Adiabatic Quantum Computation

An alternative approach to a quantum computer

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Literature:

- E. Farhi, J. Goldstone, S. Gutmann, J. Lapan, A. Lundgren, D. Preda
  "A Quantum Adiabatic Evolution Algorithm Applied to Random Instances of an NP-Complete Problem."

- Jérémie Roland and Nicolas J. Cerf
  "Quantum search by local adiabatic evolution"
  Physical Review A, Volume 65, 042308
Alternative approach to quantum computation.

- Encode problem in a constructed Hamiltonian.
Alternative approach to quantum computation.

- Encode problem in a constructed Hamiltonian.
- Encode solution in ground state of this Hamiltonian.
Figure: Three frustrated spins with ferromagnetic and anti-ferromagnetic coupling.
Spin glass

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Spin glass

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- Frustrations lead to many local minima.
- Minima are separated by large potential walls.
- Ground state not reachable by cooling.
Adiabatic Theorem

**Theorem**

A physical system remains in its instantaneous eigenstate if a given perturbation is acting on it slowly enough and if there is a gap between the eigenvalue and the rest of the Hamiltonian’s spectrum.
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- A system in the ground state remains in the ground state.
- The perturbation does not have to be small.
- Can switch Hamiltonian: $H(t) = (1 - \frac{t}{T})H_0 + \frac{t}{T}H_P$
Runtime of an adiabatic algorithm

Figure: Eigenvalues of the time-dependent Hamiltonian. $E_0$ and $E_1$ avoid crossing each other.
Runtime of an adiabatic algorithm

For probability $1 - \epsilon^2$ of remaining in the ground state:

$$g_{\text{min}} = \min_{0 \leq t \leq T} [E_1(t) - E_0(t)]$$  \hspace{1cm} (1)

$$D_{\text{max}} = \max_{0 \leq t \leq T} |\langle E_1; t | \frac{dH}{dt} | E_0; t \rangle|$$  \hspace{1cm} (2)

Condition for the adiabatic Theorem:

$$\frac{D_{\text{max}}}{g_{\text{min}}^2} \leq \epsilon$$  \hspace{1cm} (3)
String of $n$ bits $z_1, z_2...z_n$ satisfying a set of clauses of the form $z_i + z_j + z_k = 1$.
Determining a string satisfying all clauses involves checking all $2^n$ assignments, and is a NP-complete problem.
Define energy for a clause:

\[ h_C(z_{i_C}, z_{j_C}, z_{k_C}) = \begin{cases} 0 & \text{if } z_{i_C} + z_{j_C} + z_{k_C} = 1 \\ 1 & \text{if } z_{i_C} + z_{j_C} + z_{k_C} \neq 1 \end{cases} \]  

(4)

Define total energy:

\[ h = \sum_{C} H_C \]  

(5)
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Define total energy:

\[ h = \sum_C H_C \] (5)

The energy \( h \geq 0 \) and \( h(z_1, z_2, ... z_n) = 0 \) only if the string satisfies all clauses.
Problem Hamiltonian

Use spin-$\frac{1}{2}$ qubits labeled by $|z_1\rangle$ where $z_1 = 0, 1$.

$$|0\rangle = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \text{ and } |1\rangle = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$ (6)
Use spin-$\frac{1}{2}$ qubits labeled by $|z_1\rangle$ where $z_1 = 0, 1$.

$$|0\rangle = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \text{ and } |1\rangle = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$ \hspace{1cm} (6)

Define operator corresponding to clause $C$

$$H_{P,C}(|z_1\rangle...|z_n\rangle) = h_C(z_{i_C}, z_{j_C}, z_{k_C})|z_1\rangle...|z_n\rangle$$ \hspace{1cm} (7)
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(7)

Problem Hamiltonian is given by

$$H_P = \sum_C H_{P,C}$$

(8)
Use $x$ basis for initial state

$$
|x_i = 0\rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ 1 \end{pmatrix} \text{ and } |x_i = 1\rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ -1 \end{pmatrix}
$$

(9)

Define operator

$$
H_B^{(i)} |x_i = x\rangle = \frac{1}{2} (1 - \sigma_x^{(i)}) |x_i = x\rangle = x |x_i = x\rangle
$$

(10)
Initial Hamiltonian is given by

\[ H_P = \sum_{i=1}^{n} d_i H_B^{(i)} \]  \hspace{1cm} (11)

where \( d_i \) is the number of clauses involving bit \( i \).

The ground state is given by

\[ |x_1 = 0\rangle \ldots |x_n = 0\rangle = \frac{1}{2^{n/2}} (|z_1 = 0\rangle + |z_1 = 1\rangle) \ldots (|z_n = 0\rangle + |z_n = 1\rangle) \]  \hspace{1cm} (12)
Figure: Median time to achieve success probability of 1/8 for different sized problems.
Conclusion

- Alternative, non-gate based approach to quantum computation.
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- Encode solution in ground state of a Hamiltonian.
- Adiabatic theorem provides way to reach the ground state.