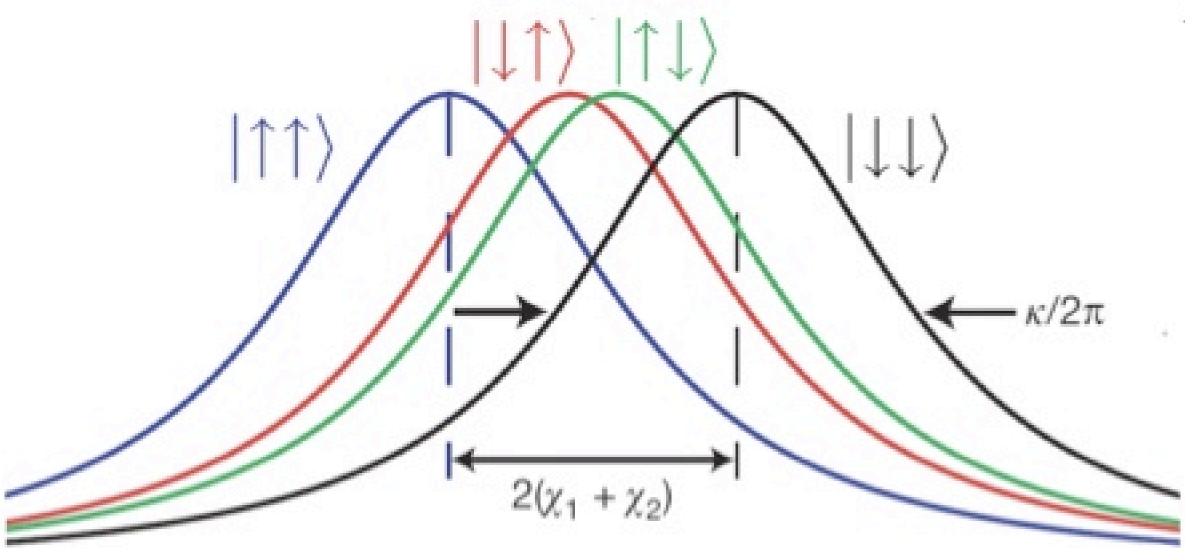

Coupling of Superconducting Qubits using Circuit Quantumelectrodynamics

Andreas Fragner, Tobias Frey
ETH Zurich, 11-26-2007

"Quantum Systems for Information Processing"
and "Topics in Quantum Information Processing"

Coupling of Superconducting Qubits using Circuit Quantumelectrodynamics

1. Introduction

Quantum computation with solid-state qubits: milestones so far, realization of gates and local coupling

2. Theory Primer

single- and multi-qubit Hamiltonians and eigenstates, dispersive and strong-coupling limits, qubit-resonator entanglement

3. Selective Strong Coupling of Single Qubits

strong coupling limit measurements, vacuum Rabi splitting, determination of coupling strengths, coherent selective control of single qubits

4. Coherent Two-Qubit Interactions

dispersive limit measurements, transfer of quantum information via virtual photon exchange, multiplexed simultaneous read-out, 2-qubit Bell states

5. Perspectives and Conclusion

ETH
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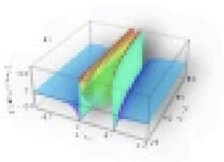
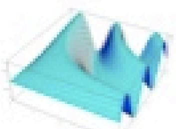
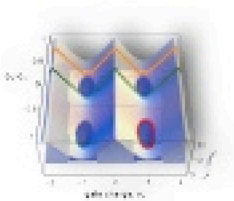
Department of Physics, Quantum Device Lab

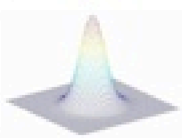
Introduction

Superconducting Quantum Information Processing - Requirements









Realization of Gates

- Multiple Qubit System fulfilling de Vincenzo Criteria

  → e.g. cNOT requires min. 2 qubits

- Selective, controllable interaction of single qubits

  → addressability for operation

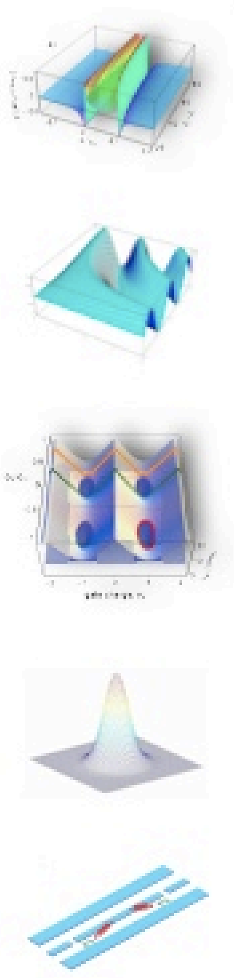
- Strong coupling of qubits to information carrier over large distances, coherent information transfer and exchange

  → Ion traps: phonons

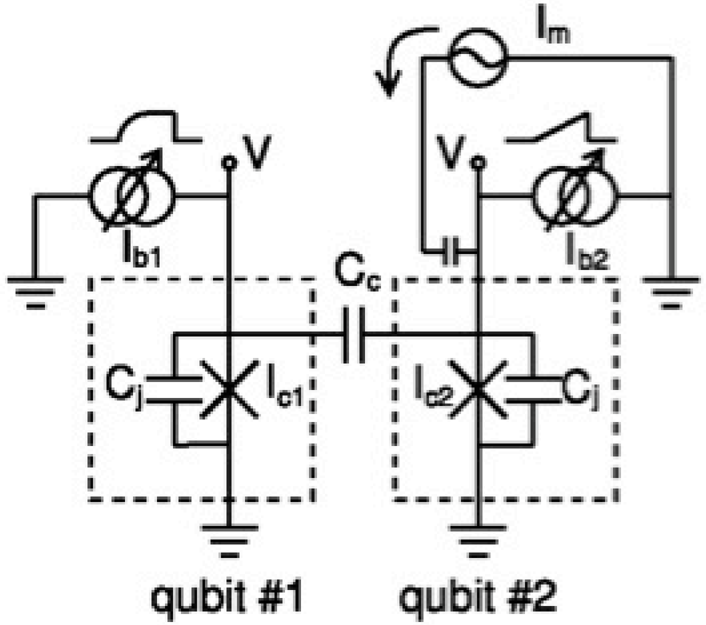
  → Solid-state qubits: photons

Introduction

Milestones so far



Bell States with two superconducting qubits



Circuit diagram of two coupled phase qubits

- pair of superconducting phase qubits

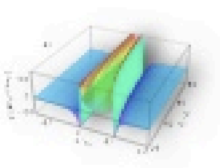
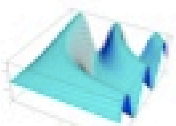
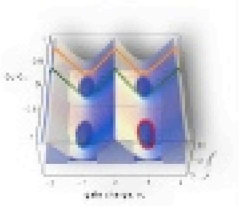
- current biased

- coupled capacitively with fixed strength

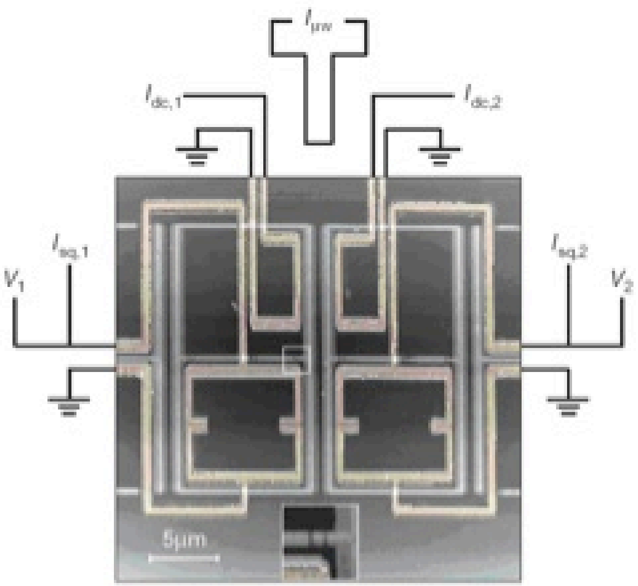
- realization of macroscopic entangled states

A.J. Berkley et al., Science **300**, 1548 (2003)

Introduction
Milestones so far

cNOT gate with superconducting flux qubits



AFM of two 8-shaped flux qubits

- pair of superconducting flux qubits

  - coupled magnetically with fixed strength

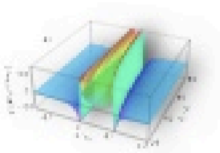
  - four level system, qubits tunable with individual flux bias

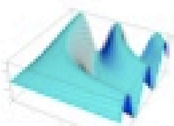
  - realization of controlled NOT gate

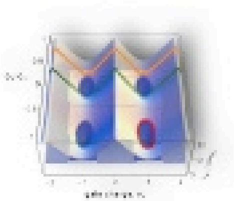
J.H. Plantenberg, P.C. de Groot, C.J.P.M. Harmans, J.E. Mooij Nature **447**, 836 (2007)

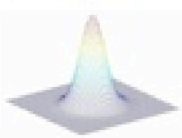
Introduction

Milestones so far









cNOT gate with charge qubits

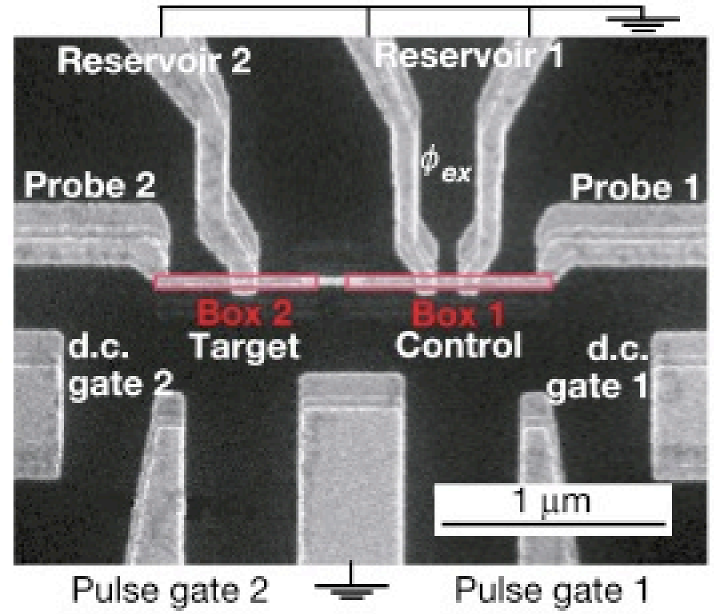
- System of two Cooper Pair Boxes

- coupled by on-chip capacitor

- input states prepared with pulse technique

- qubits addressed individually

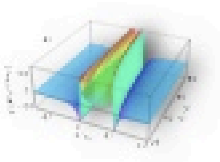
- realization of macroscopic entangled states

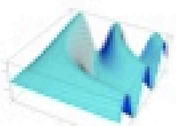


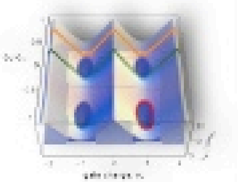
SEM graph of two qubit architecture

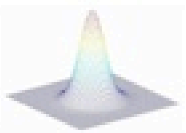
T. Yamamoto et al., Nature **425**, 941 (2003)

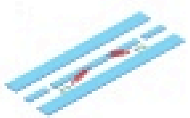
Introduction











Non-local entanglement and quantum information transfer between two distant qubits

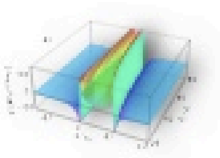
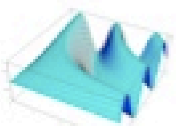
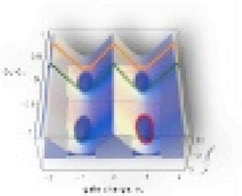
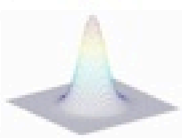
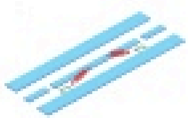
- So far: only local coupling of solid-state qubits
- Lumped circuit elements used for coupling

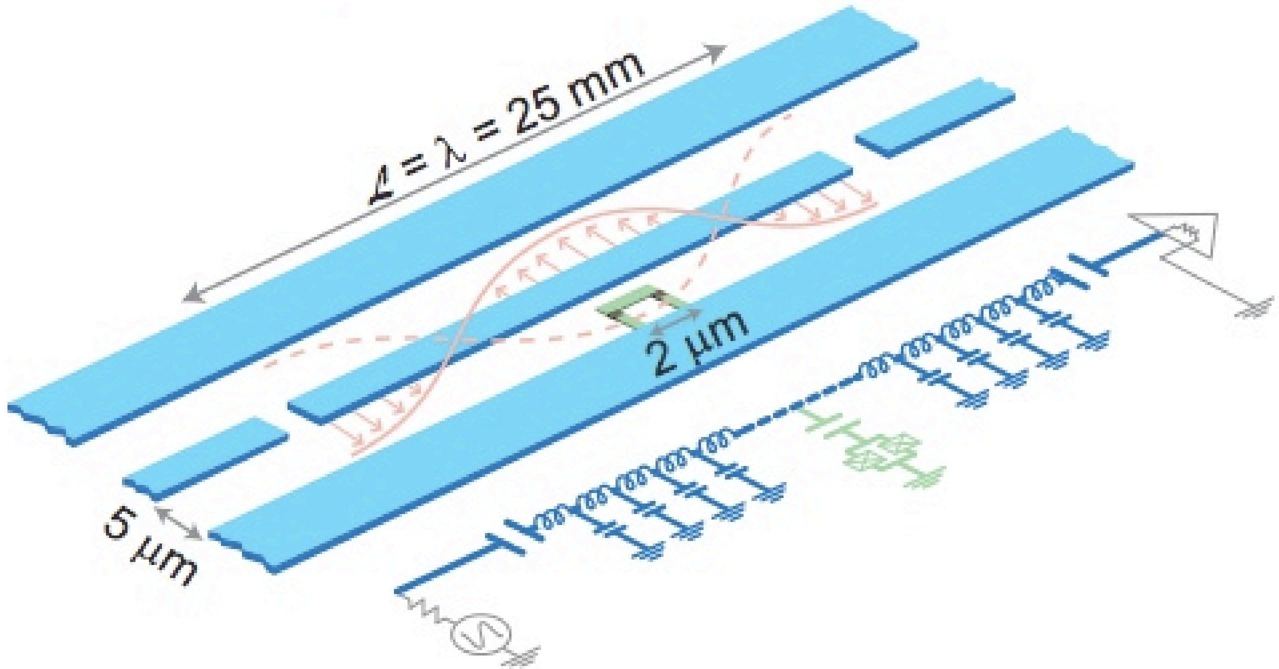
→ use of distributed circuit elements

→ non-local coupling of two superconducting qubits over macroscopic distances

J. Majer, J.M. Chow, J.M. Gambetta, J. Koch, B.R. Johnson, J.A. Schreier, L. Frunzio, D.I. Schuster, A.A. Houck, A. Wallraff, A. Blais, M.H. Devoret, S.M. Girvin & R.J. Schoelkopf, Nature **449**, 443 (2007)

Single Qubit Circuit QED



Total Hamiltonian

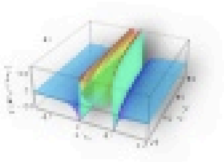
$$
\begin{aligned}
H &= H_R + H_a + H_I \\
&= \hbar\omega_R a^\dagger a + \hbar\frac{\omega_a}{2}\sigma^z - \hbar g(a^\dagger\sigma^- + \sigma^+ a)
\end{aligned}
$$

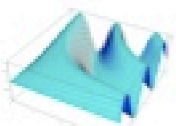
Unperturbed Eigenstates

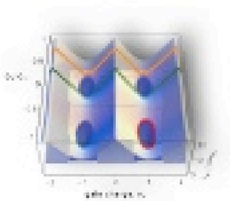
$$
H_R\,|n\rangle = \hbar\omega_R n\,|n\rangle \quad , \quad H_a\,|\updownarrow\rangle = \pm\frac{\hbar\omega_a}{2}\,|\updownarrow\rangle
$$

Single Qubit Circuit QED

Degenerate perturbation theory: zeroth order solution







- Eigenstates total qubit-resonator Hamiltonian

$$|+,n\rangle = \cos\Theta_n\,|\downarrow,n\rangle + \sin\Theta_n\,|\uparrow,n+1\rangle$$
$$|-,n\rangle = -\sin\Theta_n\,|\downarrow,n\rangle + \cos\Theta_n\,|\uparrow,n+1\rangle$$

$$\Theta_n = \frac{1}{2}\arctan\left(\frac{2g\sqrt{n+1}}{\Delta}\right)$$

- Eigenenergies

$$E_{\pm,n} = \hbar\omega_r(n+1) \pm \frac{\hbar}{2}\sqrt{4g^2(n+1)+\Delta^2}$$

$$E_{\uparrow,0} = -\frac{\hbar\Delta}{2} \quad \Delta = \omega_a - \omega_R$$

- Maximally Entangled States

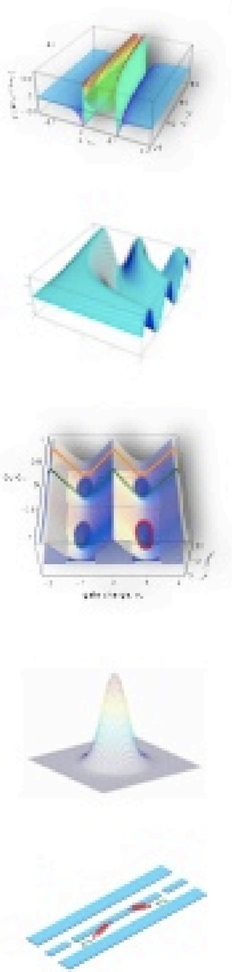
Limit $\Delta \to 0$

$$|\pm,n\rangle = \frac{1}{\sqrt{2}}\left(|\uparrow,n+1\rangle \pm |\downarrow,n\rangle\right)$$

→ Strong coupling of qubit and cavity

Single Qubit Circuit QED

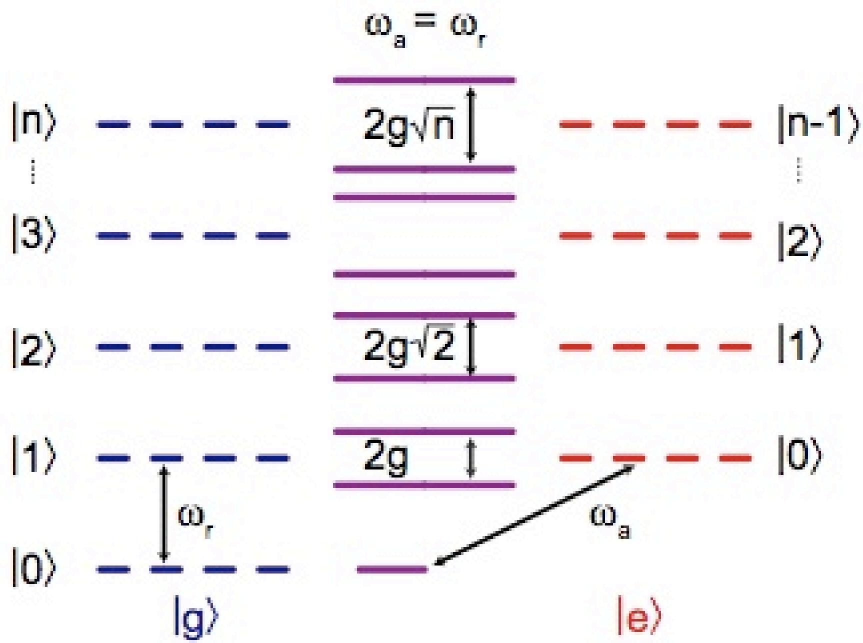
Energy Level Spectrum



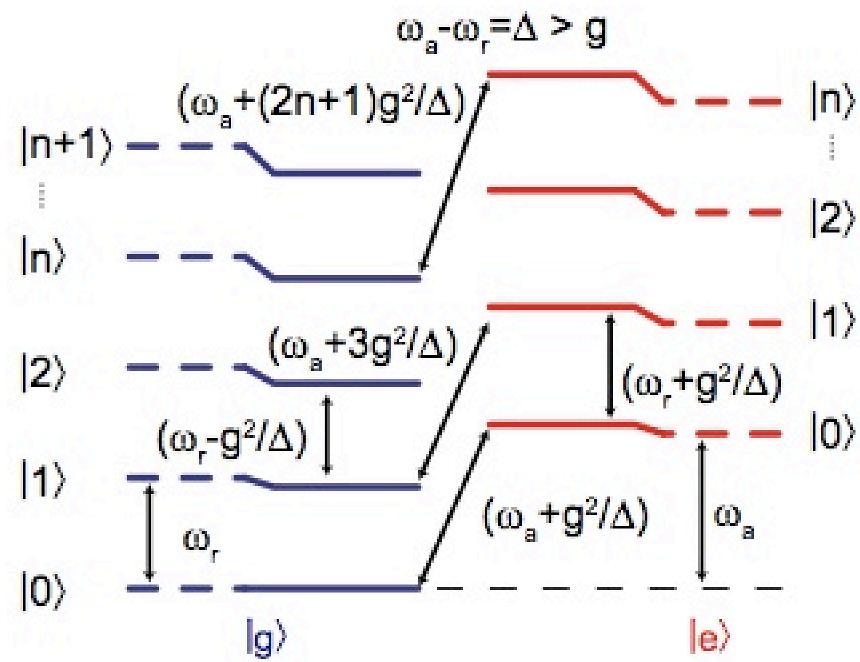
- Strong coupling limit $\Delta \to 0$

  Non-Linear Energy Spectrum

  $\Delta E_n = 2\hbar g \sqrt{n}$

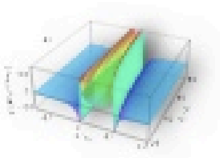
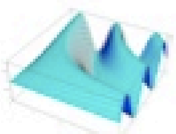
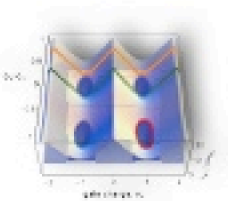


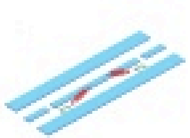
- Dispersive limit $g \ll \Delta$

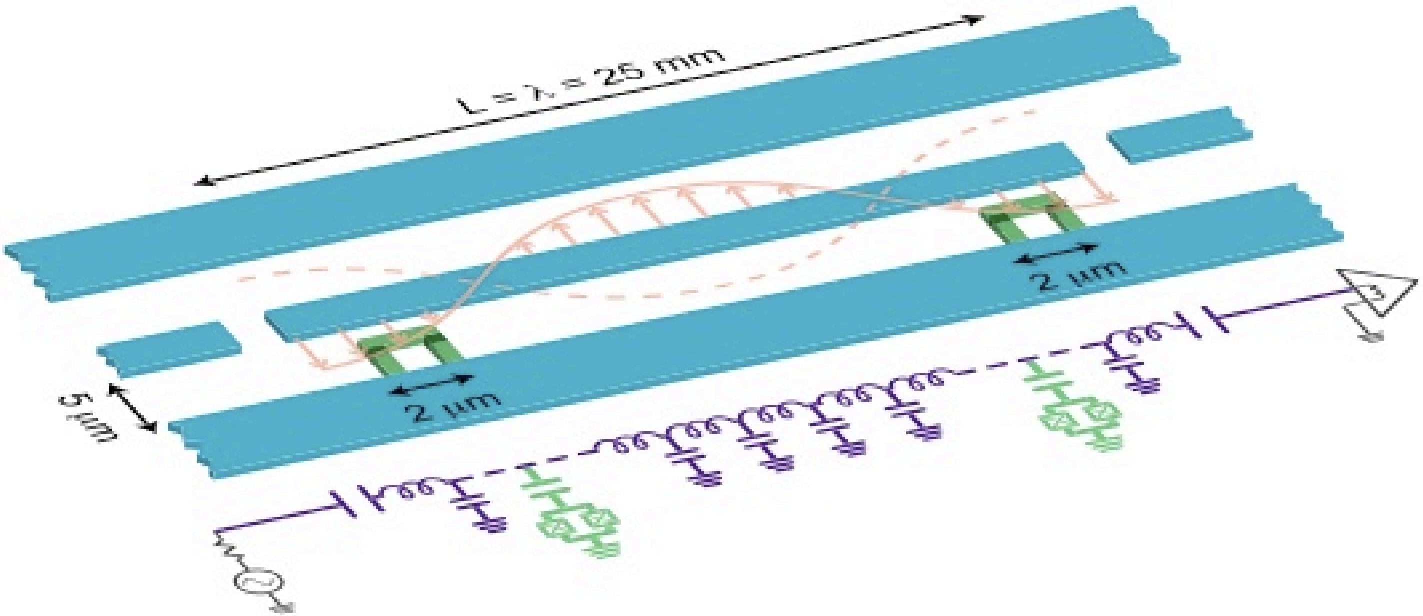


→ Rabi splitting as evidence for strong coupling of qubit and resonator

Multiple Qubit Circuit QED





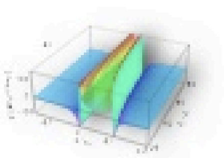
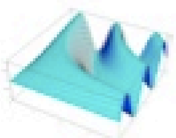
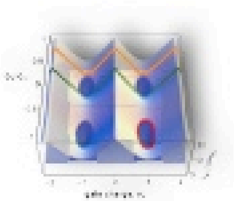
n-Qubit-Cavity Hamiltonian

$$
\begin{aligned}
H &= H_R + \sum_i H_a^{(i)} + \sum_{i\neq j} H_{IQ}^{(ij)} \\
&= \hbar\omega_R + \sum_i \hbar\frac{\omega_{ai}}{2}\sigma^z + \sum_{i\neq j}\hbar J_{ij}(\sigma_i^-\sigma_j^+ + \sigma_j^-\sigma_i^+)
\end{aligned}
$$

qubit-qubit interactions

$$
J_{ij} = \frac{g_i g_j}{2}\left(\frac{1}{\Delta_i}+\frac{1}{\Delta_j}\right) \qquad \Delta_i = \omega_{ai} - \omega_R
$$

Two Qubit Circuit QED

2-Qubit-Cavity Hamiltonian
(dispersive limit, 2nd order diagonalization)

$$H_{JC} = \sum_{i=1,2} \frac{\hbar\omega_i}{2}\sigma_i^z + \hbar\omega_R a^\dagger a + \sum_{i=1,2} \hbar\frac{g_i^2}{\Delta_i}\sigma_i^z a^\dagger a + \hbar J(\sigma_1^- \sigma_2^+ + \sigma_2^- \sigma_1^+)$$

qubit-qubit interaction:

degeneracy $\omega_1 = \omega_2$

$$J = \frac{g_1 g_2}{\Delta}$$

→ excitations can be transferred bw. qubits via virtual photons

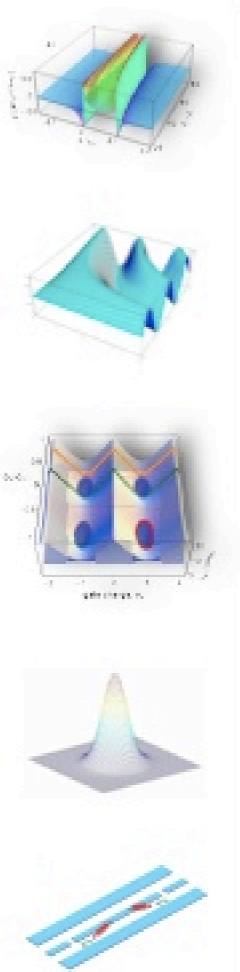
non-degeneracy $|\omega_1 - \omega_2| >> J$

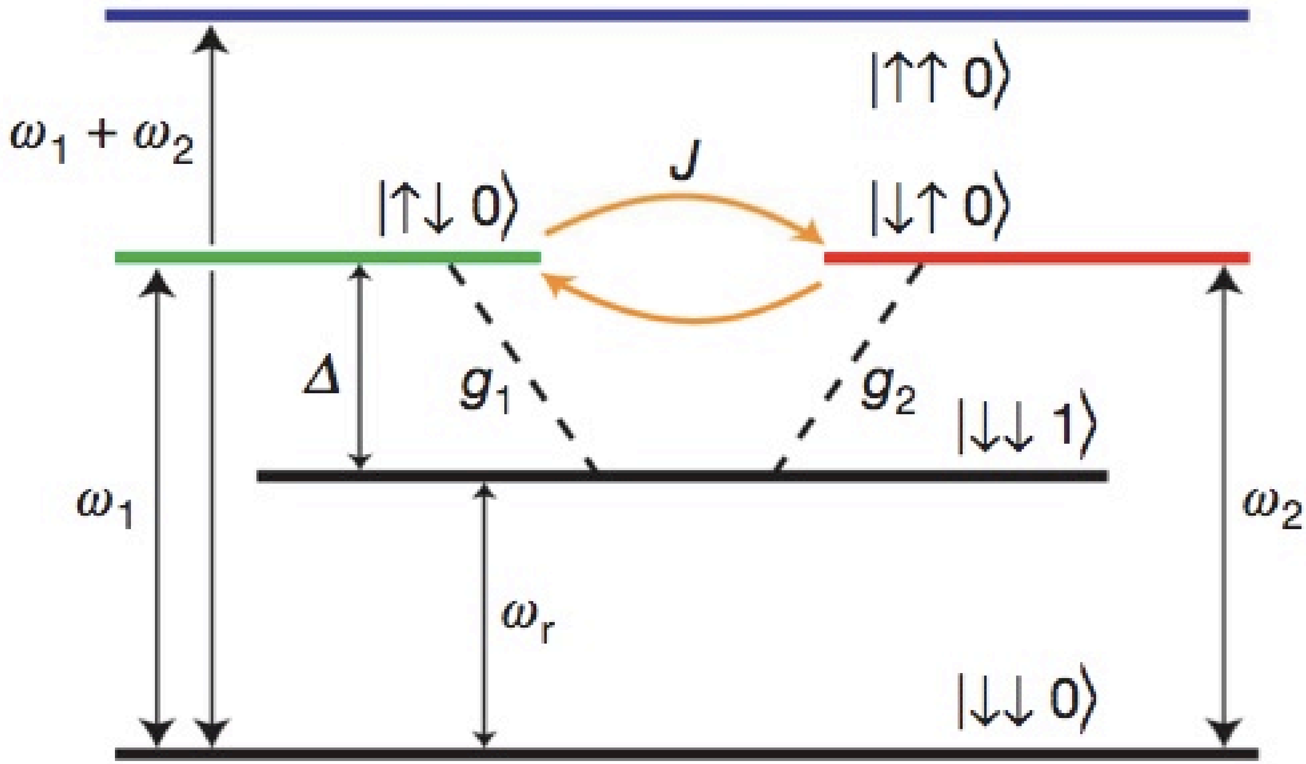
violation of energy conservation

→ process suppressed

→ tuning of transition frequencies = effective decoupling mechanism

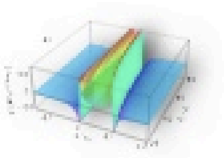
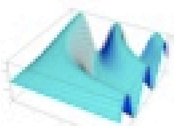
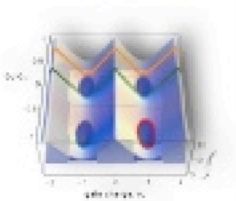
Two Qubit Circuit QED





J. Majer et al., Nature **449**, 443 (2007)

Vacuum Rabi Splitting
Communication with each individual qubit

Each qubit can be individually fluxed tuned

→ single qubits can be strongly coupled to resonator

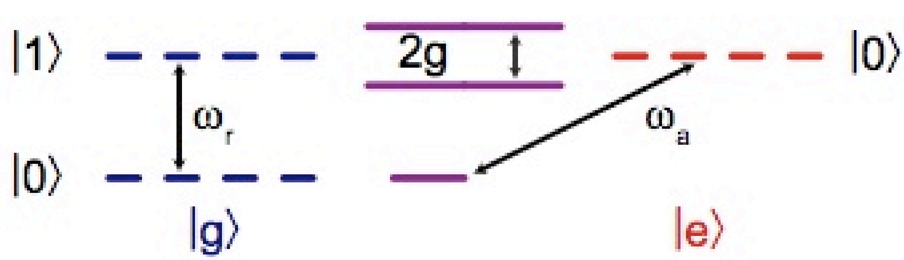
→ experimental proof of coherent information exchange of single qubit and cavity: Rabi Splitting

Assume Qubit 1 in resonance, Qubit 2 strongly detuned

$$E_{\pm,n} = \hbar\omega_r(n+1) \pm \frac{\hbar}{2}\sqrt{4g^2(n+1)+\Delta^2}$$

limit $\Delta_1 \to 0$

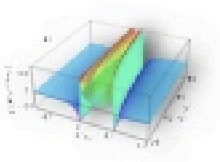
$$E_{\pm,n} = (n+1)\hbar\omega \pm \hbar g\sqrt{n+1}$$

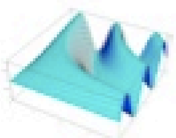


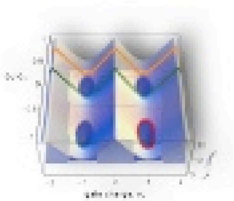
→ two well-resolved peaks in transmission spectrum

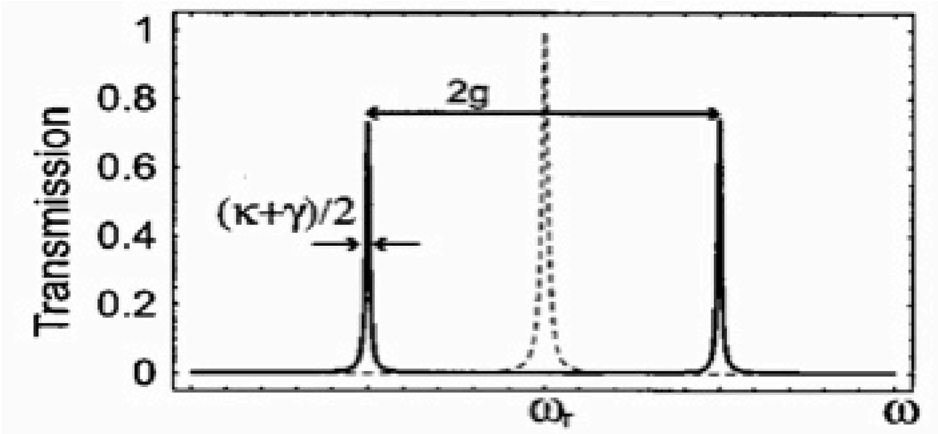
Vacuum Rabi Splitting

Communication with each individual qubit









$1/\gamma$ qubit lifetime

$1/\kappa$ cavity lifetime

ω_r cavity resonance

- peaks correspond to superposition of qubit and cavity photon states

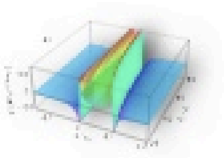
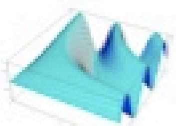
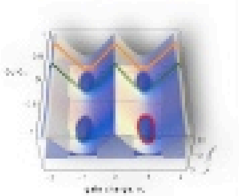
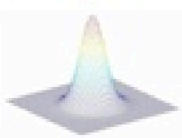
- coherent exchange of energy/information between quantized EM field and a quantum two-level system

$$|\pm,0\rangle = \frac{1}{\sqrt{2}}(|e,0\rangle \pm |g,1\rangle)$$

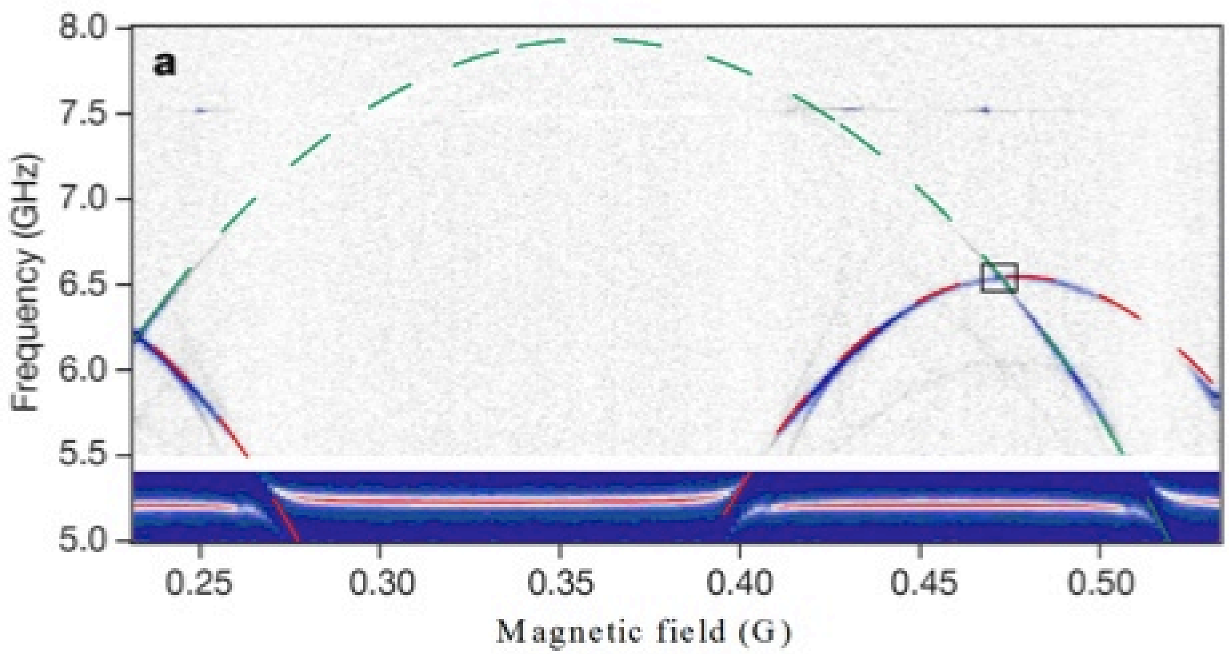
A. Blais et. al., Phys.Rev. A **69**, 062320 (2004)

A. Wallraff et. al., Nature **431**, 162 (2004)

Vacuum Rabi Splitting
Communication with each individual qubit


Vacuum Rabi Splitting in a 2-Qubit system



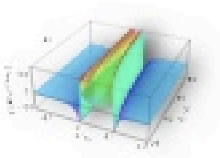
→ difference in frequency: determination of coupling strength for each qubit $g_{1,2} \approx 105\,\mathrm{MHz}$

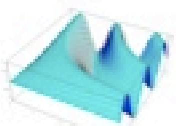
demonstration that both qubits can individually exchange information with cavity

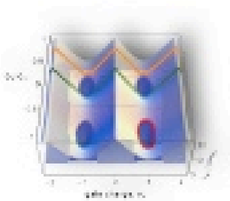
J. Majer et al., Nature **449**, 443 (2007)

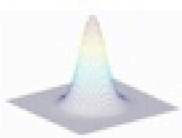
Coherent Two Qubit Interactions

Maximally entangled two-qubit states









$$H_{JC} = \sum_{i=1,2} \frac{\hbar\omega_i}{2}\sigma_i^z + \hbar\omega_R a^\dagger a + \sum_{i=1,2} \hbar\frac{g_i^2}{\Delta_i} a^\dagger a + \hbar J(\sigma_1^- \sigma_2^+ + \sigma_2^- \sigma_1^+)$$

a) $|\omega_1 - \omega_2| >> J \quad g \ll \Delta$

Truncated Hamiltonian

$$H_{JC} = \sum_{i=1,2} \frac{\hbar\omega_i}{2}\sigma_i^z + \hbar\omega_R a^\dagger a + \sum_{i=1,2} \hbar\frac{g_i^2}{\Delta_i} a^\dagger a$$

Excited Eigenstates

$|\uparrow\downarrow 0\rangle \quad |\downarrow\uparrow 0\rangle$

b) $\omega_1 = \omega_2 \quad g \ll \Delta$

Eigenenergies

$$E_\pm = \pm\hbar J$$

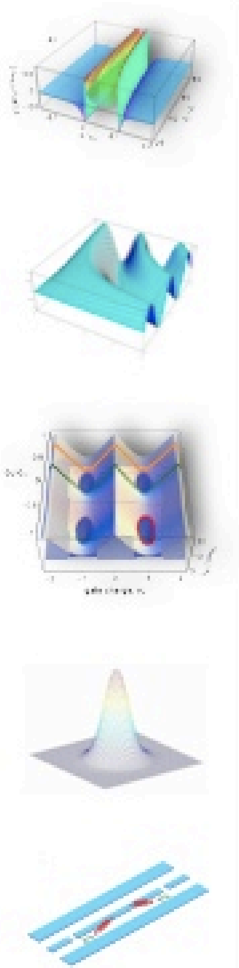
$+$ triplett state

$-$ singlet state

Eigenstates = Bell States

$$|\Psi_\pm\rangle = \frac{1}{\sqrt{2}}(|\downarrow\uparrow 0\rangle \pm |\uparrow\downarrow 0\rangle)$$

Coherent Two Qubit Interactions

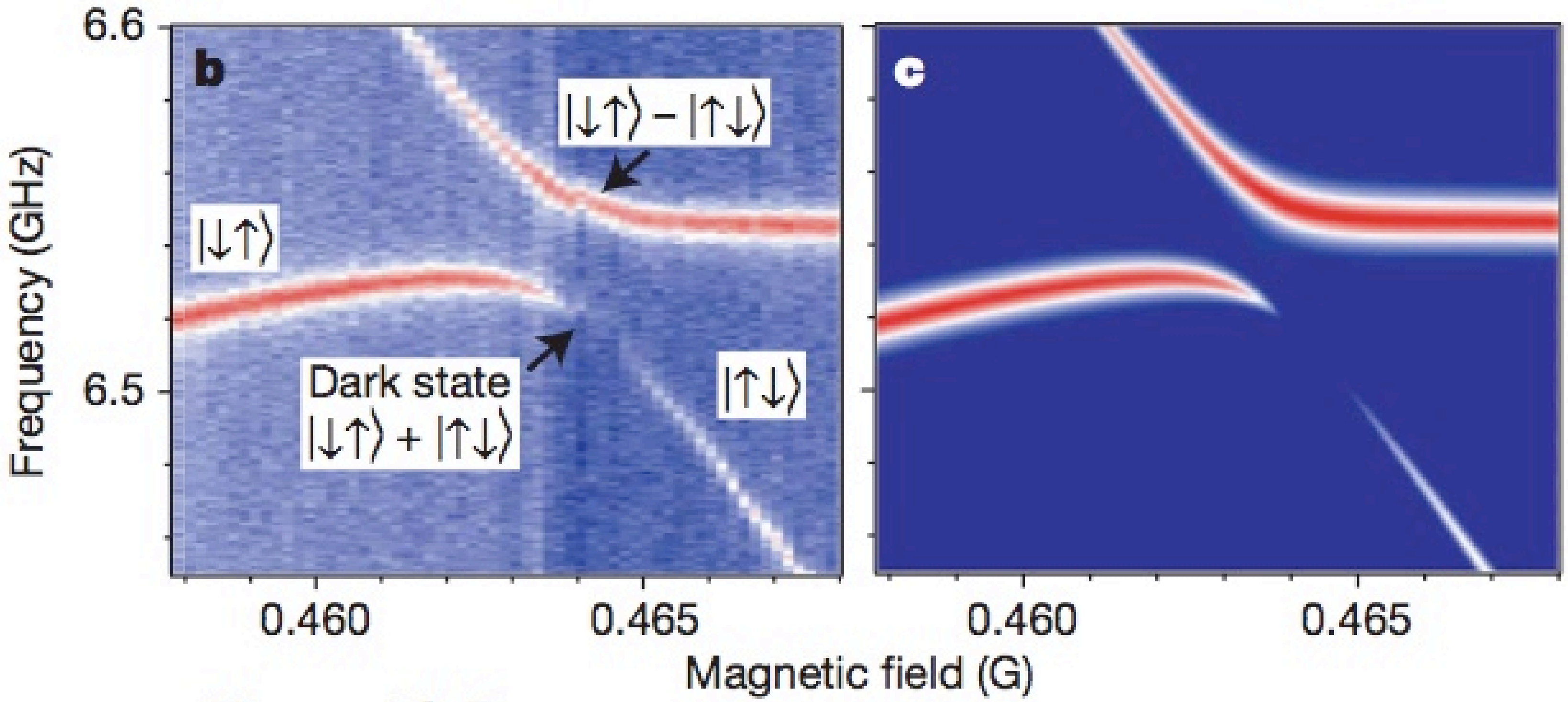


Direct Entanglement Observation

→ tune qubits through resonance $\omega_{1,2} = \omega_{1,2}^{\max}\sqrt{|\cos(\pi\Phi/\Phi_0)|}$

$$|\uparrow\downarrow 0\rangle \quad |\downarrow\uparrow 0\rangle \quad \rightarrow \quad |\Psi_\pm\rangle = \frac{1}{\sqrt{2}}(|\downarrow\uparrow 0\rangle \pm |\uparrow\downarrow 0\rangle)$$

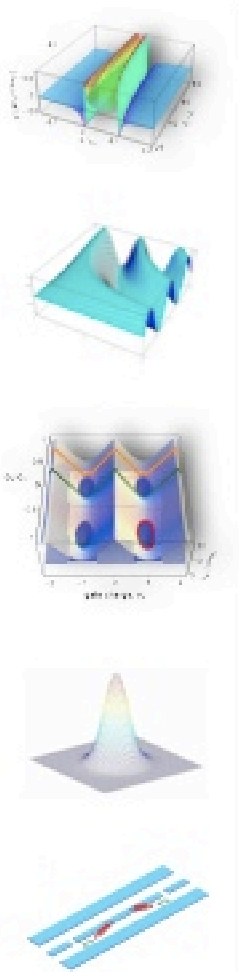
weak spectroscopic pulses so that $n \approx 0$ $\nu_s \neq \nu_R$



$$E_\pm = \pm\hbar J$$

J. Majer et al., Nature **449**, 443 (2007)

Multiplexed Readout of Qubits

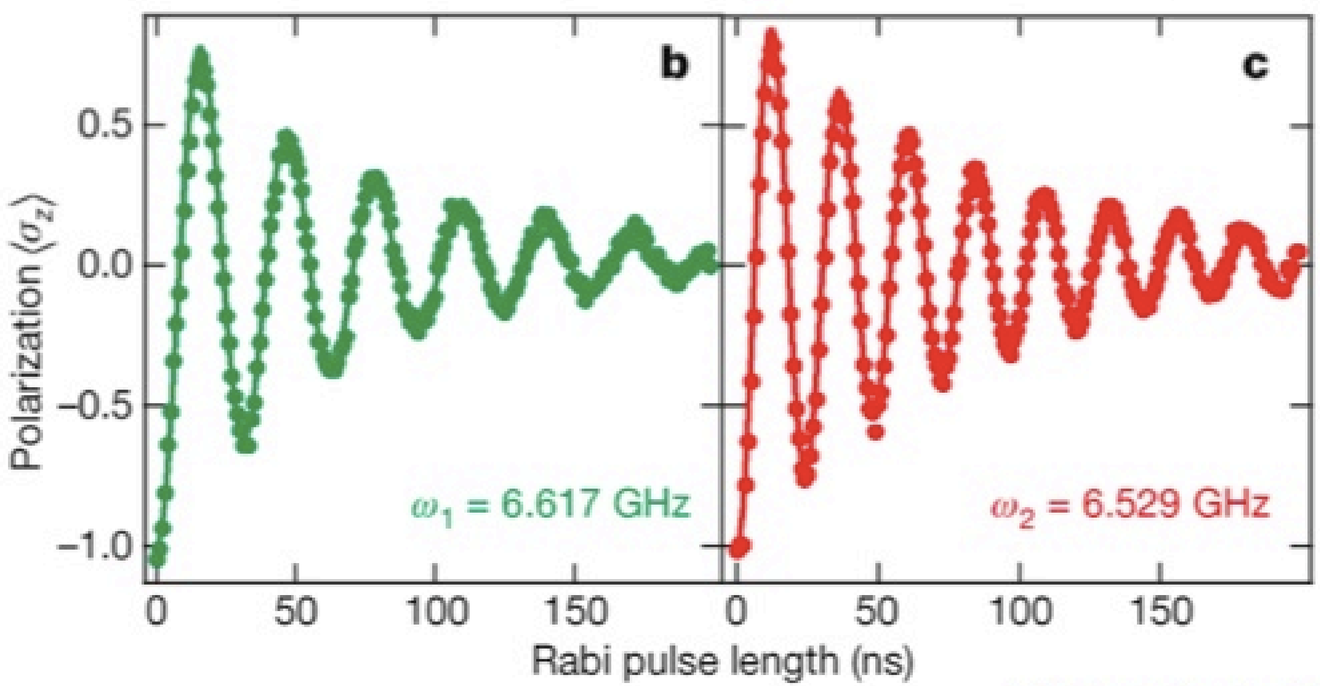


- Multiplex: information from multiple qubits can be obtained using only a single channel

- Required: possibility to address qubits independently

⟶ use flux bias: two qubits 88 MHz apart

⟶ qubit-qubit interaction negligible



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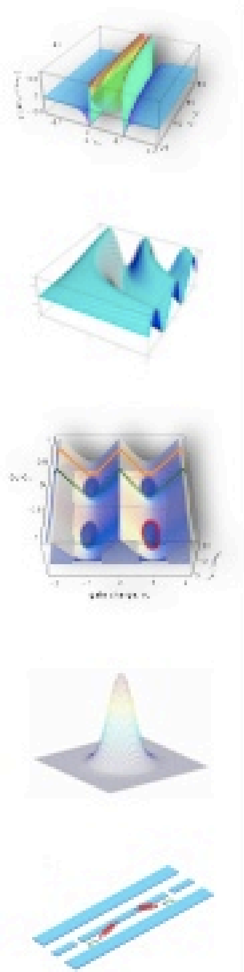
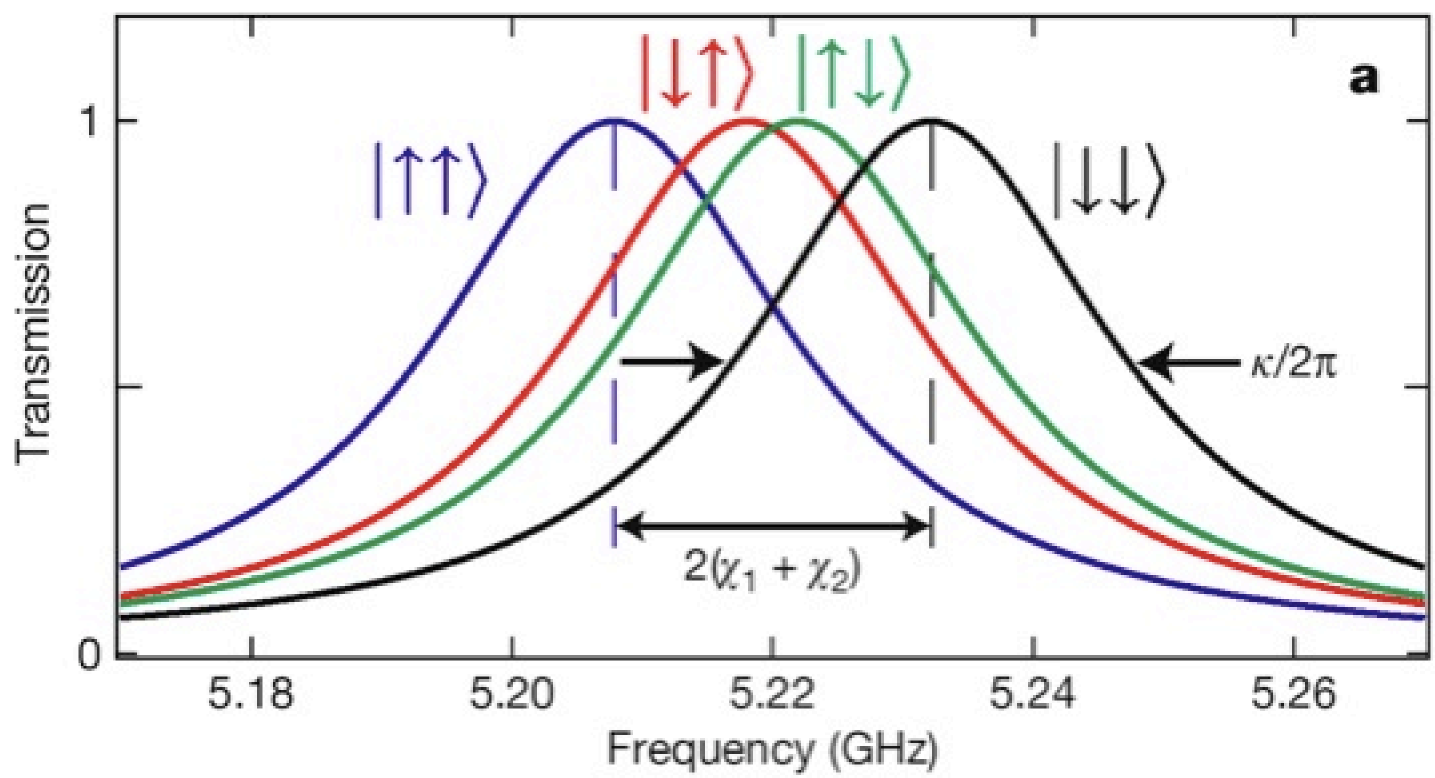
Multiplexed Readout of Qubits



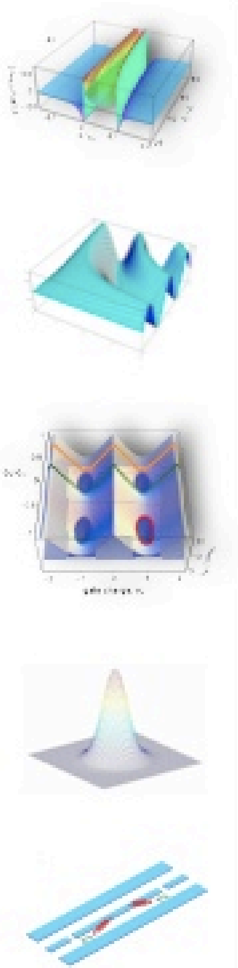
- qubits and cavity are dispersively coupled

→ qubit-state dependent frequency of resonator $\pm\chi_{1,2} = \pm\frac{g^2}{\Delta}$

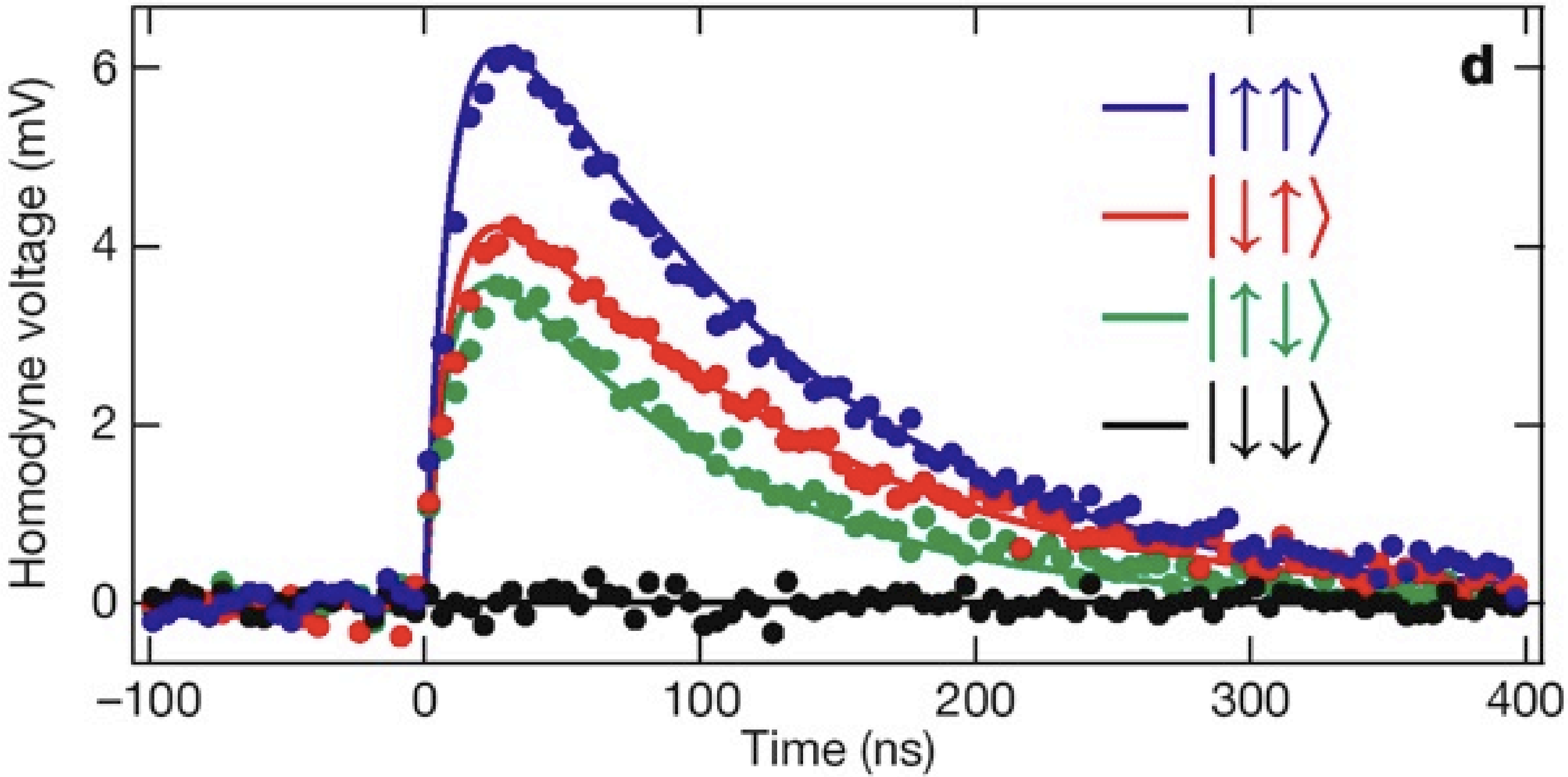
→ different cavity frequency shifts for the two qubits $(\chi_1 \neq \chi_2)$



Multiplexed Readout of Qubits



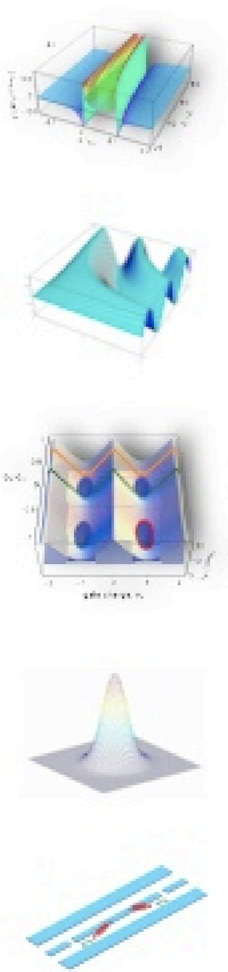
Homodyne response of cavity after π pulse on qubits



J. Majer et al., Nature **449**, 443 (2007)

Fast Coherent State Transfer between Qubits

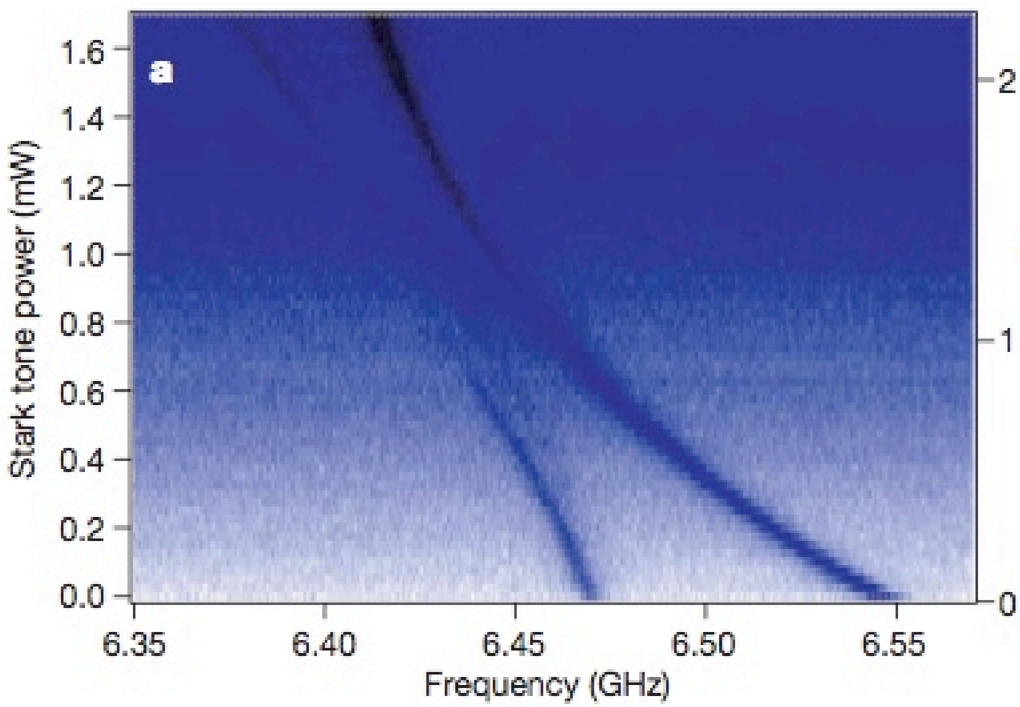
In practice: Φ switching of coupling very slow



use strongly detuned RF-pulses for fast state transfer $H = H_{JC} + H_D$

$$\Delta_a = \omega_a - \omega_d \quad \Delta_r = \omega_r - \omega_d \;\rightarrow\; \langle a^\dagger a \rangle \sim 0$$

Hamiltonian: off-resonant drive $H_z \approx \hbar \Delta_r a^\dagger a + \frac{\hbar}{2} \underbrace{\left(\widetilde{\omega}_a - \omega_d + \frac{2\varepsilon^2 g^2}{\Delta_r^2 \Delta_a} \right)}_{\omega_a'} \sigma_z$

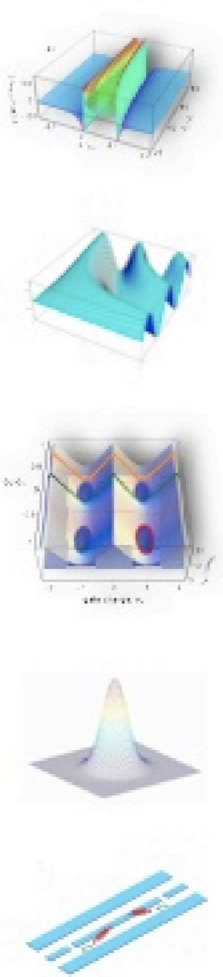


$$\omega''_{a_j} = \omega_{a_j} + \frac{\Omega_{R_j}^2}{2\Delta_{a_j}} + 2\frac{g_j^2}{\Delta_j'}\left(\langle a^\dagger a \rangle + \frac{1}{2} \right),$$

A. Blais et al., Phys. Rev. A **75**, 032329 (2007)

J. Majer et al., Nature **449**, 443 (2007)

Realization of $\sqrt{i\mathrm{SWAP}}$ gate



2-Qubit Hamiltonian dispersive limit $g \ll \Delta$

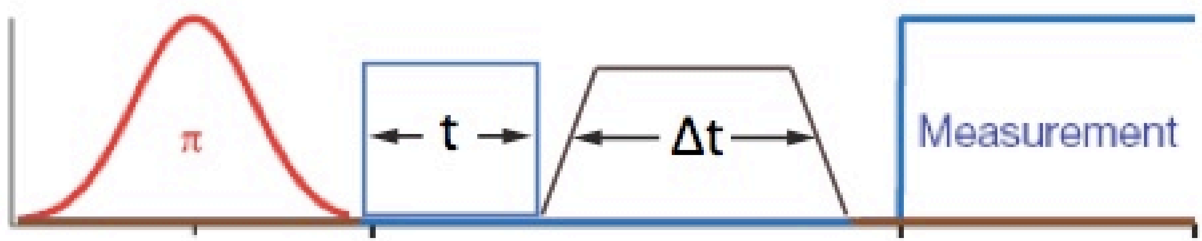
$$H_{JC} = \sum_{i=1,2} \frac{\hbar\omega_i}{2}\sigma_i^z + \hbar\omega_R a^\dagger a + \sum_{i=1,2} \hbar\frac{g_i^2}{\Delta_i}\sigma_i^z a^\dagger a + \hbar J(\sigma_1^- \sigma_2^+ + \sigma_2^- \sigma_1^+)$$

$$J = \frac{g_1 g_2(\Delta_1 + \Delta_2)}{2\Delta_1\Delta_2} \quad , \quad |\uparrow\downarrow 0\rangle \quad |\downarrow\uparrow 0\rangle \; ,$$

time evolution → let system evolve for time t

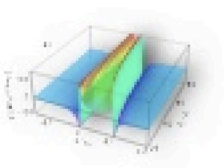
$$U_{\mathrm{JC}}(t,0) = \exp\left[-\frac{i}{\hbar}H_{JC}t\right] \qquad t = \pi\frac{\Delta_1\Delta_2}{2g_1g_2(\Delta_1+\Delta_2)}$$

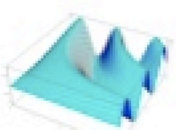
start

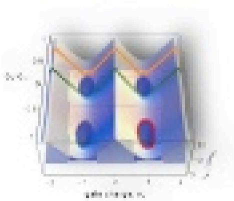


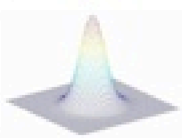
$|\downarrow\downarrow\rangle$ $\quad$ $|\downarrow\uparrow\rangle$ $\quad$ $|\uparrow\downarrow\rangle$ $\quad$ $|\uparrow\downarrow\rangle$

Perspectives & Conclusion









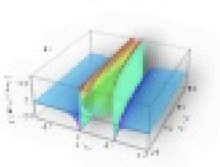
What has been achieved

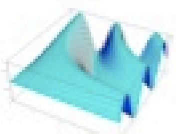
- strong coupling limit, Rabi oscillations, Stark shift, entanglement of qubit and resonator
- demonstration of universal gate operations and entanglement with locally coupled qubits
- realization of coherent information exchange bw. 2 qubits by non-local coupling over macroscopic distances
- increase of de-phasing and decoherence times (e.g. transmons)

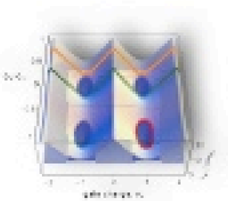
What remains to be done

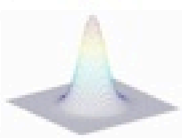
- further increase decoherence times (comparable to ion traps)
- realization of multi qubit algorithms
- demonstration of scalability strengths, e.g. coupling of multiple qubit-resonator systems
- reproduce-ability in fabrication
- hybrid quantum computation: investigate coupling possibilities with ion traps and quantum dots
- complete understanding of underlying quantum optics (e.g. Lambshift observation)

Perspectives & Conclusion









Advantages

- excellent scalability properties: in principle arbitrary qubit numbers possible
- based on existing chip fabrication technology
- good degrees of select-ability of individual qubits
- based on electronics rather than photonics, in analogy to classical computers
- QND readouts, well-established signal processing

Disadvantages

- fairly low coherence times, e.g. in comparison to ion traps
- so far only two qubit coupling achieved (e.g. compare NMR)
- currently limited to microwave regime, thus operation times limited to a few ns
- # of operations within one coherence time: currently not enough for quantum error correction