ION TRAPS – STATE OF THE ART QUANTUM GATES

Silvio Marx & Tristan Petit
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I. Fault-tolerant computing & the Mølmer-Sørensen gate with ion traps

II. Quantum Toffoli gate
I. Fault-tolerant computing & the Mølmer-Sørensen gate

Towards fault-tolerant quantum computing with ion traps,
J. Benhelm et al., Nature 2008,
doi:10.1038/nphys961
**Mølmer-Sørensen Gate**

- **Motivation**
  - Ion traps are a promising candidate for universal quantum computation
  - Fault tolerant computing only if the errors are small
  - High fidelity single & multi qubit gates are needed
  - Single qubit gates have low error rates
  - Multi qubit gates are more difficult to perform
  - Error range $\sim 10^{-2} - 10^{-4}$
  - Recently shown: 2 qubit entangling gate with high fidelity
    “Mølmer-Sørensen gate“
MOŁMER-SØRENSEN GATE

- Properties of the gate
  - Scalable multi qubit entangling gate
  - Performs collective “spin“ flips

- Experimental setup
  - Paul trap w/ two $^{40}$Ca$^+$ ions
  - Bichromatic laser field
    \[ \omega_{+/-} = \omega_0 +/- \delta, \delta > \upsilon \]
    \[ \upsilon \text{ the phonon frequency} \]
Mølmer-Sørensen Gate

- Paul trap at the University of Innsbruck
Mølmer-Sørensen Gate

Procedure

- Doppler cooling & optical pumping with a laser for initialization to the ground state $|SS>$
- Applying the bichromatic laser field (gate)
- Readout with the CCD camera
Mølmer-Sørensen Gate

- Energy scheme for the $^{40}\text{Ca}^+$ ions

![Diagram of energy levels for $^{40}\text{Ca}^+$ ions with transitions indicated by arrows and wavelengths.]
**Mølmer-Sørensen Gate**

- **Gate mechanism**

  - $\omega_{+/} = \omega_0 +/\!/- \delta$, $\delta > \nu$, $\omega_0$: single ion excitation frequency
  - Gate operation: interference of 4 2-photon-processes
Mølmer-Sørensen Gate

- Final measurements

- $|SS\rangle \rightarrow \tau_{\text{gate}} \rightarrow e^{i\phi} |SS\rangle |DD\rangle \rightarrow \tau_{\text{gate}} \rightarrow |DD\rangle$
  - Black: probability $p_2$ of finding 2 ions in state $|S\rangle$
  - Red: probability $p_1$ of finding 1 ion in state $|S\rangle$
  - Blue: probability $p_0$ of finding 0 ions in state $|S\rangle$
**Mølmer-Sørensen Gate**

- Max. entanglement for $\tau = m \cdot \tau_{\text{gate}}$, $m=1,3,...$
- “Spin“ flip for $m=2,4,...$

- Test the fidelity of the gate after multiple operations

- 21 successive gate operations shown, $\tau_{\text{gate}} = 50\mu s$
MØLMER-SØRENSEN GATE

Gate imperfections as function of pulse length

- **Blue**: state populations of $p_0 + p_2$
- **Red**: resulting Bell state fidelity
- **Black**: magnitude of coherence of the system
Conclusions

- High Bell state fidelity of $F=99.3(1)\%$ achieved
- Infidelity is less than $10^{-2}$ threshold
- Further advances needed
- Good candidate for multi qubit entangling gates with single laser interaction for more than 2 qubits
Mølmer-Sørensen Gate

References

- Towards fault-tolerant quantum computing with ion traps, J. Benhelm et al., Nature 2008, doi:10.1038/nphys961
- Deterministic entanglement of ions in thermal states of motion, G. Kirchmair et al., arXiv:0810.0670v1
- Optimierung verschränkender Quantengatter für Experimente mit Ionenfallen, Volckmar Nebendahl, Diplomarbeit 2008, Universität Hamburg
II. TOFFOLI GATE

Realization of the quantum Toffoli gate with trapped ions,
Monz, T; Kim, K; Haensel, W; et al. (not yet published)
PLAN

1. What is a Toffoli gate?
2. Why use a single gate?
3. General Principle
4. Results
5. Conclusion
1. **What is a Toffoli Gate?**

- Performs a NOT operation on a target qubit depending on the state of two control qubits

\[
\begin{pmatrix}
0 & 0 & 0 \\
0 & 0 & 1 \\
0 & 1 & 0 \\
0 & 1 & 1 \\
1 & 0 & 0 \\
1 & 0 & 1 \\
1 & 1 & 1 \\
1 & 1 & 0 \\
\end{pmatrix}
= 
\begin{pmatrix}
1 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 \\
0 & 0 & 1 & 0 \\
0 & 0 & 0 & 1 \\
\end{pmatrix}
\]

- Application in quantum error correction
2. Why use a single gate?

- Could be done with concatenated two-qubit gates

- Advantages of a single gate:
  - Simplify complex quantum operations
  - Higher fidelity
  - Faster
3. **General Principle**

- System = string of $^{40}\text{Ca}^+$ ions confined in a linear Paul trap

- Ground state: \( S_{1/2}(m=-1/2) = |S\rangle \equiv |1\rangle \)
- Excited state: \( D_{5/2}(m=-1/2) = |D\rangle \equiv |0\rangle \)

- Use of the centre-of-mass (COM) vibrational mode of the ion string as intermediate
3. General Principle (II)

- 3 major steps:

1. Encoding of the joint quantum information of the control qubits $|c_1\rangle$ and $|c_2\rangle$ in the vibrational COM mode

2. NOT operation on the target qubit controlled by the vibrational mode

3. Decoding of the qubits (reversal of the encoding step)
3. **GENERAL PRINCIPLE (III)**

- Ideal unitary map implemented:

\[
U_T = \exp(-i\pi \frac{1}{2\sqrt{2}} \sigma_{Z,t}) = \begin{pmatrix}
1 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 1 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 1 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 1 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 1 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & -i \\
0 & 0 & 0 & 0 & 0 & 0 & i
\end{pmatrix}
\]

In the basis

\[\{|c_1, c_2, t\rangle = \{|DDD\rangle, |DDS\rangle, |DSD\rangle, |DSS\rangle, |SDD\rangle, |SDS\rangle, |SSD\rangle, |SSS\rangle\}\]
3. **Encoding Section**

State of the quantum qubits at the end:

\[
\begin{align*}
|SS, 0\rangle & \rightarrow |DD, 2\rangle \\
|DS, 0\rangle & \rightarrow \sin \frac{\pi}{2\sqrt{2}} |DD, 1\rangle + \cos \frac{\pi}{2\sqrt{2}} |DS, 0\rangle \\
|SD, 0\rangle & \rightarrow \cos \frac{\pi}{2\sqrt{2}} |DD, 1\rangle - \sin \frac{\pi}{2\sqrt{2}} |DS, 0\rangle \\
|DD, 0\rangle & \rightarrow |DD, 0\rangle
\end{align*}
\]
3. **Information in the COM mode**

- Initially it contains no phonons

  $$|\text{vib}\rangle = |n = 0\rangle$$

- Encoding 2 phonons (\( |c_1 c_2\rangle = |SS\rangle \)) or 1 phonon (other cases)

- Removal of one phonon

  $$\begin{align*}
  (c_1 \text{ AND } c_2) = 1 &\Rightarrow |n = 1\rangle \\
  (c_1 \text{ AND } c_2) = 0 &\Rightarrow |n = 0\rangle
  \end{align*}$$
4. RESULTS

- Probabilities of 81 (± 5)% that the ion ends up in the correct output state.
4. COMPARAISON OF X-MATRIX

- Obtained by quantum process tomography

Basis: $\sigma_c^{1} \otimes \sigma_c^{2} \otimes \sigma_t \in \{I \otimes I \otimes I, I \otimes I \otimes X, \ldots, Z \otimes Z \otimes Z\}$

- Mean fidelity of approximately 71%
4. SOURCES OF INFIDELITY

- Mainly due to technical imperfections:
  - Rabi frequency imprecisions (12%)
  - Temperature changes and voltage fluctuations (7%)
  - Initialization of the COM mode in the ground state (1%)
  - Laser linewidth and magnetic field fluctuations (1%)
  - Ion state initialization (0.5% per ion)

  - Statistical uncertainties in the tomographic measurements

- Decoherence time of quantum information stored in the vibrational mode: 85ms
5. **Comparison with CNOT Gates Based Realization**

- Need of 6 CNOT gates to make a Toffoli gate

- Fidelity: $F = (92.6\%)^6 \sim 63\%$

- Duration of the gate = 3 times longer than the 3-qubits gate (1.5ms)
QUESTIONS?
## Pulse sequence

<table>
<thead>
<tr>
<th>Pulse</th>
<th>Comment</th>
<th>Logical part</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 $R_1^+ (\pi, \frac{3\pi}{2})$</td>
<td>Encode first target qubit onto motion</td>
<td>Encoding</td>
</tr>
<tr>
<td>2 $R_2^+ \left( \frac{\pi}{\sqrt{2}}, \frac{3\pi}{2} \right)$</td>
<td>Encode second target qubit onto motion</td>
<td></td>
</tr>
<tr>
<td>3 $R_1^+ \left( \frac{\pi}{2\sqrt{2}}, \frac{\pi}{2} \right)$</td>
<td>Composite pulse to remove one phonon</td>
<td></td>
</tr>
<tr>
<td>4 $R_1^+ (\pi, 0)$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>5 $R_1^+ \left( \frac{\pi}{2\sqrt{2}}, \frac{\pi}{2} \right)$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>6 $R_3 \left( \frac{\pi}{2}, 0 \right)$</td>
<td>Prepare target qubit for motion controlled NOT</td>
<td>controlled NOT</td>
</tr>
<tr>
<td>7 $R_3^+ \left( \frac{\pi}{2}, 1 \right)$</td>
<td>Composite phase gate</td>
<td></td>
</tr>
<tr>
<td>8 $R_3^+ \left( \sqrt{2}\pi, \frac{\pi}{2} \right)$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>9 $R_3^+ \left( \frac{\pi}{2}, 0 \right)$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>10 $R_3 \left( \frac{\pi}{2}, \left( \frac{1}{\sqrt{2}} - 1 \right)\pi \right)$</td>
<td>Complete motion controlled NOT on target qubit</td>
<td></td>
</tr>
<tr>
<td>11 $R_1^+ \left( \frac{\pi}{2\sqrt{2}}, \left( -\frac{1}{2} + \frac{1}{\sqrt{2}} \right)\pi \right)$</td>
<td>Undo encoding algorithm</td>
<td>Decoding</td>
</tr>
<tr>
<td>12 $R_1^+ \left( \pi, \left( -1 + \frac{1}{\sqrt{2}} \right)\pi \right)$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>13 $R_1^+ \left( \frac{\pi}{2\sqrt{2}}, \left( -\frac{1}{2} + \frac{1}{\sqrt{2}} \right)\pi \right)$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>14 $R_2^+ \left( \frac{\pi}{\sqrt{2}}, \left( \frac{1}{2} + \frac{1}{\sqrt{2}} \right)\pi \right)$</td>
<td>Decoding finished for second control qubit</td>
<td></td>
</tr>
<tr>
<td>15 $R_1^+ \left( \pi, \left( \frac{1}{2} + \frac{1}{\sqrt{2}} \right)\pi \right)$</td>
<td>Decoding finished for first control qubit, Toffoli complete</td>
<td></td>
</tr>
</tbody>
</table>