

QSIT 2010 - Questions 2

9. March 2012, HIT F 13

1. State preparation

Any single qubit state can be prepared by applying a sequence of unitary operations onto the initial state. Assuming that the system is initially in its ground state, $|\psi_i\rangle = |0\rangle$, determine the unitary matrix (sequence) that results in the following final states:

- (a) $|\psi_f\rangle = |1\rangle$
- (b) $|\psi_f\rangle = (|0\rangle - |1\rangle)/\sqrt{2}$
- (c) $|\psi_f\rangle = \sin \frac{3\pi}{8}|1\rangle - \cos \frac{3\pi}{8}|0\rangle$
- (d) $|\psi_f\rangle = e^{i\pi/4} \sin \frac{3\pi}{8}|1\rangle - \cos \frac{3\pi}{8}|0\rangle$

2. Measurement

A measurement of the Pauli matrices on a spin-1/2 particle corresponds to a measurement of its polarization.

- (a) Calculate the expectation value of the observables σ_x and σ_z , when the particle is in the state $|1\rangle$ or $(|0\rangle + |1\rangle)/\sqrt{2}$, respectively. Calculate also the standard deviation of these measurements.
- (b) Find the observable that measures the polarization along the axis $\vec{n} = (\cos \phi \sin \theta, \sin \phi \sin \theta, \cos \theta)$. Which states give a standard deviation of zero for this measurement.

3. Quantum State Tomography.

To determine the state of a quantum system a specific number of measurements have to be performed on identically prepared systems. From the results of such a complete set of measurements the state can then be fully characterized.

- (a) How many measurements do you need to determine the quantum state of the system?
- (b) Write down explicitly, what measurements can be used and how you can infer the state from the results of these measurements.
- (c) How is the number of required measurements related to the normalization of the state? What does it mean, if the state is found to be not normalized?