QSIT 2015 - Questions 1

27. March 2015, HIT F 13

1. **State Space in Quantum Mechanics** What is the relevant Hilbert space that represents the dynamics of the following systems. Write down their basis states.

   (a) A neutron in a static magetic field.

   (b) A neutron in a magnetic gradient field.

   (c) A small mirror attached to a spring.

   (d) A small mirror attached to a spring exposed to laser radiation .

   (e) A ground-state hydrogen atom at room temperature.

   (f) A ground-state hydrogen atom exposed to laser radiation at a wavelength of 121 nm.

   (g) An ensemble of N hydrogen atoms in the ground state at room temperature.

2. **Bloch Sphere** Any quantum state of a spin-1/2 (or two-level system) can be represented on the *Bloch sphere*. Calculate the polar and azimuthal angles of the following states and draw the states on the Bloch sphere.

   (a) $|\psi\rangle = \frac{1}{\sqrt{3}}\left(|0\rangle + \sqrt{2}|1\rangle\right)$

   (b) $|\psi\rangle = \frac{1}{\sqrt{3}}\left(|0\rangle - i\sqrt{2}|1\rangle\right)$

   (c) $|\psi\rangle = \frac{1}{\sqrt{3}}\left(\sqrt{2}|0\rangle - i2|1\rangle\right)$

   (d) $|\psi\rangle = \frac{e^{i\pi/4}}{\sqrt{3}}\left(|0\rangle - i\sqrt{2}|1\rangle\right)$

3. **Rabi Oscillations**

   A spin-1/2 particle is placed in a magetic field of magnitude B_z pointing in the z-direction . At time t_0 an additional field B_x is applied in the x-direction. Calculate the expected excited state population as a function of time and draw a diagram. Assume that $B_x \gg B_z$ and that the particle is initially in its ground state. What changes, if the additional magnetic field points in the y-direction instead?

4. **State preparation**

   Any single qubit state can be prepared by applying a sequence of unitary operations onto the inital state. Assuming that the system is initially in its ground state, $|\psi_i\rangle = |0\rangle$, determine the unitary matrix (sequence) that results in the following final states:

   (a) $|\psi_f\rangle = |1\rangle$

   (b) $|\psi_f\rangle = (|0\rangle - |1\rangle)/\sqrt{2}$

   (c) $|\psi_f\rangle = \sin\frac{3\pi}{8}|1\rangle - \cos\frac{3\pi}{8}|0\rangle$

   (d) $|\psi_f\rangle = e^{i\pi/4}\sin\frac{3\pi}{8}|1\rangle - \cos\frac{3\pi}{8}|0\rangle$