit-specific measurement capability. ✓

plus two criteria requiring the possibility to transmit information:

- #6. The ability to interconvert stationary and mobile (or flying) qubits. ✓
- #7. The ability to faithfully transmit flying qubits between specified locations. ✓