Dispersive Regime for Quantum Computation
Non-Resonant (Dispersive) Interaction

approximate diagonalization: \( |\Delta| = |\omega_a - \omega_r| \gg g \):

\[
H \approx \hbar \left( \omega_r + \frac{g^2}{\Delta} \sigma_z \right) a^\dagger a + \frac{\hbar}{2} \left( \omega_a + \frac{g^2}{\Delta} \right) \sigma_z
\]

\[
\text{//}
\]

cavity frequency shift and qubit ac-Stark shift

\[
\text{Lamb Shift}
\]

\[
|0\rangle \quad |1\rangle \quad |2\rangle
\]

Transmission (arb. units)

\[
\omega_r - \frac{g^2}{\Delta} \quad \omega_r + \frac{g^2}{\Delta}
\]

A. Blais et al., PRA 69, 062320 (2004)
Circuit QED – read out of qubit state

- transmission measurement to determine qubit state:

  Phase sensitive measurement of transmitted microwave:

Voltage signal:

\[ A(t) \sin(\omega_m t + \phi(t)) \equiv I(t) \sin \omega_m t + Q(t) \cos \omega_m t \]
Circuit QED – read out of qubit state

• transmission measurement to determine qubit state:

\[ H = \hbar (\omega_r + \chi \sigma_z) a^\dagger a + \frac{\hbar}{2} (\omega_a + \chi) \sigma_z \]

state-dependent frequency shift \( \rightarrow \sigma_z \) determined
extendable to more qubits
excite qubit at $t<0$
measure transmitted field quadratures ($I$, $Q$) with microwave drive at resonance ($\omega_m = \omega_r - \chi$)
qubit in ground state: full resonator transmission (rise time given by $\kappa$)
qubit in excited state: only partial transmission until qubit decays to ground state

[Time dependent measurements]

[Bianchetti et al. PRA 80 (2009)]
Population reconstruction

Area between curves is proportional to qubit state population:
Coherent Single Qubit Control
Qubit control

apply microwave signal through resonator input
or through side-gate

time-dependent Hamiltonian for state manipulation

$$\hat{H} = \frac{1}{2} \hbar \omega_a \hat{\sigma}_z + \hbar \Omega_R \cos(\omega_b t + \phi_R) \hat{\sigma}_x$$
Coherent Control of a Qubit in a Cavity

- qubit state represented on a Bloch sphere
- vary length, amplitude and phase of microwave pulse to control qubit state
Qubit Control and Readout

initialize

$|e\rangle$

$|g\rangle$

control

$B_z$

$x$

$B_\perp \sin(\omega t)$

decay

$|e\rangle$

$|g\rangle$

$10^6$ averages

$I [mV]$ vs. $Time [\mu s]$

$1/\kappa \sim 0.1 \mu s$

$T_1 \sim 1 \mu s$
Coherent population transfer – Rabi Oscillations

Drive system at its resonance frequency with varying drive strength:

- High visibility (~99%)
- Well characterized and understood measurement
- Good control accuracy
State reconstruction single qubit

3 measurements for 3 coefficients $r_x$, $r_y$, $r_z$ of

$$\rho = \frac{1}{2}(\text{id} + r_x \sigma_x + r_y \sigma_y + r_z \sigma_z)$$
State reconstruction single qubit

3 measurements for 3 coefficients $r_x$, $r_y$, $r_z$ of

$$\rho = \frac{1}{2}(\text{id} + r_x \sigma_x + r_y \sigma_y + r_z \sigma_z)$$

Measurement along z-axis: $r_z = \langle \sigma_z \rangle = \text{Tr}[\rho \sigma_z]$
State reconstruction single qubit

3 measurements for 3 coefficients $r_x, r_y, r_z$ of

$$\rho = \frac{1}{2}(\text{id} + r_x \sigma_x + r_y \sigma_y + r_z \sigma_z)$$

Measurement along z-axis:  
$r_z = \langle \sigma_z \rangle = \text{Tr}[\rho \sigma_z]$

Rotation + measurement:  
$r_x = \langle \sigma_x \rangle = \text{Tr}[\left(\frac{\pi}{2}\right)_y \rho \left(\frac{\pi}{2}\right)_{-y} \sigma_z]$
State reconstruction single qubit

3 measurements for 3 coefficients $r_x$, $r_y$, $r_z$ of

$$\rho = \frac{1}{2} \left( \text{id} + r_x \sigma_x + r_y \sigma_y + r_z \sigma_z \right)$$

Measurement along z-axis: $r_z = \langle \sigma_z \rangle = \text{Tr}[\rho \sigma_z]$

Rotation + measurement: $r_x = \langle \sigma_x \rangle = \text{Tr}[\left(\frac{\pi}{2}\right)_y \rho \left(\frac{\pi}{2}\right)_y \sigma_z]$ $\cdot \sigma_z$

Rotation + measurement: $r_y = \langle \sigma_y \rangle = \text{Tr}[\left(\frac{\pi}{2}\right)_x \rho \left(\frac{\pi}{2}\right)_x \sigma_z]$
Control and Tomographic Read-Out of Single Qubit

Rabi rotation pulse sequence:

Delta t

\[ \pi/2 \ x \]
\[ \pi/2 \ y \]
\[ \pi \ x \]

Measure

Experimental Bloch vector:

Experimental density matrix:

\[ \Delta t = 0 \text{ ns} \]

L. Steffen et al., Quantum Device Lab, ETH Zurich (2008)