QSIT 2010 - Questions 5

3. November 2010

1. **Partial Trace**
   A two-qubit system is in the state $|\Psi(\theta)\rangle = \cos\theta|0\rangle_A|1\rangle_B + \sin\theta|1\rangle_A|1\rangle_B$.

   (a) Calculate the reduced state for subsystem A, $\rho_A = \mathrm{tr}_B\left[|\Psi(\theta)\rangle\langle\Psi(\theta)|\right]$, via the partial trace.

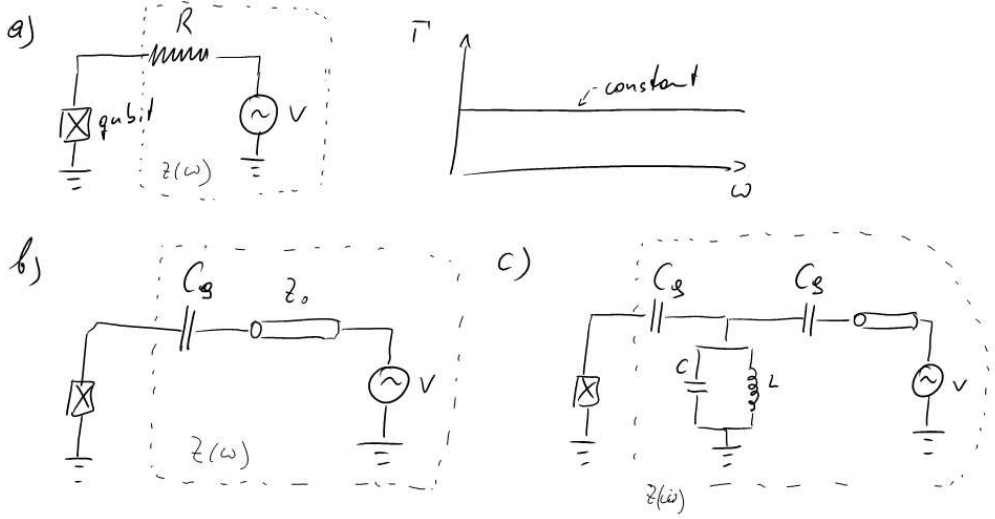
   (b) For which values of θ is ρ_A a pure state? For what θ is ρ_A maximally mixed and how does this relate to the entanglement of the total state $|\Psi\rangle$?

   (c) For physical consistency, the definition of the reduced state ρ_A has to yield the correct measurement statistics for measurements on subsystem A. Show that

   $$\langle\Psi|(\hat{M}_A \otimes id)|\Psi\rangle = \mathrm{Tr}[\hat{M}_A \rho_A]$$

   for the measurement operator $\hat{M}_A$ acting on system A only.

2. **Energy relaxation of a qubit**
   In analogy to the harmonic oscillator, the energy decay time of a qubit is given by $T_1 = RC$, where C is the intrinsic capacitance of the qubit. R denotes the effective resistance $R = 1/\mathrm{Re}[Y(\omega)]$ obtained from the impedance of the environment $Z(\omega) = 1/Y(\omega)$ as seen from the position of the qubit.



If the impedance of the environment is purely resistive, e.g. $Z(\omega) = 50\ \Omega$, the decay rate $\Gamma = 1/T_1$ is frequency independent (see Figure a).

(a) Derive the impedance of a Cooper-pair box qubit that is capacitively coupled to a transmission line ($Z_0 = 50\ \Omega$) via a gate capacitance C_g (Figure b). Sketch the decay rate Γ as a function of frequency.

(b) What is the spectral shape of Γ for a coupling to an LC oscillator (Figure c)?