QIP II: Implementations, FS 2017 - Questions 2

March 6, 2017

1) Rabi oscillations:

A single atom in a laser field interacts with the light via a dipole interaction and can be described by the Hamiltonian $H = H_{atom} + H_{light} + H_{int}$ with the interaction given by

$$H_{int} = -\hat{\boldsymbol{d}} \cdot \mathcal{E} = -\hat{d}\,\mathcal{E}_0 \cos(\omega_l t), \tag{1}$$

where $\hat{d}$ is the dipole operator for the atom and $\mathcal{E}_0$ is the electric field strength of the laser field. When the laser frequency is on resonance with only two levels of the atom, the rotating wave approximation can be applied and the system can be described by the Hamiltonian:

$$H = \frac{\Omega}{2}\sigma_x, \tag{2}$$

where σ_x is the Pauli x operator. The same Hamiltonian also describes a superconducting qubit connected to a microwave drive.

- Argue how to get from Eq. (1) to Eq. (2)
- Show that the time evolution operator can be written on the form

$$U(t) = \begin{bmatrix} \cos\frac{1}{2}\Omega t & -i\sin\frac{1}{2}\Omega t \\ -i\sin\frac{1}{2}\Omega t & \cos\frac{1}{2}\Omega t \end{bmatrix} \tag{3}$$

- If you start in the ground state, $|0\rangle$, what is qubit state at a later time, t? Why is Ω called the Rabi frequency?
- Illustrate this on the Bloch sphere. What do you think a π-pulse is?
- Bonus question: What happens if σ_x is replaced by σ_y?
- Bonus question: In an experiment the qubit may be detuned from the drive, which is described by Hamiltonian is

$$H = \frac{\Omega}{2}\sigma_x + \frac{\Delta}{2}\sigma_z \tag{4}$$

  with σ_z the Pauli z operator. What happens in this case?

2) Ramsey fringes:

A particular quantum interference effect can appear when two $\pi/2$-pulses are applied to a qubit sequentially. Since this interference relies on quantum coherence, it is very useful tool to measure the decoherence time of a qubit. This method is known as a Ramsay experiment and consists of three steps:

1. Starting from the ground state $|0\rangle$, the state $|+\rangle = (|0\rangle + |1\rangle)/\sqrt{2}$ is prepared by a Hadamard gate,

$$H = \frac{1}{\sqrt{2}}\begin{bmatrix} 1 & 1 \\ 1 & -1. \end{bmatrix} \tag{5}$$

   Discuss how you would do this using the results of the previous exercise.

2. Wait for a time τ.

3. Apply another Hadamard gate.

Now, assume that the Hadamard gates are perfect and that there is a detuning, Δ, of the qubit. The Hamiltonian during step 2 is then described as

$$H = \frac{\Delta}{2}\sigma_z. \tag{6}$$

Show that after the sequence, the ground state population is $p_0(\tau) = \frac{1}{2}[1+\cos(\Delta t)]$. Illustrate on the Bloch sphere. Discuss the effect of decoherence.