Quantum Information Processing with Photons

Q: Why have the first quantum information processing experiments been performed with photons?

A:
- generation of polarization entangled photons
- manipulation of single photon polarization
- single photon detection

$\Rightarrow$ are all well developed for photons

Demonstration:
- Super Dense Coding
- Teleportation

+ Introduction to Bell Inequalities

Experimental Realization of Super Dense Coding

① Preparation of initial entangled state using parametric down conversion (PDC)

$$|\psi^+\rangle = \frac{1}{\sqrt{2}}\left(|HV\rangle + |VH\rangle\right)$$

② Generation of all 4 maximally entangled 2 photon polarization states

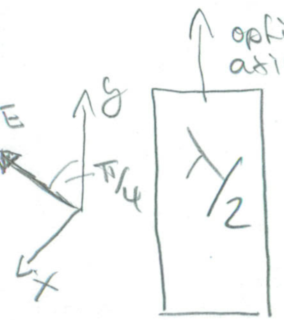
realized using:

$$|\psi^+\rangle \xrightarrow{I_2} |\psi^+\rangle$$

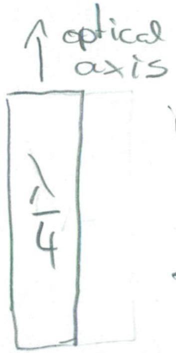
$$|\psi^+\rangle \xrightarrow{X_2} \frac{1}{\sqrt{2}}\left(|HH\rangle + |VV\rangle\right) = |\phi^+\rangle$$ half wave plate ($\lambda/2$)

$$|\psi^+\rangle \xrightarrow{Z_2} \frac{1}{\sqrt{2}}\left(|HV\rangle - |VH\rangle\right) = |\psi^-\rangle$$ quarter wave plate ($\lambda/4$)

$$|\psi^+\rangle \xrightarrow{Z_2 X_2} \frac{1}{\sqrt{2}}\left(|HH\rangle - |VV\rangle\right) = |\phi^-\rangle$$ $\lambda/4$ & $\lambda/2$ plate



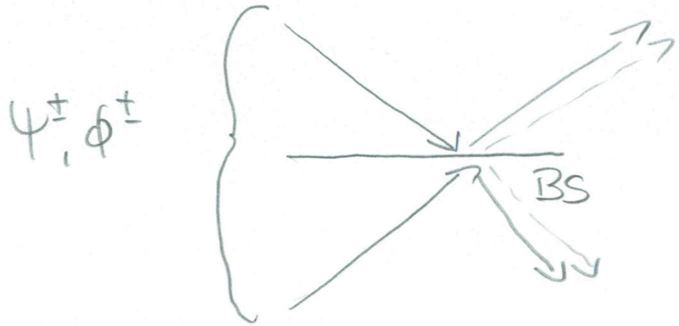
retardation plate induces a phase shift of π between ordinary and extra-ordinary beam in a optically active medium: $H \to V$ and $V \to H$



induces phase shift of $\pi/2$ between e and eo beams. turns linear into circular polarization at $\pi/4$ incidence to optical axis

③ Bell state measurement using beam splitters

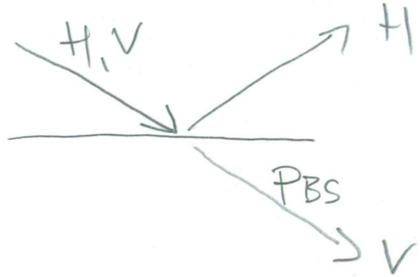
<u>A</u>: distinguish symmetric $|\psi^+\rangle, |\phi^+\rangle, |\phi^-\rangle$ from antisymmetric $|\psi^-\rangle$ state using a beam splitter (BS)



bunching for sym. states

anti-bunching for anti-symm. states

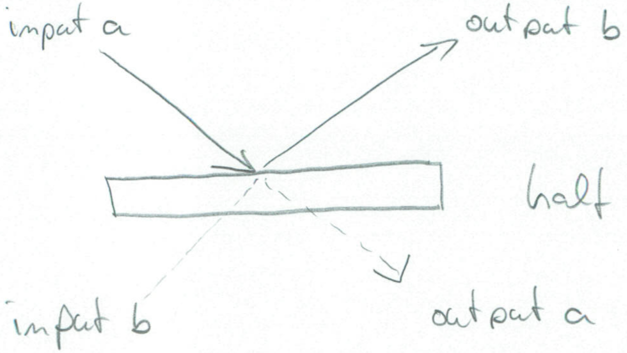
<u>B</u>: distinguish polarization state using polarizing beam splitter



<u>outcomes</u>:

$\|\psi^+\rangle$:	coincidences	D_H & D_V or $D_{H'}$ & $D_{V'}$	sym.
$\|\psi^-\rangle$:	"	$D_{H'}$ & D_V or D_H & $D_{V'}$	a-sym.
$\|\phi^+\rangle \|\phi^-\rangle$:	2 photons in	$D_H, D_V, D_{H'}, D_{V'}$	sym.

Two Photon Interference at a Beam Splitter



half silvered mirror $\equiv$ beam splitter (polarization independent)

action of a beam splitter on a single photon impinging from input a

$$a^+ \xrightarrow{BS} \frac{1}{\sqrt{2}}\left(a^+ + i\, b^+\right)$$

valid for both polarizations a^+_H, a^+_V

π phase shift for reflected beam

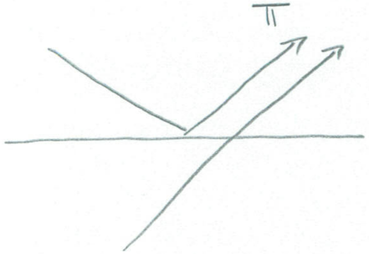
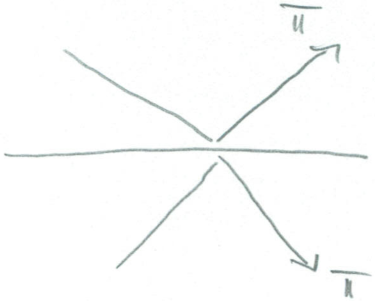
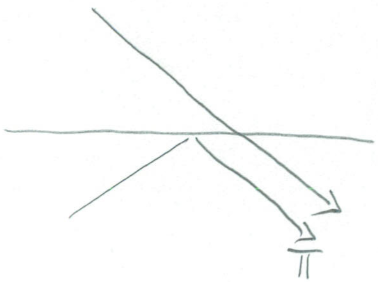
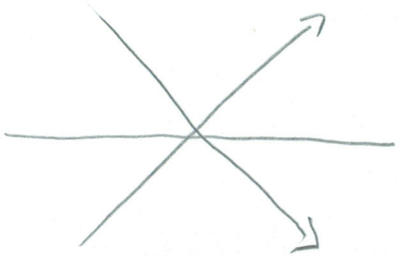
equivalent for input b:

$$b^+ \xrightarrow{BS} \frac{1}{\sqrt{2}}\left(b^+ + i\, a^+\right)$$

$\Rightarrow$ photon from either input will be scattered into either output with probability of 50 % each

$\Rightarrow$ What happens if two photons impinge simultaneously from two different sides on the beam splitter?

4 different possibilities

			symmetric spatial wave function	anti-symmetric spatial wave function
①		bunching	☑	☐
②		anti-bunching	☐	☑
③		bunching	☑	☐
④		anti-bunching	☐	☑

Symmetric or Anti-Symmetric Two-Photon States at Beam Splitter

$$\frac{1}{\sqrt{2}}\left(a_H^+ b_V^+ \overset{\text{anti-symm.}}{\underset{\text{symm.}}{\mp}} a_V^+ b_H^+\right) \xrightarrow{BS}$$

"apply beam-splitter transformation to each mode"

$$\frac{1}{2}\frac{1}{\sqrt{2}}\Big((a_H^+ + i\, b_H^+)(b_V^+ + i\, a_V^+) \mp (a_V^+ + i\, b_V^+)(b_H^+ + i\, a_H^+) \Big)$$

$$= \frac{1}{2\sqrt{2}}\left(a_H^+ b_V^+ + i\, b_H^+ b_V^+ + i\, a_H^+ a_V^+ - b_H^+ a_V^+ \mp a_V^+ b_H^+ \mp i\, b_V^+ b_H^+ \mp i\, a_V^+ a_H^+ \pm b_V^+ a_H^+ \right)$$

for antisymmetric spatial wave function (−) "$\sim\sim$"

$$= \frac{1}{\sqrt{2}}\left(a_H^+ b_V^+ - a_V^+ b_H^+\right) \quad \Rightarrow \text{ anti-bunching}$$

for symmetric spatial wave function (+) "___"

$$= i\,\frac{1}{\sqrt{2}}\left(a_H^+ a_V^+ + b_H^+ b_V^+\right) \quad \Rightarrow \text{ bunching}$$

similar for other symmetric spatial wave functions

$$\frac{1}{\sqrt{2}}\left(a_H^+ b_H^+ \pm a_V^+ b_V^+\right)$$

⇒ go back to first detection slide to explain measurement results!

Teleportation of Photon State using Bell - Measurement

① Preparation of initial states

$$|\psi_{23}^-\rangle = \frac{1}{\sqrt{2}}\left(|HV\rangle - |VH\rangle\right)$$ from PDC source

$$|\psi_1\rangle = \alpha|H\rangle + \beta|V\rangle$$ from PDC source using second photon for trigger; α, β adjusted using polarizer

② joint state

$$|\psi\rangle = |\psi_1\rangle|\psi_{23}^-\rangle = \frac{1}{\sqrt{2}}\left(\alpha|HHV\rangle - \alpha|\underline{HV}H\rangle + \beta|\underline{VH}V\rangle - \beta|VVH\rangle\right)$$

projected onto $|\psi_{12}^-\rangle = \frac{1}{\sqrt{2}}\left(|HV\rangle - |VH\rangle\right)$ with probability $\frac{1}{4}$

③ post measurement state

$$|\psi'\rangle = |\psi_{12}^-\rangle\left(\alpha|H\rangle + \beta|V\rangle\right)$$

qubit 3 assumes initial state of qubit 1