Cavity QED with Superconducting Circuits

coherent quantum mechanics with individual photons and qubits ...

... in superconducting circuits:

circuit quantum electrodynamics

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Circuit Quantum Electrodynamics

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elements
- the cavity: a superconducting 1D transmission line resonator with large vacuum field $E_0$ and long photon life time $1/\kappa$
- the artificial atom: a Cooper pair box with large $E_J/E_C$ with large dipole moment $d$ and long coherence time $1/\gamma$

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Vacuum Field in 1D Cavity

voltage across resonator in vacuum state \((n = 0)\)

\[ V_{0,\text{rms}} = \sqrt{\frac{\hbar \omega_r}{2C}} \approx 1 \mu V \]

\[ E_0 = \frac{V_{0,\text{rms}}}{b} \approx 0.2 V/m \]

for \(\omega_r/2\pi \approx 6 \text{ GHz} \) \((C \approx 1 \text{ pF})\), \(b \approx 5 \mu m\)

harmonic oscillator

\[ H_r = \hbar \omega_r \left( a^\dagger a + \frac{1}{2} \right) \]

\(\times 10^6\) larger than \(E_0\)
in 3D microwave cavity

Resonator Quality Factor and Photon Lifetime

resonance frequency:

\[ \nu_r = 6.04 \text{ GHz} \]

quality factor:

\[ Q = \frac{\nu_r}{\delta \nu_r} \approx 10^1 \]

photon decay rate:

\[ \frac{k}{2\pi} = \frac{\nu_r}{Q} \approx 0.8 \text{ MHz} \]

photon lifetime:

\[ T_\kappa = \frac{1}{k} \approx 200 \text{ ns} \]
Qubit/Photon Coupling in a Circuit

\[ \hbar g = e V_{0,\text{rms}} \frac{C_g}{C_\Sigma} \]

\[ \nu_{\text{vac}} = \frac{g}{\pi} \approx 1 \ldots 300 \text{ MHz} \]

\( g \gg [\kappa, \gamma] \) possible!

large effective dipole moment

\[ d = \frac{\hbar g}{E_0} \sim 10^2 \ldots 10^4 e a_0 \]

Circuit QED with One Photon

superconducting cavity QED circuit

Resonant Vacuum Rabi Mode Splitting ...

... with one photon ($n = 1$):

very strong coupling:

$$g_{ge}/\pi = 308 \text{ MHz}$$

$$\kappa, \gamma < 1 \text{ MHz}$$

$$g_{ge} \gg \kappa, \gamma$$

forming a 'molecule' of a qubit and a photon

$$|1\pm\rangle = (|g, 1\rangle \pm |e, 0\rangle)/\sqrt{2}$$

this data: J. Fink et al., *Nature (London)* 454, 315 (2008)

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How to Measure Single Microwave Photons

- average power to be detected

$$\rightarrow \langle n = 1 \rangle \hbar \omega_r \kappa/2 \approx P_{RF} = -140 \text{ dBm} = 10^{-17} \text{ W}$$

- efficient with cryogenic low noise HEMT amplifier ($T_N = 6 \text{ K}$)
- prevent leakage of thermal photons (cold attenuators and circulators)
Measurement Setup

sample mount

Microwave electronics

20 mK cryostat

cold stage

Read-Out ...

... of a superconducting charge qubit
Qubit Read Out

<table>
<thead>
<tr>
<th>QUBIT</th>
<th>ON</th>
<th>OFF</th>
</tr>
</thead>
<tbody>
<tr>
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\[ |0\rangle \quad \text{on state} \]

\[ |1\rangle \quad \text{off state} \]

desired:
- good on/off ratio
- no relaxation in on state (QND)

Read Out Strategies

demolition measurements (switching/latching measurements)

Quantronium (Saclay, Yale)
Flux Qubit (TU Delft, NEC)
Phase Qubit (NIST, UCSB)

quantum non-demolition (QND) measurements

Yale (circuit QED)
also: Chalmers, Delft, Yale (JBA)
Non-Resonant Qubit-Photon Interaction

approximate diagonalization in the dispersive limit $|\Delta| = |\omega_a - \omega_r| \gg g$

\[
H \approx \hbar \left( \omega_r + \frac{g^2}{\Delta} \sigma_z \right) a^\dagger a + \frac{1}{2} \hbar \left( \omega_a + \frac{g^2}{\Delta} \right) \sigma_z
\]

\begin{align*}
&\text{cavity frequency shift} & &\text{Lamb shift} \\
&|g\rangle & &|1\rangle \\
&|e\rangle & &|0\rangle \\
\end{align*}

\[
\omega_r + \frac{g^2}{\Delta} \quad |1\rangle \\
\omega_r - \frac{g^2}{\Delta} \quad |0\rangle
\]

\[
|g\rangle : \epsilon > 1 \\
|e\rangle : \epsilon < 1
\]

A. Blais et al., PRA 69, 062320 (2004)
Qubit Spectroscopy with Dispersive Read-Out ...

... additional material

Measurement Technique

- measurement of microwave transmission amplitude $T$ and phase $\phi$

- intra-cavity photon number controllable from $n \sim 10^3$ to $n \ll 1$
**Dispersive Shift of Resonance Frequency**

Sketch of qubit level separation:

\[ \Delta = 2\pi \delta > g \]

\[ g/\pi = \nu_{\text{vac}} = 11 \text{ MHz} \]

\[ \Delta(n_g = 1)/2\pi = 66 \text{ MHz} \]

\[ n = 10 \]

**Qubit Spectroscopy with Dispersive Read-Out**

Qubit in ground state
**CW Spectroscopy of Cooper Pair Box**

![Plot of phase shift vs gate charge for different frequencies](image)

- detuning $\Delta_{r,a}/2\pi \sim 100$ MHz
- extracted: $E_J = 6.2$ GHz, $E_C = 4.8$ GHz

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**Line Shape**

excited state population (steady-state Bloch equations):

$$P_e = 1 - P_g = \frac{1}{21 + (T_2\Delta_{s,a})^2 + \Omega_R^2 T_1 T_2} \Omega_R^2 T_1 T_2$$

- fixed drive $P_s \propto \Omega_R^2 = n_s \omega_v$ 
- varying $\Delta_{s,a} = \omega_s - \tilde{\omega}_a$
- weak continuous measurement ($n \sim 1$)
- at charge degeneracy ($n_g = 1$)
Excited State Population

![Graph showing excited state population vs. spec. power (μW)]

\[ P_e = 1 - P_g = \frac{1}{21 + \Omega_R^2 T_1 T_2} \]

Line Width

![Graph showing line width vs. spec. power (μW)]

\[ 2\pi \delta \nu_{\text{HWHM}} = \frac{1}{T_2} = \sqrt{\frac{1}{T_2^2} + \Omega_R^2 T_1 T_2} \]

Min(\( \delta \nu_{\text{HWHM}} \)) \( \sim \) 750 kHz \( \rightarrow T_2 > 200 \text{ ns} \)
Coherent Control of a Qubit in a Cavity

- qubit state represented on a Bloch sphere
- vary length, amplitude and phase of microwave pulse to control qubit state

Qubit Control and Readout

- continuous
- dispersive
- quantum non-demolition
- in good agreement with predictions

Wallraff, Schuster, Blais, ... Girvin, and Schoelkopf,
Varying the Control Pulse Length

2\pi rotation

\lvert g \rangle

3\pi rotation

\lvert e \rangle

Wallraff, Schuster, Blais, ..., Girvin, Schoelkopf, *PRL* 95, 060501 (2005)

High Visibility Rabi Oscillations

Rabi oscillations:

visibility 95 ± 5%

for superconducting qubits:

- high visibility
- well characterized and understood measurement
- good control accuracy

Rabi Frequency

pulse scheme:

Rabi oscillations:

Rabi frequency:

- linear dependence of Rabi frequency on microwave amplitude

Rabi rotation pulse sequence:

experimental Bloch vector:

experimental density matrix:

\[ \Delta t = 36 \text{ ns} \]
Measurements of Coherence Time

Coherence Time Measurement: Ramsey Fringes

pulse scheme:

\[ B_z \]

\[ |g\rangle \quad |e\rangle \]

\[ \text{Ramsey fringes:} \quad T_2 \approx 500 \text{ ns} \]

\[ \text{Ramsey frequency, } \gamma_{\text{Ramsey}} \text{ [MHz]} \]

\[ \text{detuning, } \Delta \omega [\text{MHz}] \]

\[ 0 \quad 40 \quad 80 \quad 120 \quad 160 \quad 200 \quad 240 \quad 280 \quad 320 \quad 360 \quad 400 \quad 440 \quad 480 \quad 520 \quad 560 \quad 600 \]

\[ 0 \quad 20 \quad 40 \quad 60 \quad 80 \]

pulse sequence:

\[
\begin{align*}
\pi/2 & \quad \Delta t & \quad \pi/2 \quad x \\
\pi/2 \quad y & \quad \pi & \quad x \\
\text{Measure} & \\
\end{align*}
\]

experimental Bloch vector:

experimental density matrix:

\[\Delta t = 54 \text{ ns}\]

Decoherence ...

... additional material
Sources of Decoherence

• remove sources of decoherence
  • improve materials

• use dynamic methods to counteract specific sources of decoherence
  • spin echo
  • geometric manipulations

• reduce sensitivity of quantum systems to specific sources of decoherence
  • make use of symmetries in design and operation

Tomography of a Spin Echo

pulse sequence:

result:

- refocusing
- elimination of low frequency fluctuations
- increased effective coherence time

Lars Steffen et al. (2009)
P. J. Leek, J. Fink et al., Science 318, 1889 (2007)

L. Steffen et al., Quantum Device Lab, ETH Zurich (2008)
Coupling Superconducting Qubits and Generating Entanglement using Sideband Transitions

2-Qubit Chip

- Two near identical superconducting qubits
- Local control of magnetic flux allows independent selection of qubit transition frequencies
- Local drive lines allow selective excitation of individual qubits

2-Qubit Circuit with Selective Control

Joint dispersive read-out

Local magnetic fields created using small inductively coupled coils

Selective qubit excitation using locally capacitively coupled drive lines


Sideband Transitions in Circuit QED

- dispersive coupling allows joint excitations to be driven
- sideband transitions forbidden to first order: use two photon transition

\[ \omega_A/2 = (\omega_R + \omega_A)/2 \]

Wallraff et al., PRL 99, 050501 (2007)
simultaneous excitation of qubit and resonator: $|g,0\rangle \rightarrow |e,1\rangle$

entangle a qubit with a photon on the bus: $|g,0\rangle \rightarrow |g,0\rangle + |e,1\rangle$

Bell State Preparation

Qubit A

$\pi$ pulse qubit A

Transfer entanglement to qubits to create $\Psi$ Bell state

Cavity

Entangle qubit B with cavity using blue sideband B

Qubit B

$|gg0\rangle \rightarrow |eg0\rangle \rightarrow \frac{1}{\sqrt{2}}(|eg0\rangle + |ee1\rangle) \rightarrow \frac{1}{\sqrt{2}}(|eg\rangle + |ge\rangle) \otimes |0\rangle$
Bell State Preparation

Qubit A

Cavity

Qubit B

π pulse qubit A to convert to \( \Phi \) Bell state

π pulse qubit A

Transfer entanglement to qubit A to create \( \Psi \) Bell state

Characterise the entanglement with cavity state using quantum state tomography with joint measurement:

\[
\frac{1}{\sqrt{2}} (|eg\rangle + |ge\rangle) \otimes |0\rangle
\]

Experimental state fidelity:

\( F = 86\% \)

Concurrence:

\( 0.541 \)

Entanglement formation:

\( 0.371 \)

Overlap with calculation:

\( F = 99\% \)

\[ |\Psi_+ \rangle = \frac{1}{\sqrt{2}} (|ge \rangle + |eg \rangle) \]

experimental state fidelity:  
F = 86%  
concurrence:  
0.518  
entanglement of formation:  
0.374  

overlap with calculation  
F = 99%  


DiVincenzo Criteria fulfilled for Superconducting Qubits

for Implementing a Quantum Computer in the standard (circuit approach) to quantum information processing (QIP):

#1. A scalable physical system with well-characterized qubits. ✔

#2. The ability to initialize the state of the qubits. ✔

#3. Long (relative) decoherence times, much longer than the gate-operation time. ✔

#4. A universal set of quantum gates. ✔

#5. A qubit-specific measurement capability. ✔

plus two criteria requiring the possibility to transmit information:

#6. The ability to interconvert stationary and mobile (or flying) qubits. ✔

#7. The ability to faithfully transmit flying qubits between specified locations. ✔