High fidelity quantum gates in trapped ions

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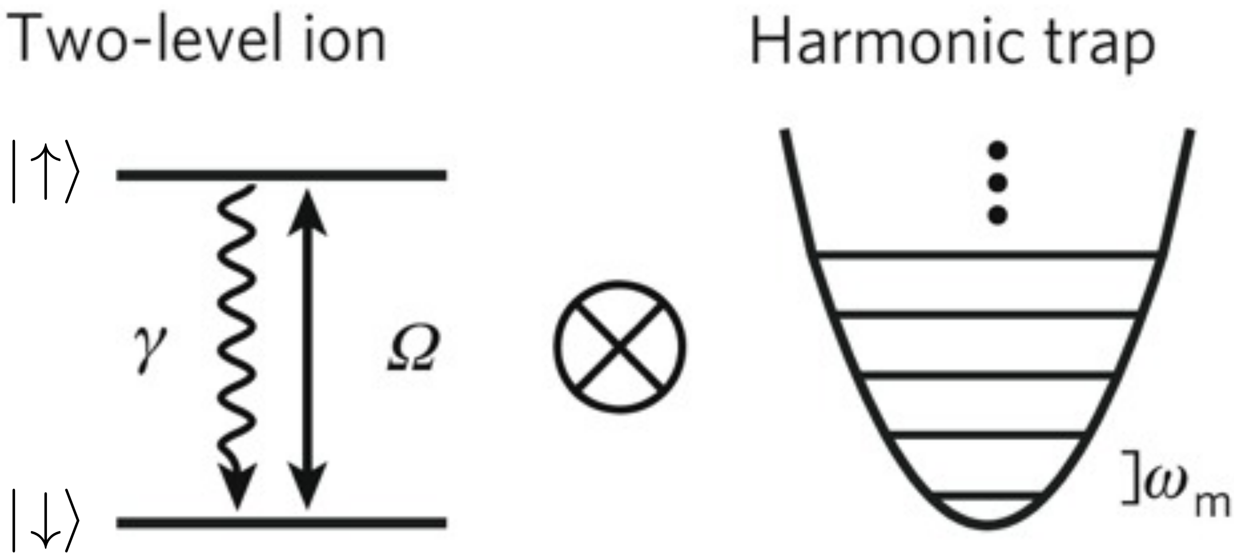
Introduction

Overview

- Introduction
- Realization of Qubit
  - Qubit
  - Trapping
  - Cooling
  - Readout
- Quantum Gates
  - Geometric Phase Gate
  - Mølmer-Sørensen Gate
- Comparison

Qubit

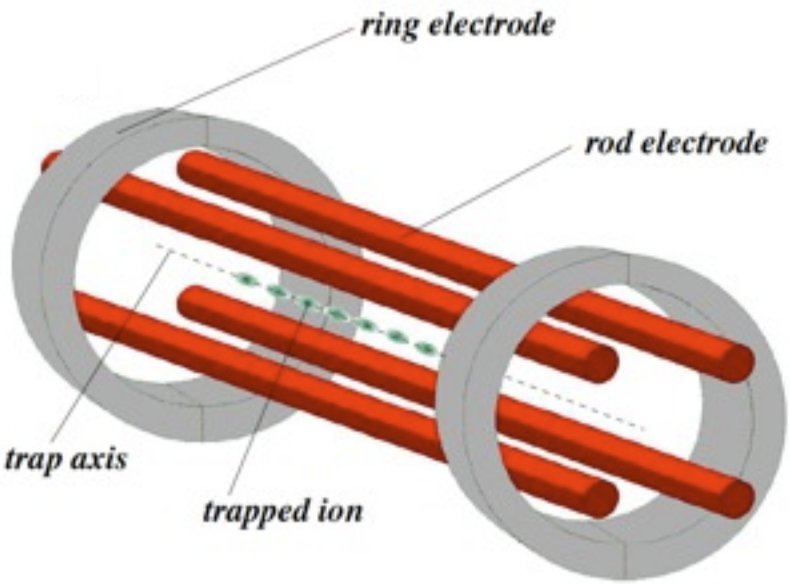
- Internal (atomic) and external (motional) degrees of freedom
- Electronic ion levels to store quantum information
- Laser field couples qubit levels

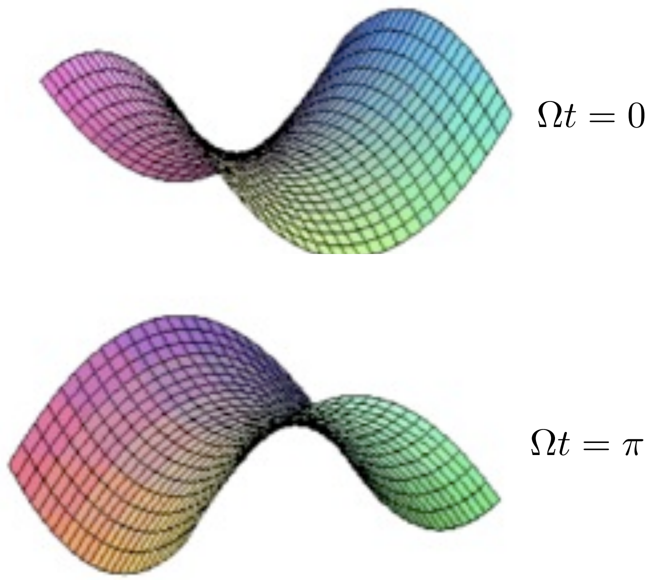


Trapping – Linear Paul Trap

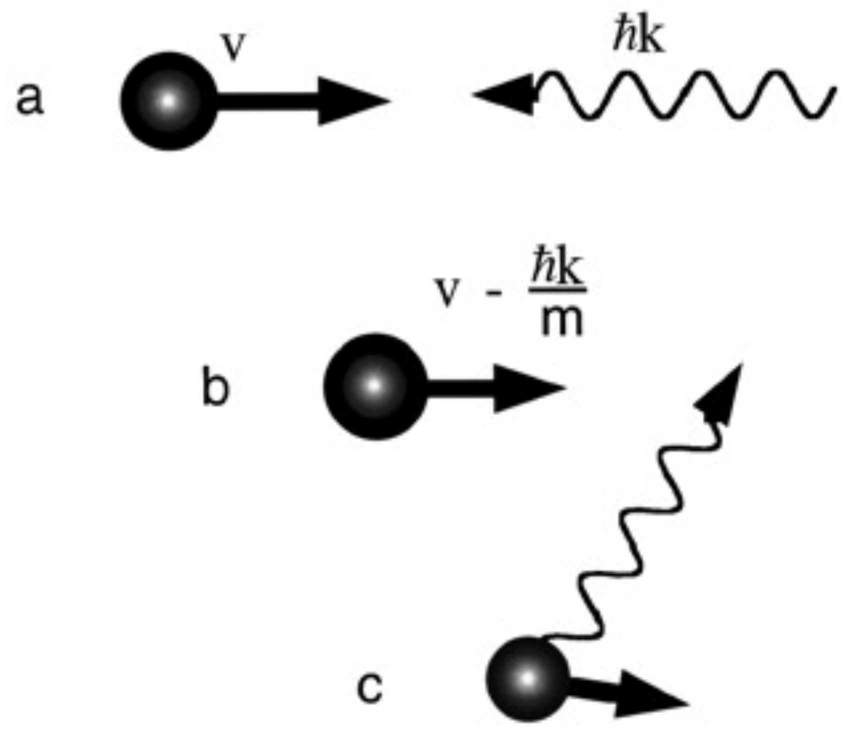
- Charged particles confined by EM fields
- oscillating potential between diagonal rods
- other rods are grounded

$$\Phi = \Phi_0 \frac{x^2 - y^2}{2r_0^2} \text{ where } \Phi_0 = U_0 + V_0 \cos \Omega t$$





Cooling



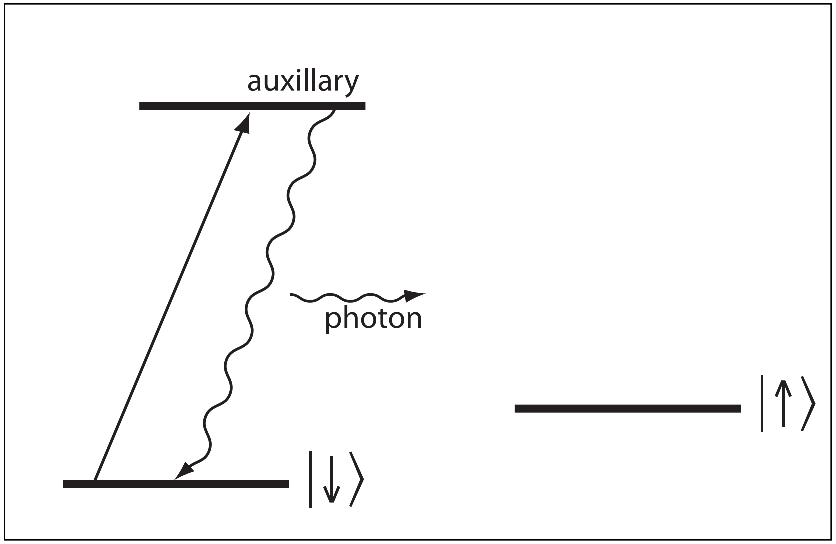
- Momentum change due to absorbed photon

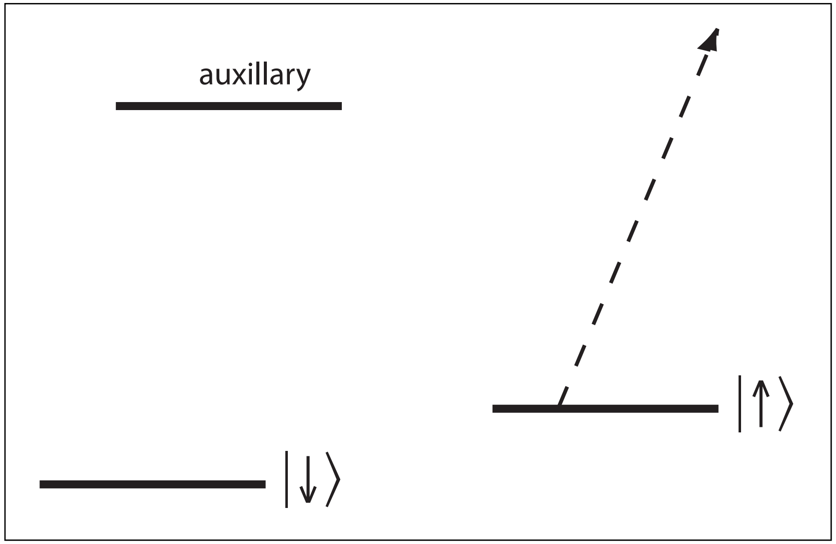
$$F = \frac{I}{\hbar\omega}\frac{\Gamma}{2}\frac{\Omega^2/2}{2\delta^2+\Omega^2/2+\Gamma^2/4}$$

- Spontaneously emitted photons randomly distributed
- Laser frequency red/blue shifted due to doppler effect
- Adjust laser frequency or transition frequency for absorption process

Readout

- Read out internal state





⇒ ion fluoresces

⇒ ion does not fluoresce

- Quantum non demolition measurement

Geometric Phase Gate

- Two qubit gate with two ions
- Acts on internal degrees of freedom

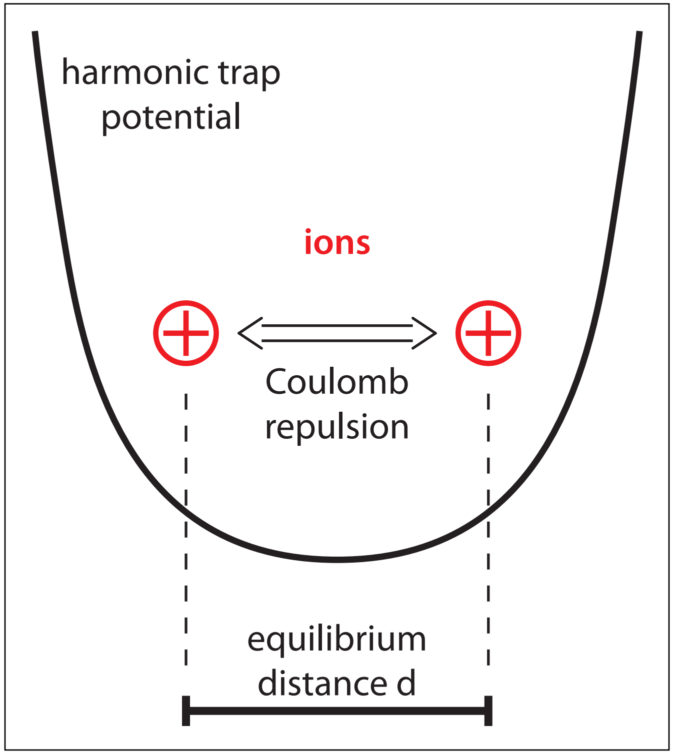
Input	Output
$\lvert\downarrow\downarrow\rangle$	$\lvert\downarrow\downarrow\rangle$
$\lvert\uparrow\uparrow\rangle$	$\lvert\uparrow\uparrow\rangle$
$\lvert\downarrow\uparrow\rangle$	$\mathbf{i}\lvert\downarrow\uparrow\rangle$
$\lvert\uparrow\downarrow\rangle$	$\mathbf{i}\lvert\uparrow\downarrow\rangle$

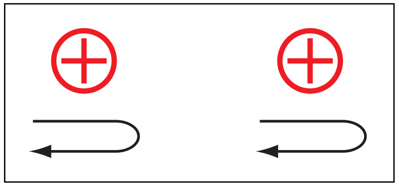
Geometric Phase Gate

- Ions in same trap
- Coulomb interaction

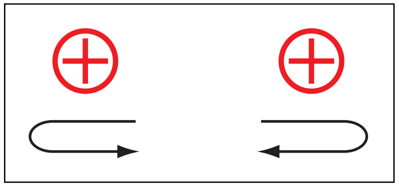
→

- Normal modes





Common mode ω_c



Stretch mode ω_s

- Ions share motional state

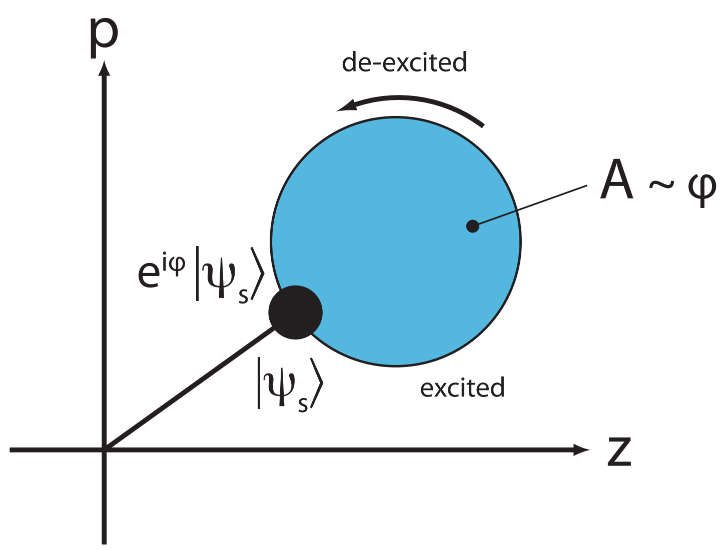
$$|m_{s_1} m_{s_2}\rangle |\psi_c\rangle |\psi_s\rangle$$

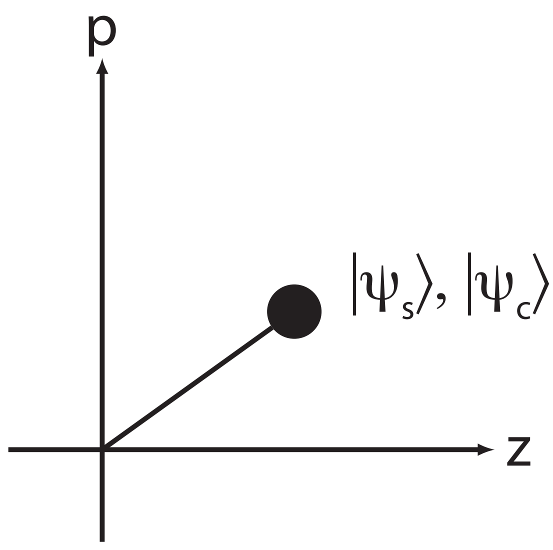
$$m_{s_1}, m_{s_2} \in \{\uparrow, \downarrow\}$$

Geometric Phase Gate

Input	Output
$\lvert\downarrow\uparrow\rangle$	$\mathbf{i}\lvert\downarrow\uparrow\rangle$
$\lvert\uparrow\downarrow\rangle$	$\mathbf{i}\lvert\uparrow\downarrow\rangle$

Input	Output
$\lvert\downarrow\downarrow\rangle$	$\lvert\downarrow\downarrow\rangle$
$\lvert\uparrow\uparrow\rangle$	$\lvert\uparrow\uparrow\rangle$



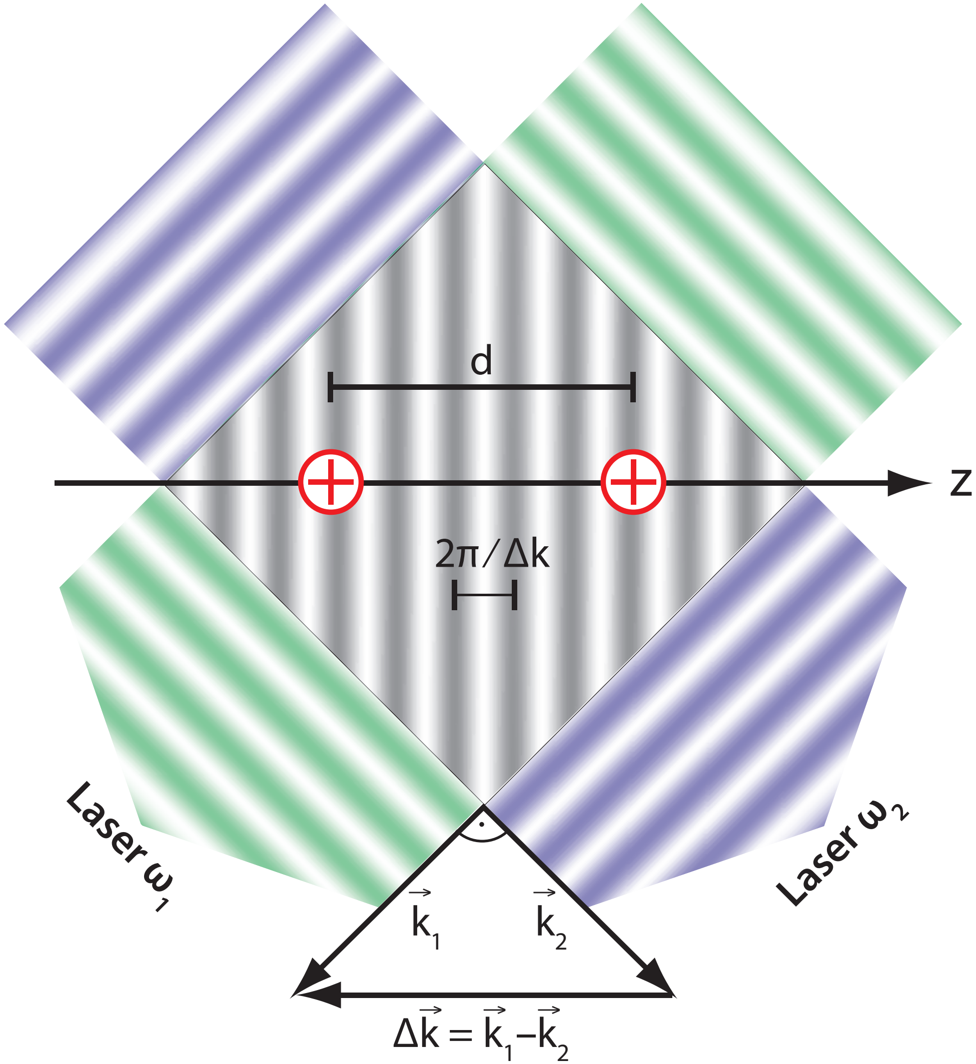


external force drives stretch mode

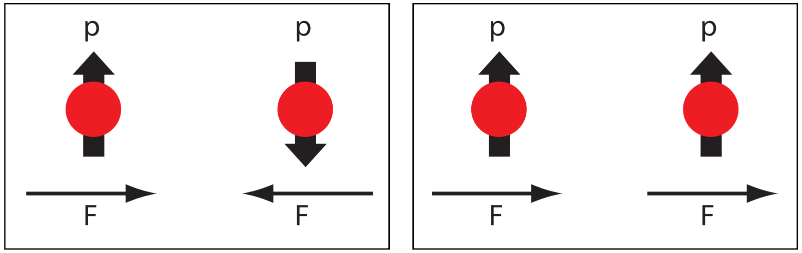
⇒ phase acquired

external force does not drive stretch mode nor common mode ⇒ no phase acquired

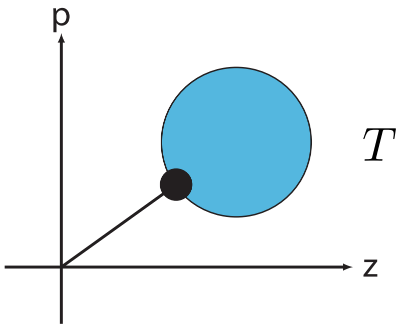
Geometric Phase Gate



- Lasers create interference pattern
- $d \in 2\pi/\Delta k\mathbf{Z}$
  ⇒ Ions „feel" same E-field
- Dipole force depends on internal state $\vec{F} = \nabla(\vec{p} \cdot \vec{E})$

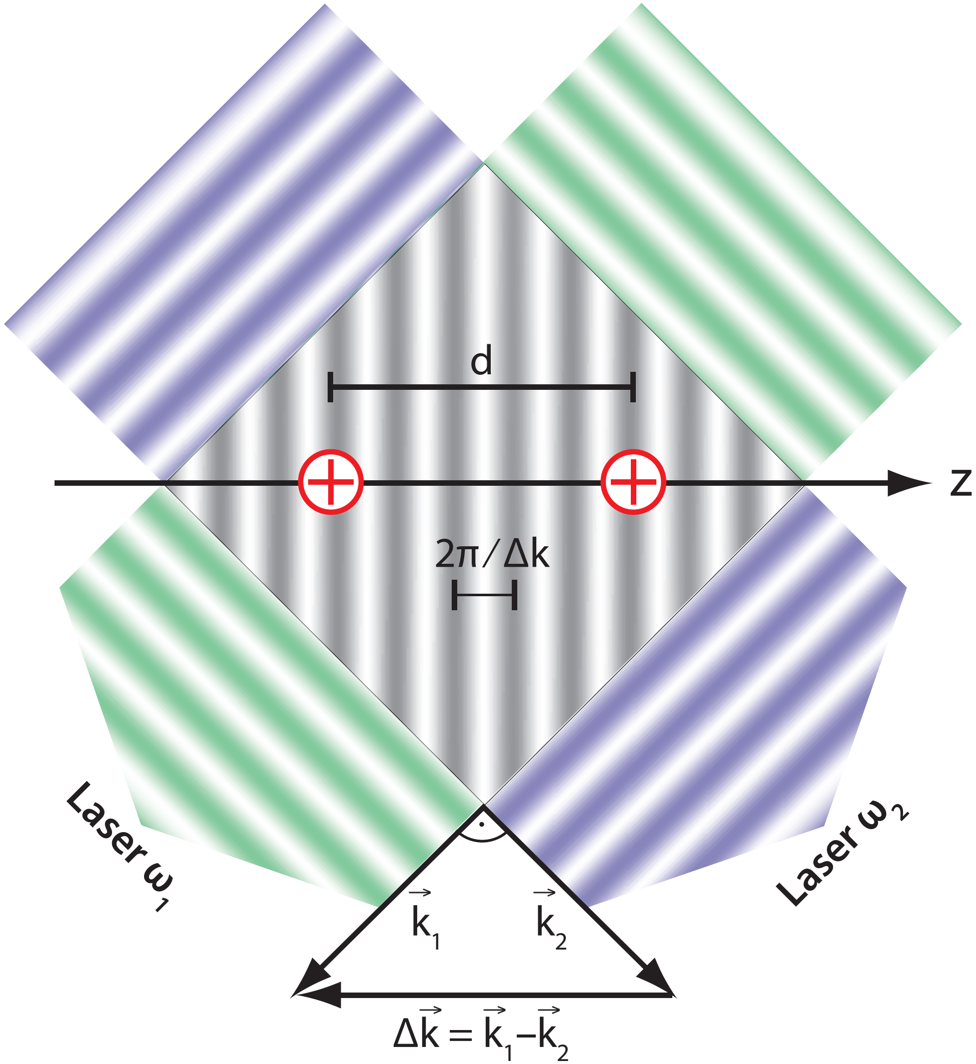


- $\omega_1 - \omega_2 = \omega_s + \delta \quad \text{with } |\delta| \ll \omega_s$



$T = 2\pi/\delta$

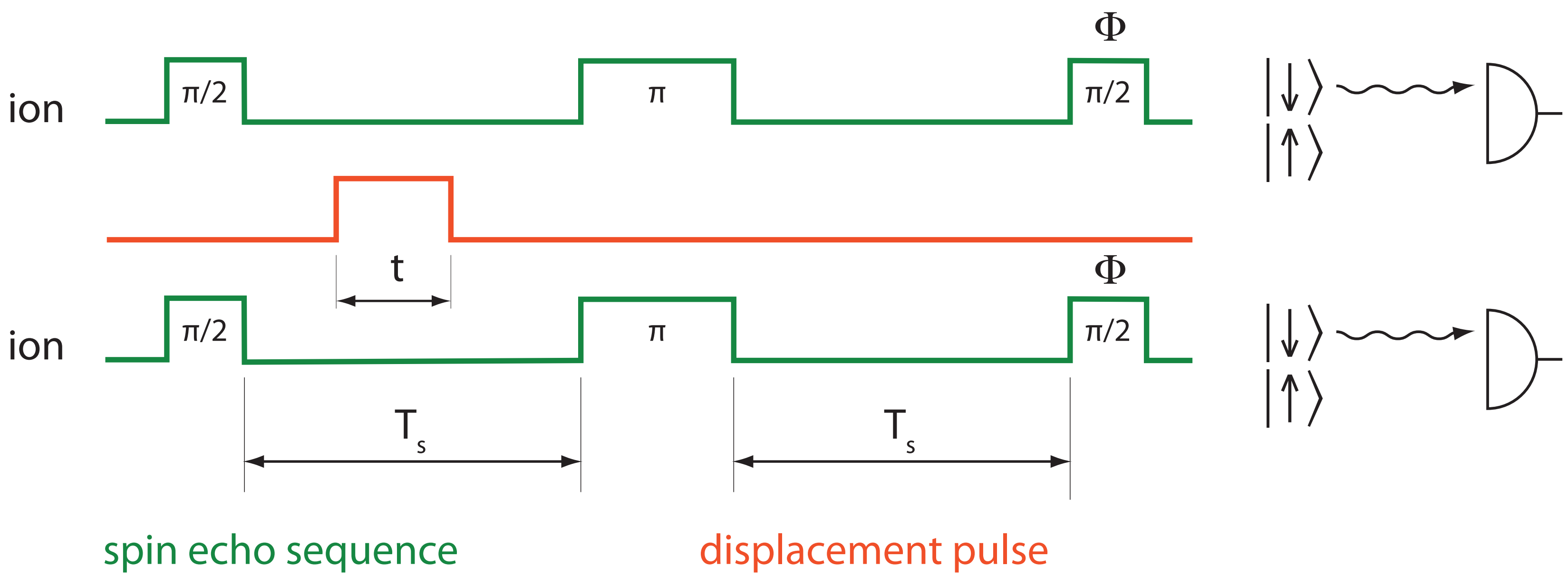
Geometric Phase Gate



- ω_1, ω_2 detuned to ions transition
  ⇒ internal states not affected

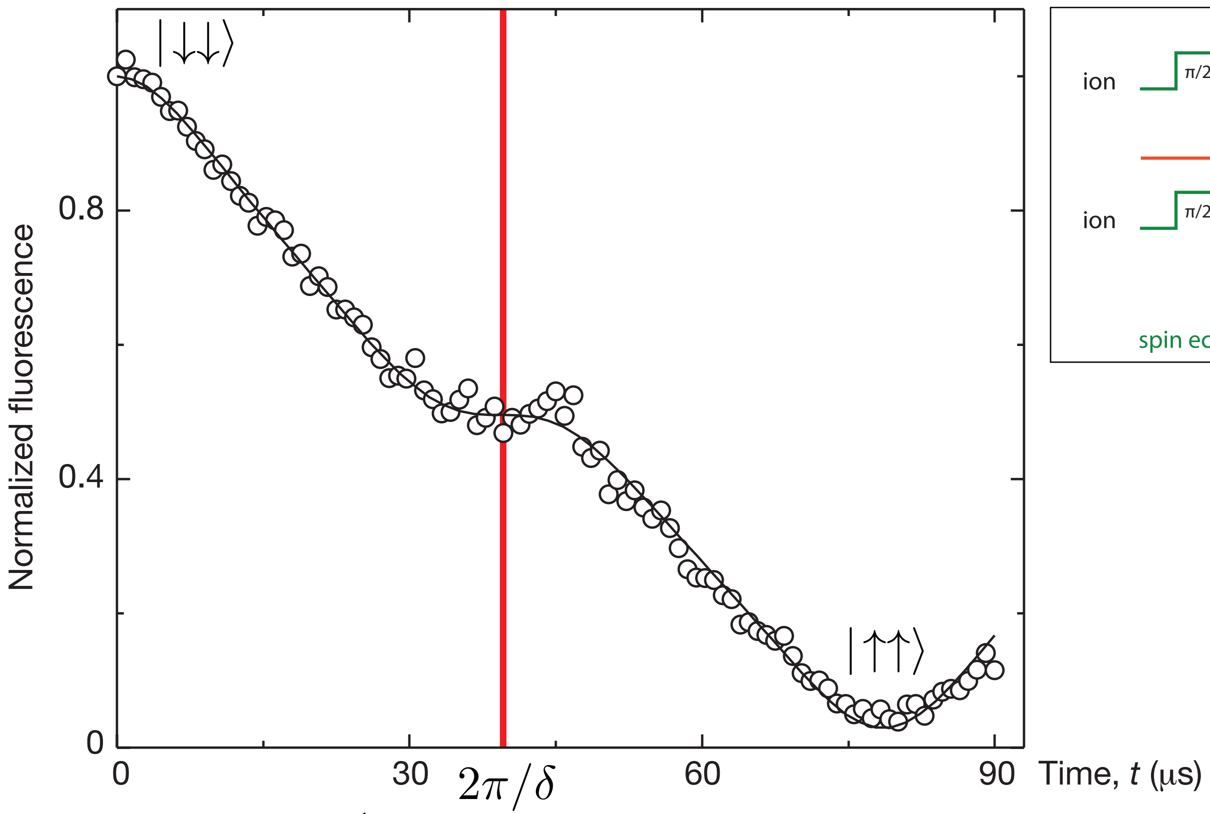
Geometric Phase Gate

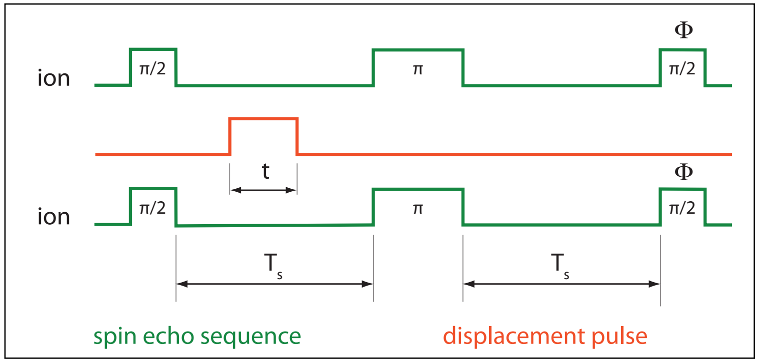
- two $^9Be^+$ ions
- initialized by cooling $|\downarrow\downarrow\rangle|n=0\rangle|n=0\rangle$
- pulse sequence



Geometric Phase Gate

- Analysis pulse phase Φ = 0



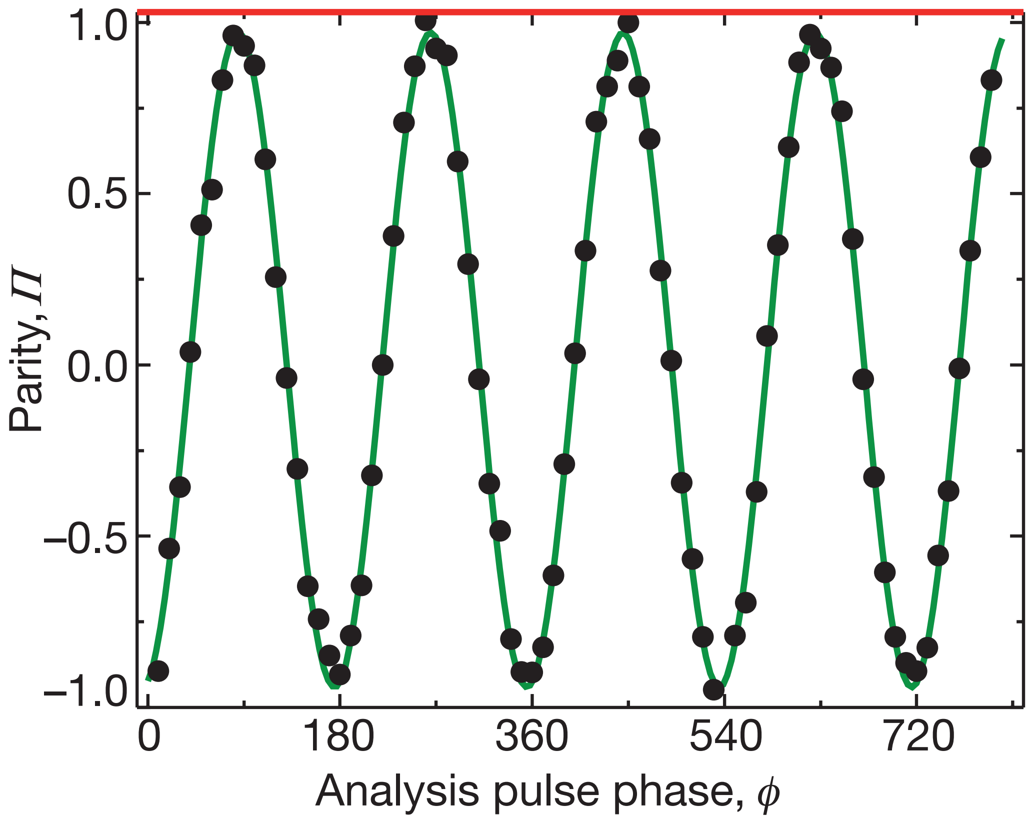


$$\frac{1}{\sqrt{2}}(|\downarrow\downarrow\rangle - \mathbf{i}|\uparrow\uparrow\rangle)$$ expected

$50\%\ |\downarrow\downarrow\rangle,\ 50\%\ |\uparrow\uparrow\rangle$ also possible

Geometric Phase Gate

- Parity $\Pi(\phi) = P_{\downarrow\downarrow}(\phi) + P_{\uparrow\uparrow}(\phi) - [P_{\uparrow\downarrow}(\phi) + P_{\downarrow\uparrow}(\phi)]$



$50\% \; |\downarrow\downarrow\rangle, \; 50\% \; |\uparrow\uparrow\rangle$

$\frac{1}{\sqrt{2}}(|\downarrow\downarrow\rangle - \mathbf{i}|\uparrow\uparrow\rangle)$

- Geometric phase gate works!
- Fidelity 0.97 ± 0.02

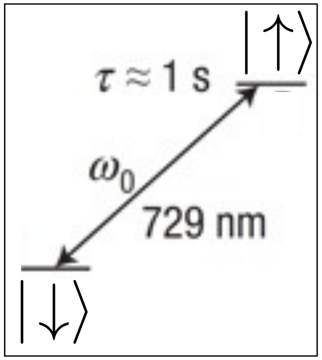
Mølmer-Sørensen Gate

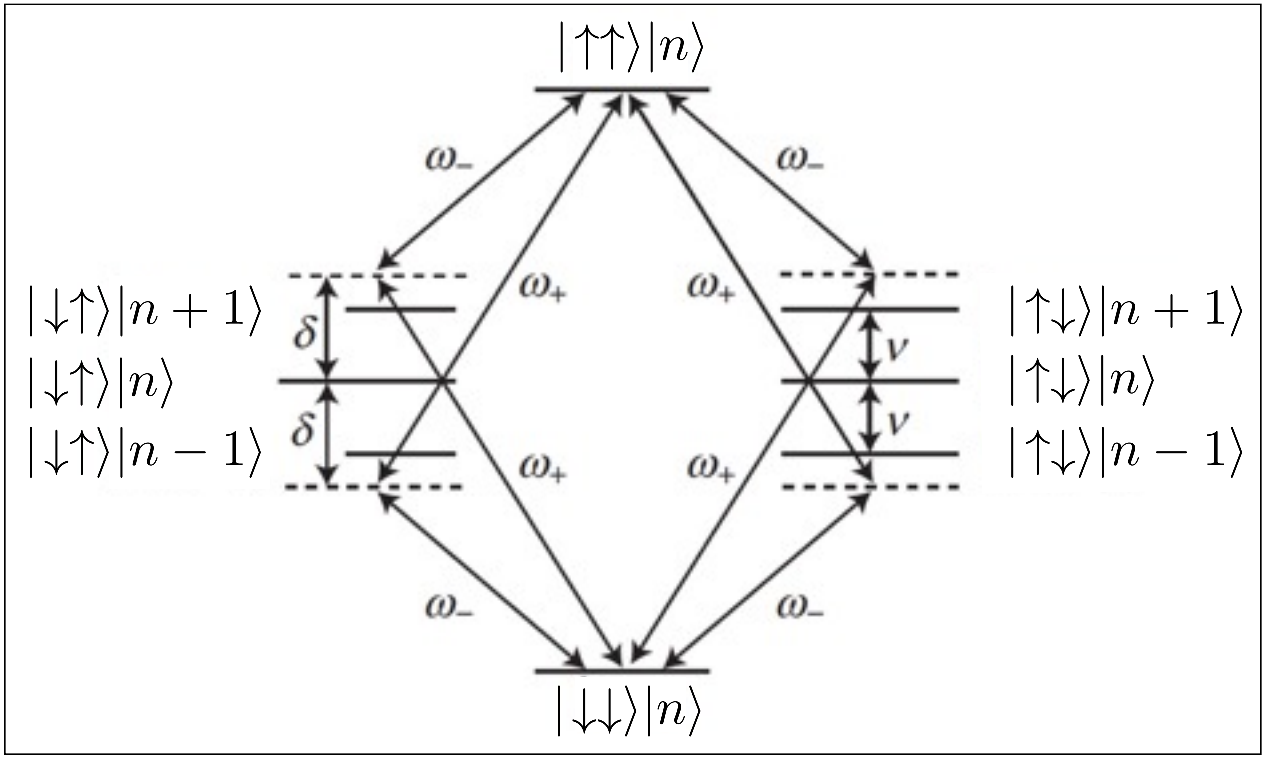
- Maps separable states onto max. entangled states

Input	Output
$\|\downarrow\downarrow\rangle$	$\frac{1}{\sqrt{2}}(\|\downarrow\downarrow\rangle + \mathbf{i}\|\uparrow\uparrow\rangle)$
$\|\uparrow\uparrow\rangle$	$\frac{1}{\sqrt{2}}(\|\uparrow\uparrow\rangle + \mathbf{i}\|\downarrow\downarrow\rangle)$
$\|\downarrow\uparrow\rangle$	$\alpha\|\downarrow\uparrow\rangle + \beta\|\uparrow\downarrow\rangle$
$\|\uparrow\downarrow\rangle$	$\alpha^*\|\downarrow\uparrow\rangle + \beta^*\|\uparrow\downarrow\rangle$

Mølmer-Sørensen Gate

- Gate operation by 4 different 2-photon processes in $^{40}Ca^+$



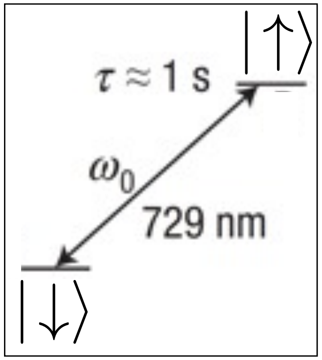


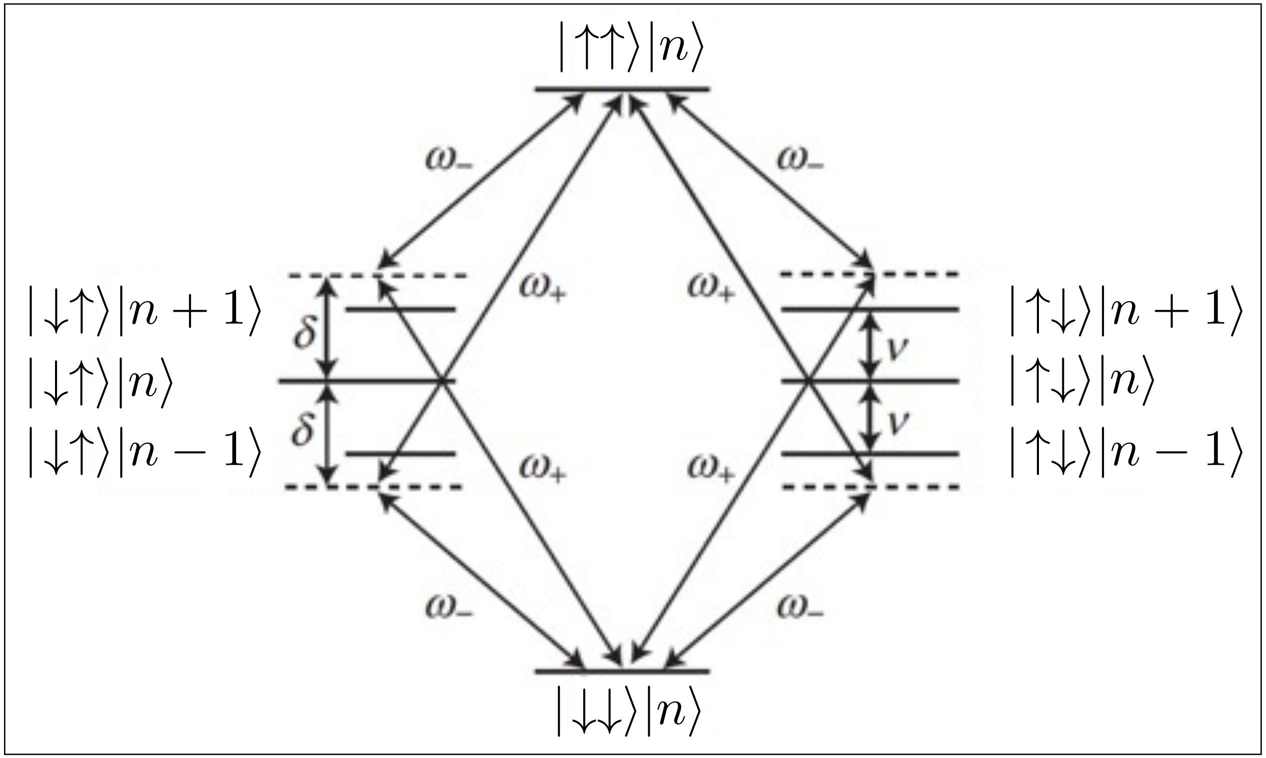
- Intermediate states enter virtually

$$|\downarrow\downarrow\rangle|n\rangle \longleftrightarrow \{|\uparrow\downarrow\rangle|n+1\rangle,\ |\downarrow\uparrow\rangle|n-1\rangle\} \longleftrightarrow |\uparrow\uparrow\rangle|n\rangle$$

Mølmer-Sørensen Gate

- Gate operation by 4 different 2-photon processes in $^{40}Ca^{+}$





$$H = \hbar\Omega \mathbf{e}^{\mathbf{i}\Phi} S_+ \left(\mathbf{e}^{-\mathbf{i}(\delta t+\zeta)} + \mathbf{e}^{\mathbf{i}(\delta t+\zeta)}\right) \mathbf{e}^{\mathbf{i}\eta\left(a\mathbf{e}^{-\mathbf{i}\nu t}+a^\dagger \mathbf{e}^{\mathbf{i}\nu t}\right)} + \mathrm{h.c.}$$

Mølmer-Sørensen Gate

- Gate operation described by propagator

$$U(t) = \mathbf{e}^{-\mathbf{i}F(t)S_x} D(\alpha(t)S_{y\psi})\mathbf{e}^{-\mathbf{i}(\lambda t + \chi \sin(\nu-\delta)t)S_{y\psi}^2}$$

- Laser intensity $\eta\Omega \approx |\delta - \nu|/4$ and $t = \tau_{\mathrm{gate}} = 2\pi/|\delta - \nu|$

$$U_{\mathrm{gate}} = \mathbf{e}^{-\mathbf{i}\frac{\pi}{8}S_y^2}$$

- Multiple application of bichromatic pulse of duration τ_{gate}

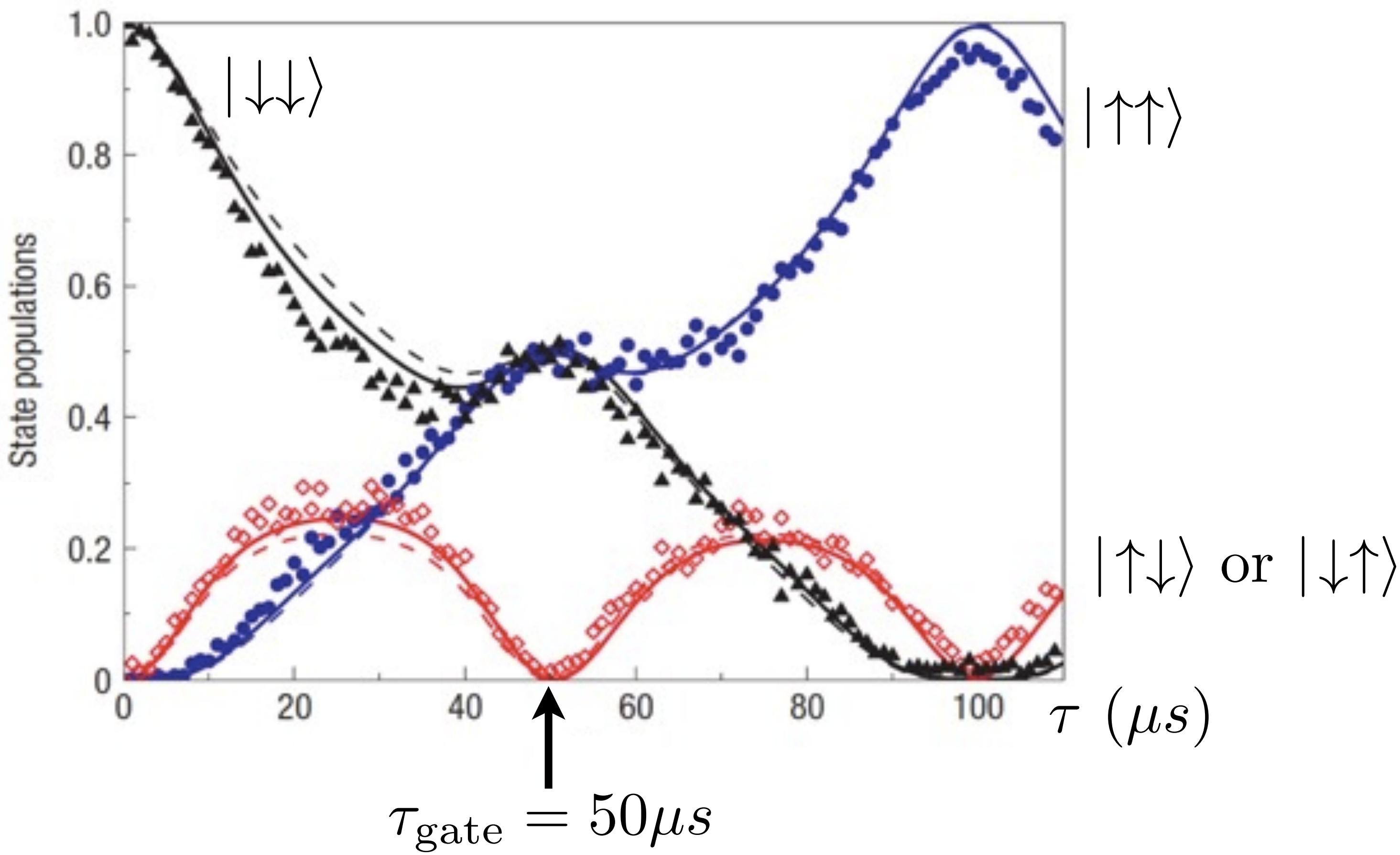
$$|{\downarrow\downarrow}\rangle \to \frac{1}{\sqrt{2}}(|{\downarrow\downarrow}\rangle + \mathbf{i}|{\uparrow\uparrow}\rangle) \to |{\uparrow\uparrow}\rangle \to \frac{1}{\sqrt{2}}(|{\uparrow\uparrow}\rangle + \mathbf{i}|{\downarrow\downarrow}\rangle) \to |{\downarrow\downarrow}\rangle$$

- Maximal entangled states for

$$t = m\tau_{\mathrm{gate}}, \quad m \in \{1, 3, \dots\}$$

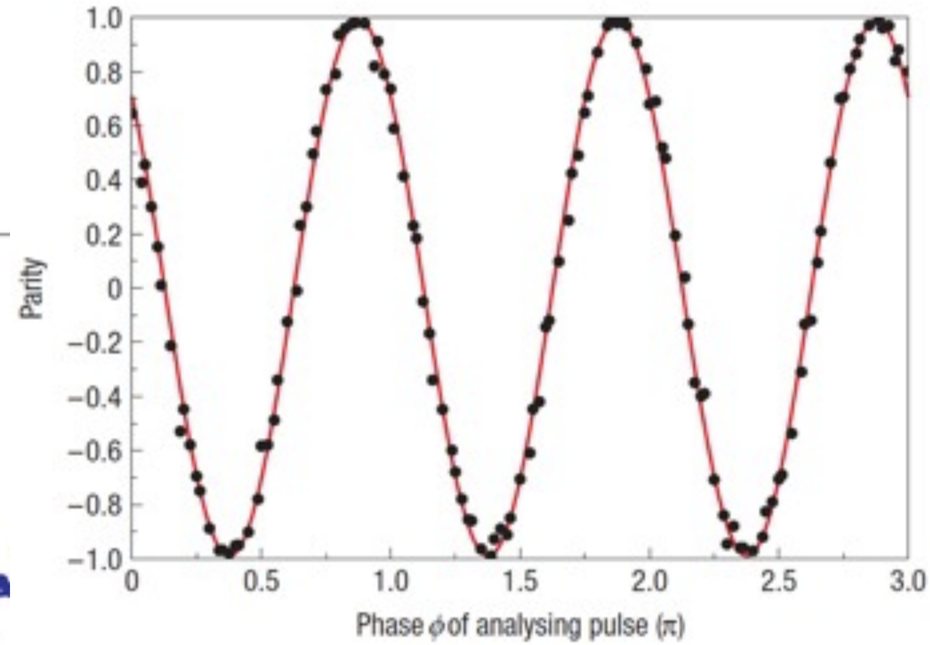
Mølmer-Sørensen Gate

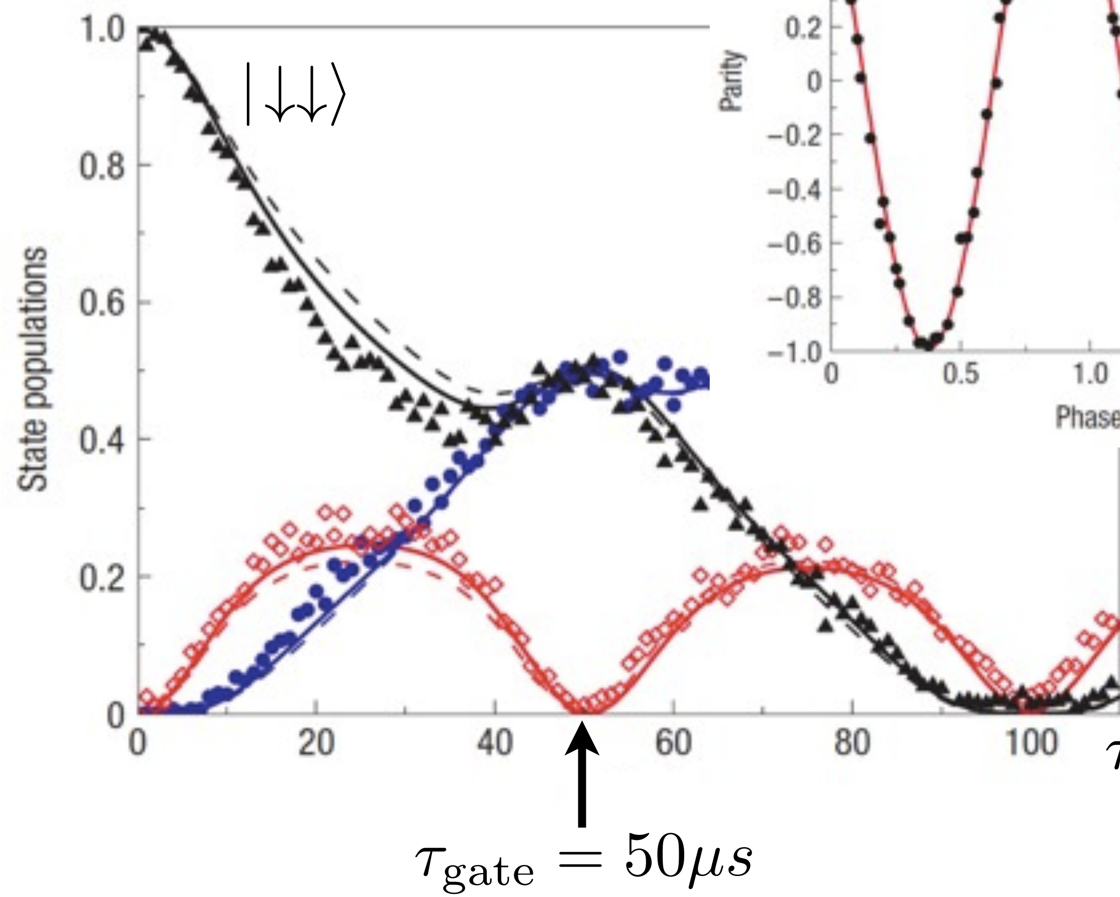
- $|\downarrow\downarrow\rangle \rightarrow \frac{1}{\sqrt{2}}(|\downarrow\downarrow\rangle + \mathbf{i}|\uparrow\uparrow\rangle) \rightarrow |\uparrow\uparrow\rangle \rightarrow \frac{1}{\sqrt{2}}(|\uparrow\uparrow\rangle + \mathbf{i}|\downarrow\downarrow\rangle) \rightarrow |\downarrow\downarrow\rangle$



Mølmer-Sørensen Gate

- $|\downarrow\downarrow\rangle \rightarrow \frac{1}{\sqrt{2}}(|\downarrow\downarrow\rangle + \mathbf{i}|\uparrow\uparrow\rangle) \rightarrow$



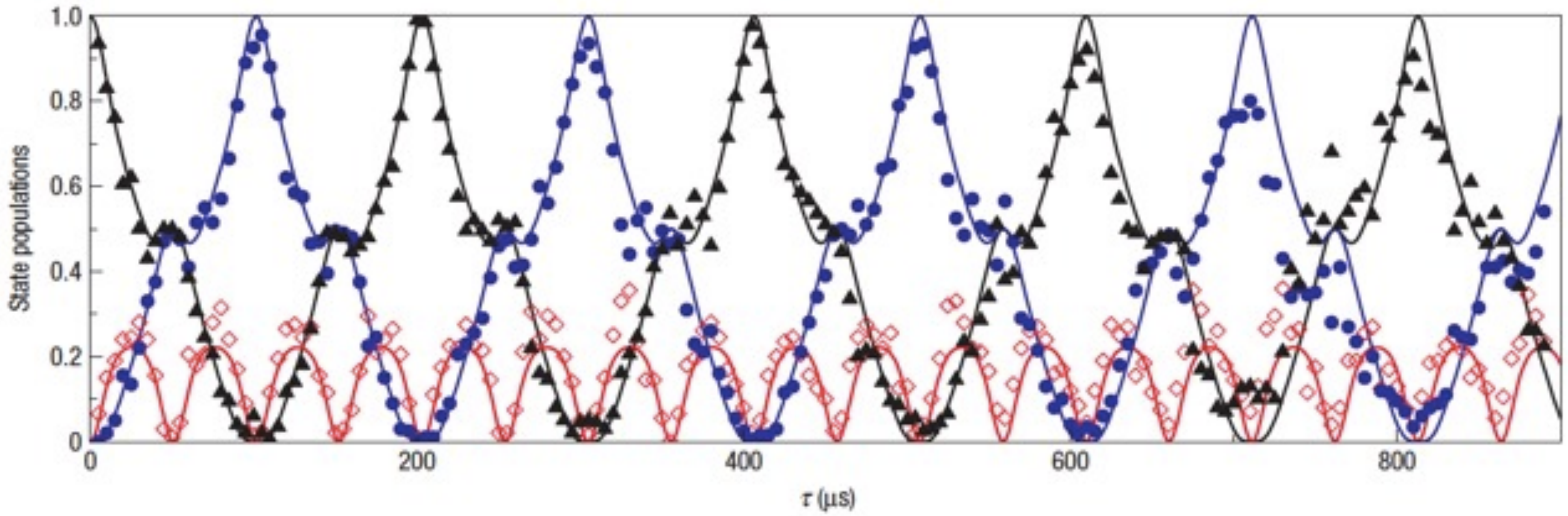


Mølmer-Sørensen Gate

- Fidelity

$$F = \langle \psi_1 | \rho^{\text{exp}} | \psi_1 \rangle = \frac{1}{2}(\rho^{\text{exp}}_{\downarrow\downarrow,\downarrow\downarrow} + \rho^{\text{exp}}_{\uparrow\uparrow,\uparrow\uparrow}) + \text{Im}(\rho^{\text{exp}}_{\downarrow\downarrow,\uparrow\uparrow})$$

- Multiple Spin-Flip Operations (21 operations)

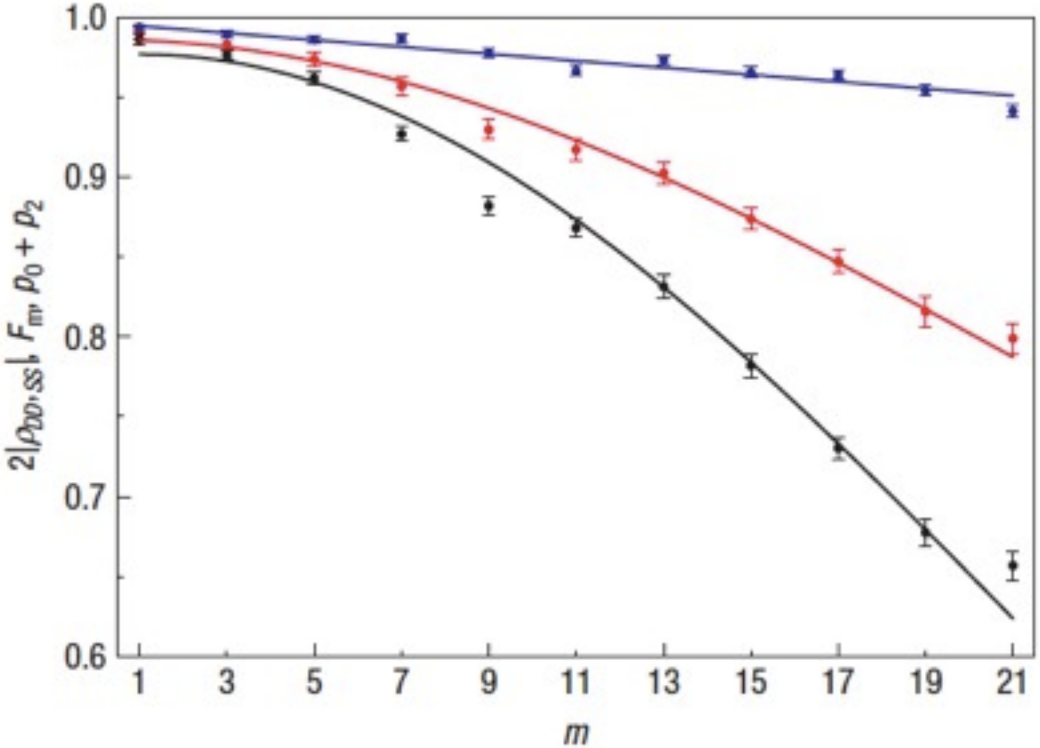


Mølmer-Sørensen Gate

- Fidelity

$$F = \langle \psi_1 | \rho^{\text{exp}} | \psi_1 \rangle = \frac{1}{2}(\rho^{\text{exp}}_{\downarrow\downarrow,\downarrow\downarrow} + \rho^{\text{exp}}_{\uparrow\uparrow,\uparrow\uparrow}) + \text{Im}(\rho^{\text{exp}}_{\downarrow\downarrow,\uparrow\uparrow})$$

- Multiple Spin-Flip Operations (m)



$$p_0 + p_1 = \frac{1}{2}(\rho_{\downarrow\downarrow,\downarrow\downarrow} + \rho_{\uparrow\uparrow,\uparrow\uparrow})$$

Combined for Φ^+

$F_{m=1} = 0.993(1)$

$F_{m=21} = 0.80(1)$

$2\rho_{\uparrow\uparrow,\downarrow\downarrow}$

Comparison

Mølmer-Sørensen Gate	Geometric Phase Gate
High fidelities	
Universal two qubit gates	
• Collective spin flip due to 2 photon process • Periodic entangled and disentangled state	• Map internal states to shared motional state • Accumulate phase by motion