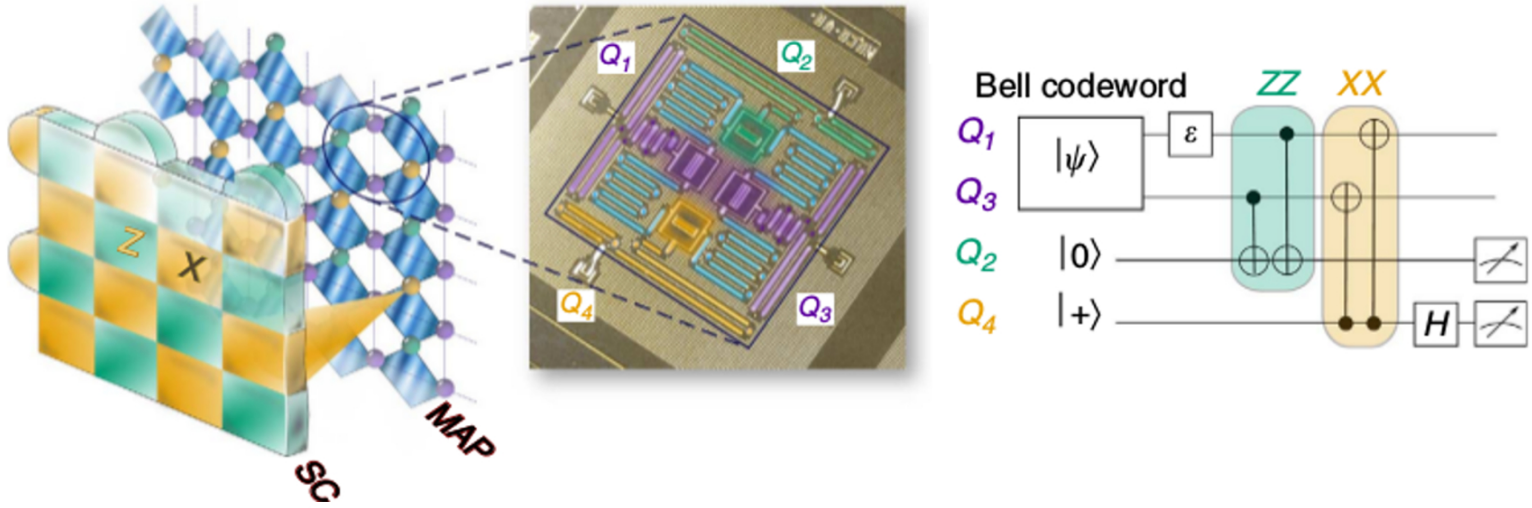

Demonstration of a quantum error detection code using a square lattice of four superconducting qubits (2015)

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Outline

- Quantum Error Correction
  - Syndrome extraction
  - Stabilizer formalism
- Experimental Design
  - Surface code
  - Transmon qubits
  - Physical realization
  - Implemented quantum error correction scheme
  - Functionality of stabilizers
  - Detection of general errors
- Results
  - Single-shot histograms and state tomography
- Conclusions

Classical error correction

- Classically, only one possible type of error: bit flip ($0 \leftrightarrow 1$)
- Easy solution: redundant coding, e. g.,

$$0_L = 000\,, \quad 1_L = 111$$

- Detect and fix by parity checks (or counting)

Quantum error correction

- Can still detect bit flips, e. g., by measuring parity with $Z_1 Z_2$ and $Z_2 Z_3$ (same bit $\rightarrow$ +1):

$$Z_1 Z_2 \left|000\right\rangle = \left|000\right\rangle$$
$$Z_1 Z_2 \left|100\right\rangle = -\left|100\right\rangle$$
$$\vdots$$

- Qubits have phase: how to correct for phase flips?

$$Z_1 Z_2 \left|000\right\rangle = (+1)\left|000\right\rangle$$
$$Z_1 Z_2 (-\left|000\right\rangle) = (+1)(-\left|000\right\rangle)$$
$$\vdots$$

Quantum error correction

- Phase flips mean we need more complex encoding schemes: this is the crucial part of QEC
- Example: Shor code,

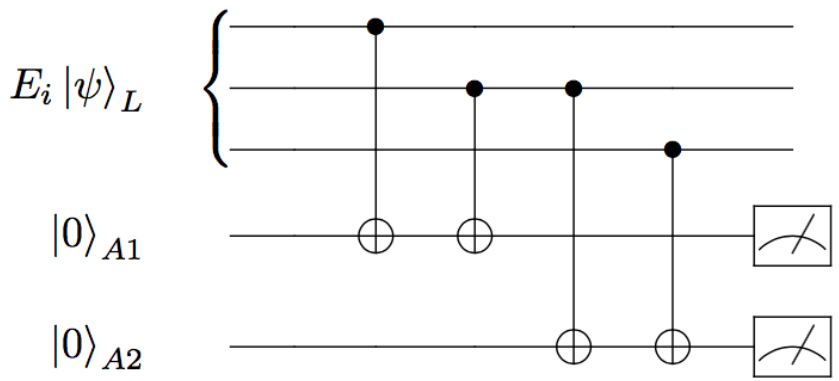
$$|0\rangle_L = \frac{1}{2\sqrt{2}}(|000\rangle + |111\rangle)(|000\rangle + |111\rangle)(|000\rangle + |111\rangle)$$

- Bit flips: $Z_i Z_{i+1}$, $Z_{i+1} Z_{i+2}$
- Phase flips: $X_1 \cdots X_6$, $X_4 \cdots X_9$

Syndrome extraction

- Another problem: measurement of quantum states is destructive
- Solution: measure correlations, output on “ancilla” qubits

Example: bit flip on $|\psi\rangle = \alpha\,|000\rangle + \beta\,|111\rangle$



Stabilizer formalism

- Stabilizer group

$$\mathcal{S} = \{S_1, S_2, \ldots, S_n\}$$

- Stabilizers commute, have eigenvalues ± 1
- Space of codewords is intersection of $\lambda = +1$ eigenspaces of S_i (stabilizers should not affect codewords)
- Allows for specific errors, whose corresponding operators anticommute with one S_i

Stabilizer formalism

- Example: 3-qubit repetition code,

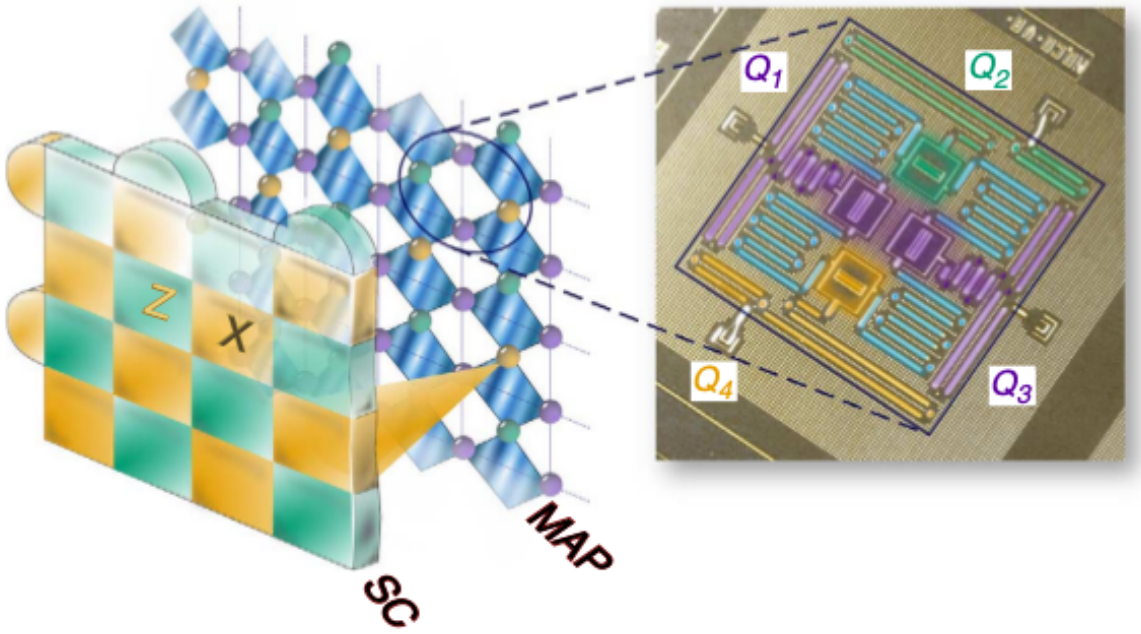
$$|0\rangle_L = |000\rangle \ , \quad |1\rangle_L = |111\rangle$$

- Stabilizer group

$$\mathcal{S} = \langle Z_1 Z_2, Z_2 Z_3 \rangle = \{ I, Z_1 Z_2, Z_2 Z_3, Z_1 Z_3 \}$$

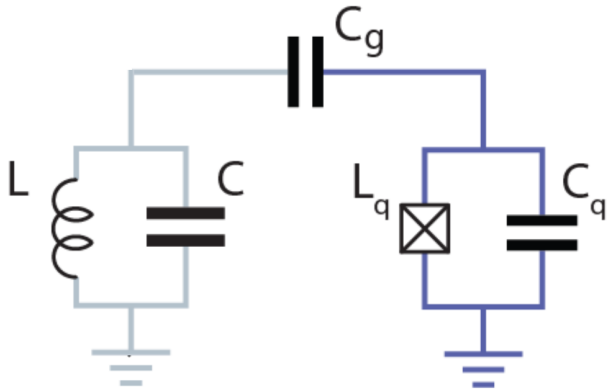
- Bit flip on qubit *i* can be detected

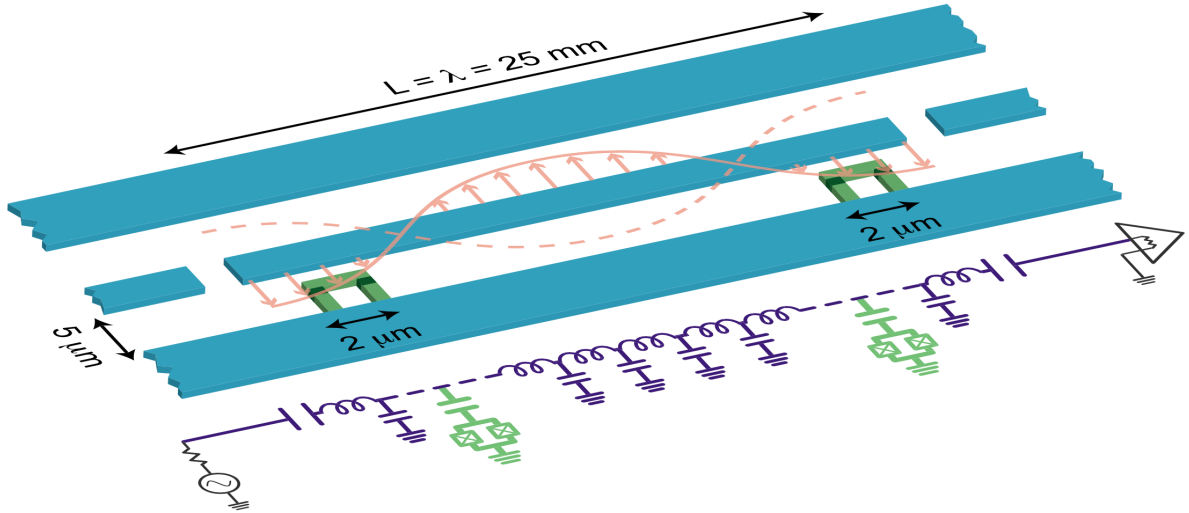
Surface code



- Code qubits (purple) and syndrome/ancilla qubits (green: bit-flip and yellow: phase-flip) at sites of 2D lattice
- Stabilizer code, nearest-neighbor connectivity, decoupled code qubits $\Rightarrow$ high fault-tolerant error thresholds, scalable geometry

Transmon qubits



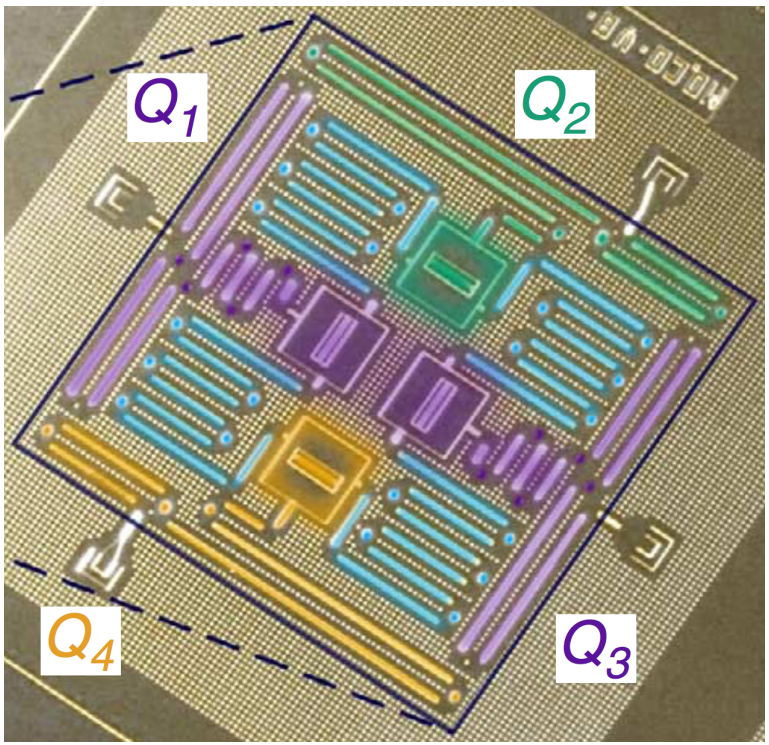


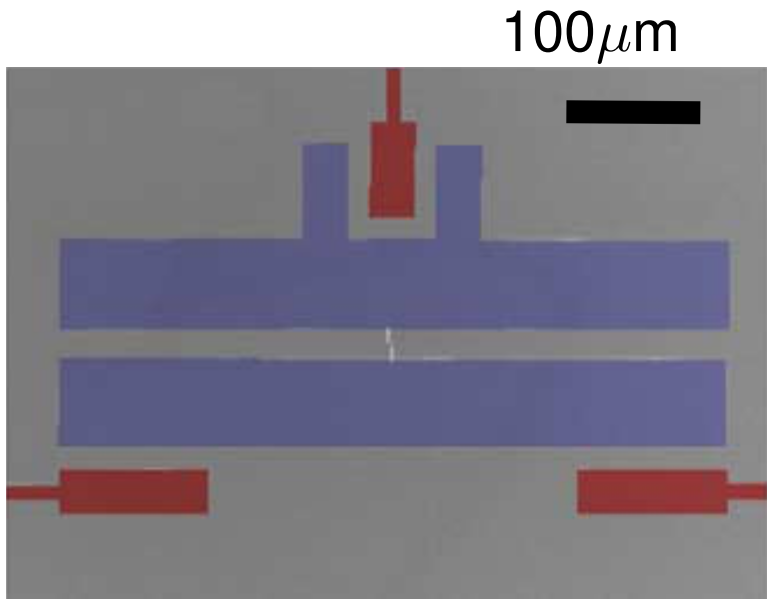
- $\{|0\rangle, |1\rangle\} \approx \{|N_g\rangle, |N_g + 1\rangle\}$ on island (Cooper pair box)
- Single-qubit gates via Jaynes-Cummings: $g_i(a\sigma_i^+ + a^\dagger\sigma_i^-)$
- 2-qubit gates implemented by virtual photon exchange: $H_{int} = \frac{1}{2}g_i g_j(\Delta_i^{-1} + \Delta_j^{-1})(\sigma_i^+\sigma_j^- + \sigma_i^-\sigma_j^+)$
- Single-shot readout of qubit state via dispersive phase shift converted via mixer to voltage signal: $\omega_r + (g_i^2/\Delta_i)\sigma_i^z$

QIP lecture slides, Blais *et al.*, Phys. Rev. A (2007)

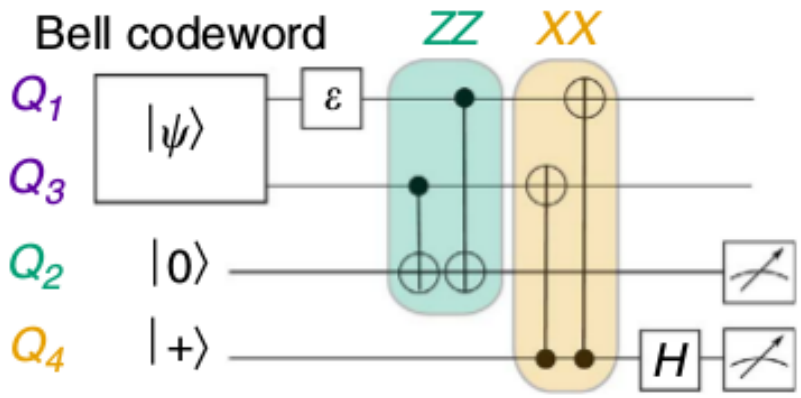
Physical realization

Primitive tile of surface code: 2 code qubits (next-nearest neighbors), each coupled to ancilla qubits via superconducting coplanar waveguides





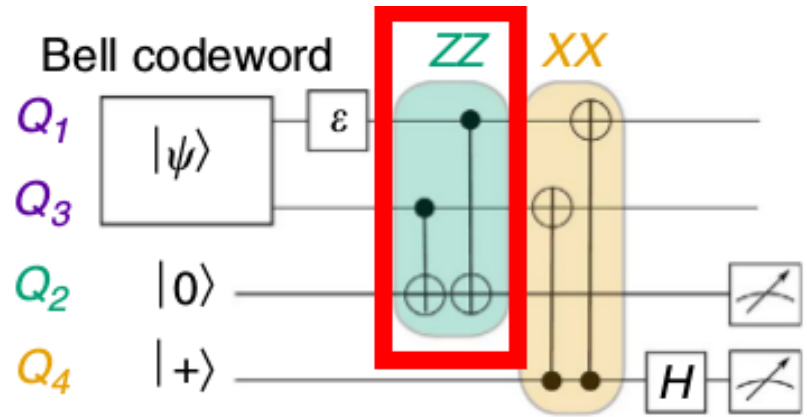
Quantum error correction scheme



[2,0,2] code

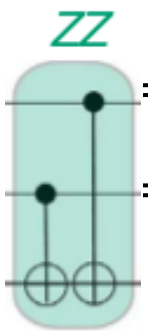
- prepare code word $|\psi\rangle = \frac{1}{\sqrt{2}}\left(|00\rangle + |11\rangle\right)$ and initialize ancilla qubits as $|0\rangle$ and $|+\rangle = \frac{1}{\sqrt{2}}\left(|0\rangle + |1\rangle\right)$
- infuse error ϵ on one code qubit
- parity check stabilizers *ZZ* and *XX* implemented with CNOT gates
- syndrome readout on ancilla qubits

Parity check stabilizers and error detection



stabilizers

$|\psi\rangle = \frac{1}{\sqrt{2}}(|00\rangle + |11\rangle)$ is eigenstate of ZZ and XX with eigenvalue 1



$$= [I \otimes |0\rangle\langle 0| \otimes I + I \otimes |1\rangle\langle 1| \otimes X] \cdot [|0\rangle\langle 0| \otimes I \otimes I + |1\rangle\langle 1| \otimes I \otimes X]$$

$$= [|0\rangle\langle 0| \otimes |0\rangle\langle 0| + |1\rangle\langle 1| \otimes |1\rangle\langle 1|] \otimes I \quad +$$

$$[|0\rangle\langle 0| \otimes |1\rangle\langle 1| + |1\rangle\langle 1| \otimes |0\rangle\langle 0|] \otimes X$$

Parity check stabilizers and error detection

action on qubits after error ϵ

error ϵ	code qubits	syndrome qubits
I	$\|00\rangle + \|11\rangle$	$\|00\rangle$
X	$\|01\rangle + \|10\rangle$	$\|10\rangle$
Z	$\|00\rangle - \|11\rangle$	$\|01\rangle$
Y	$\|01\rangle - \|10\rangle$	$\|11\rangle$

$\Rightarrow$ single-qubit error detection possible

General error detection

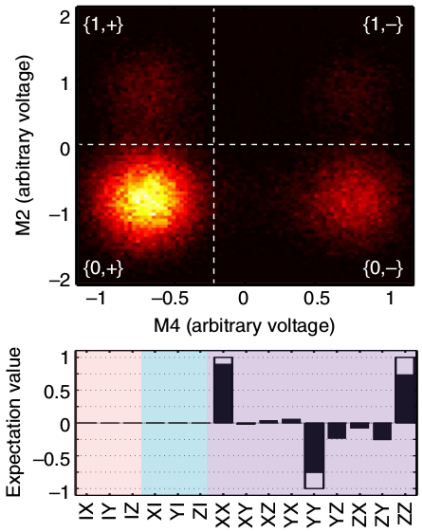
arbitrary single-qubit error U = exp $\left(-i\frac{\theta}{2}\hat{n}\cdot\vec{\sigma}\right)$

error	syndrome	probability of measuring
I	00	$\cos^2\left(\frac{\theta}{2}\right)$
X	10	$\sin^2\left(\frac{\theta}{2}\right) n_x^2$
Z	01	$\sin^2\left(\frac{\theta}{2}\right) n_y^2$
Y	11	$\sin^2\left(\frac{\theta}{2}\right) n_z^2$

- discretization of errors via projective measurements

State tomography and single-shot histograms

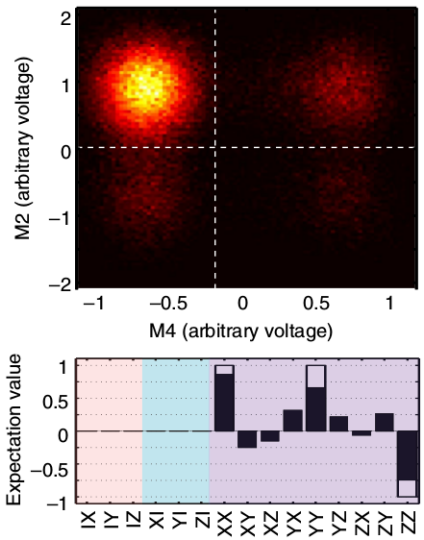
no error



$|\psi\rangle = |00\rangle + |11\rangle$

$F = 0.849$

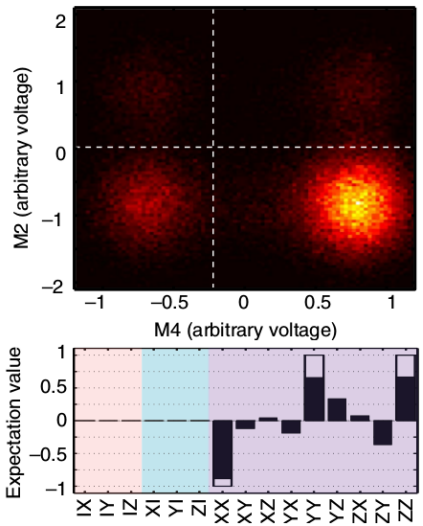
***X* error**



$|\psi\rangle = |01\rangle + |10\rangle$

$F = 0.820$

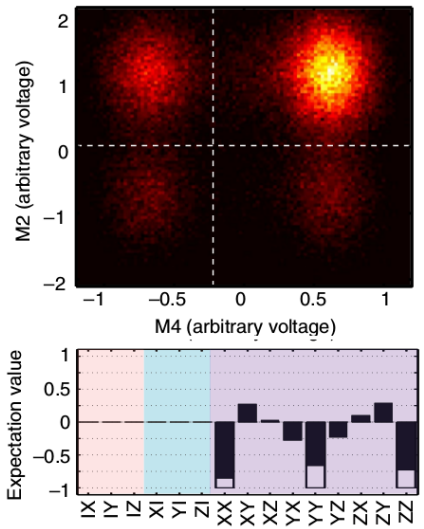
***Z* error**



$|\psi\rangle = |00\rangle - |11\rangle$

$F = 0.805$

***Y* error**

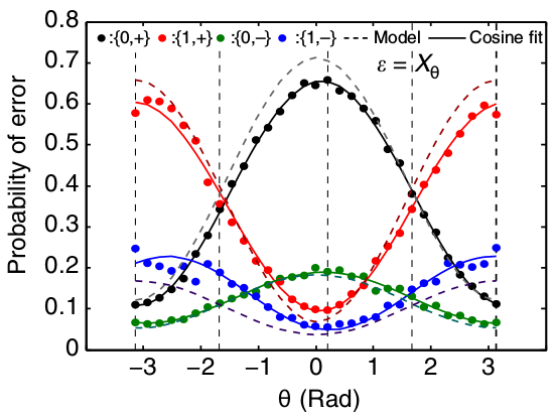


$|\psi\rangle = |01\rangle - |10\rangle$

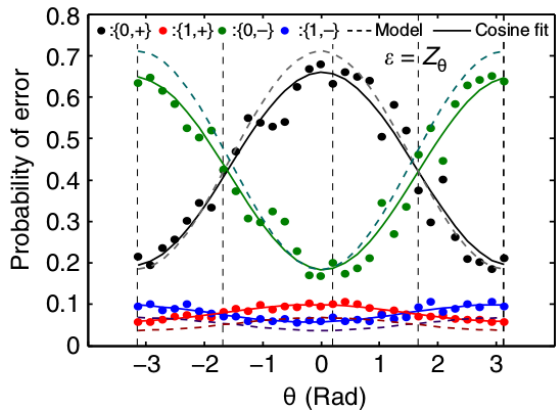
$F = 0.815$

Single-shot correlated syndrome measurements for arbitrary errors

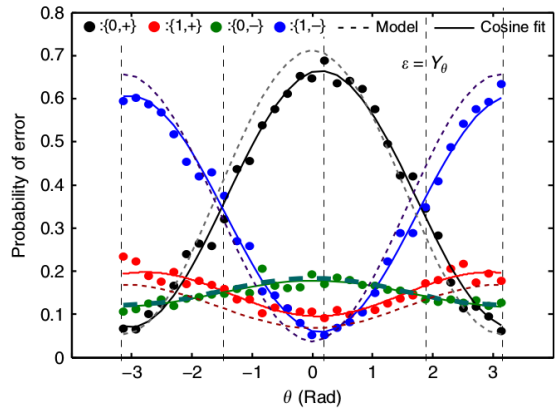
$\epsilon = X_\theta$



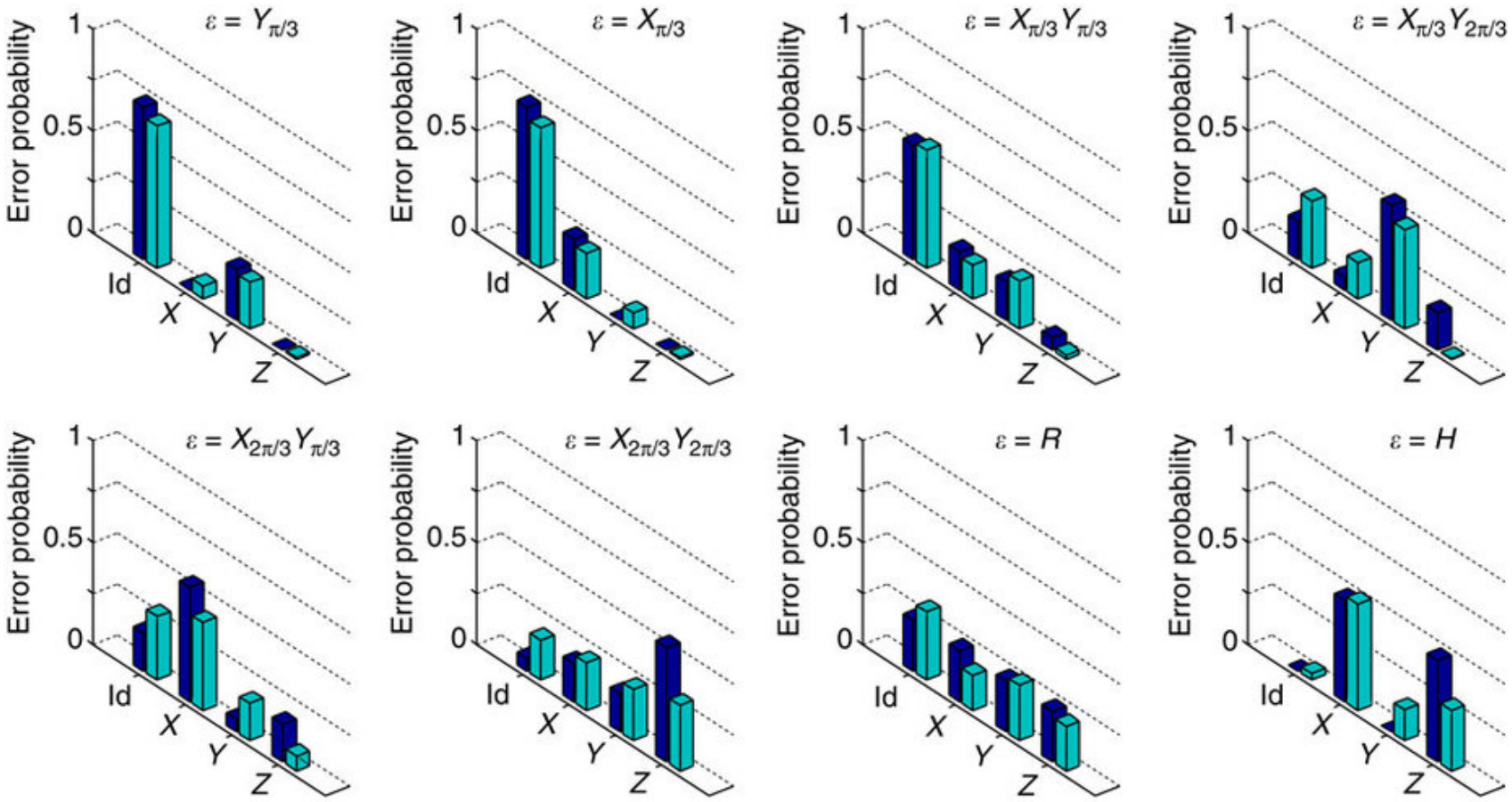
$\epsilon = Z_\theta$



$\epsilon = Y_\theta$

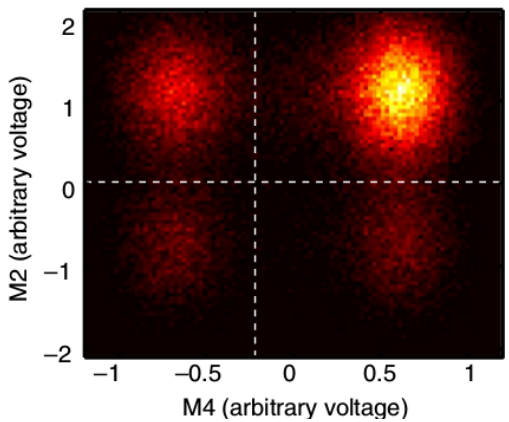


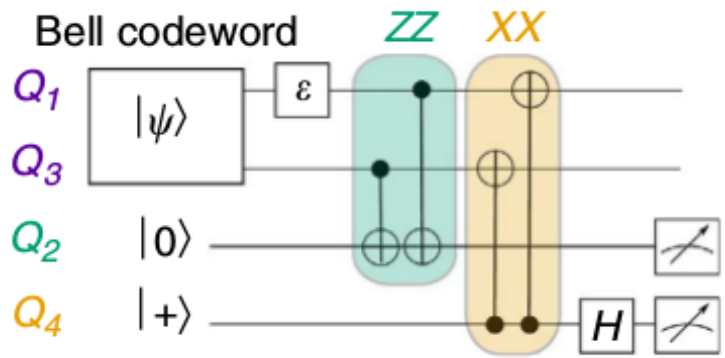
Single-shot correlated syndrome measurements for arbitrary errors



Conclusions

- implemented a scalable system using nearest-neighbor coupled superconducting qubits
- implemented high-fidelity one- and two-qubit gates
- achieved high single-shot assignment fidelities
- enabled detection of arbitrary single-qubit errors via non-demolition measurements





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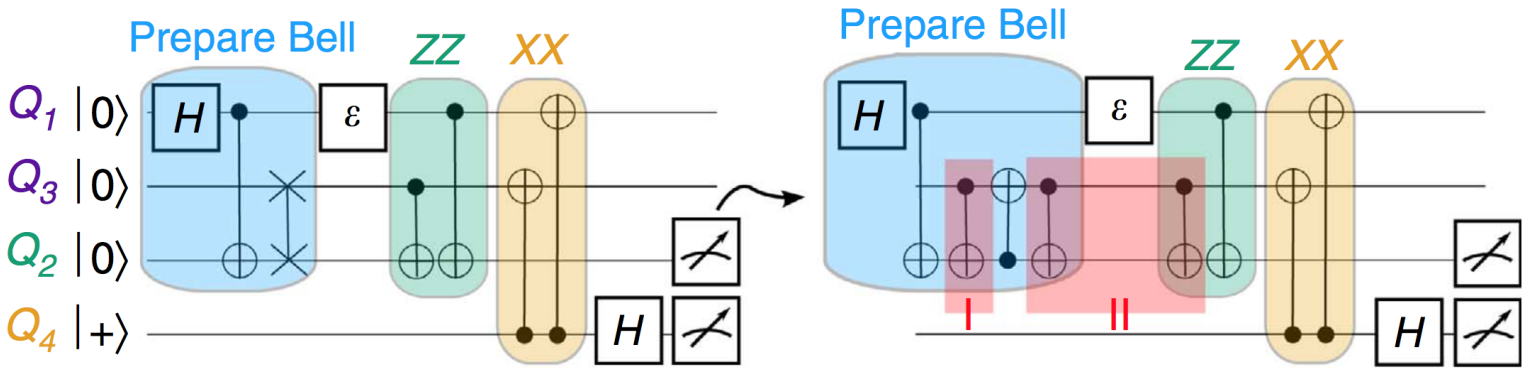
Circuit decomposition



- State preparation of next-nearest neighbor code qubits (Q_1, Q_3) is mediated by bit-flip syndrome qubit (Q_2)
- Error ε controllably introduced via single-qubit rotations about X, Y, or Z axes (*e.g.* $X_{\pi/3}$ or XY)
- Stabilizers implemented with ZZ (bit-flip) and XX (phase-flip) gates