

QSIT 2011 - Questions 4

23. March 2012, HIT F 13

1. Mixed state vs. pure state

A source emits spin-1/2 particles either in the pure state

$$|\psi\rangle = \sqrt{\frac{3}{4}}|0\rangle + \sqrt{\frac{1}{4}}|1\rangle$$

or in the mixed state

$$\rho = \frac{3}{4}|0\rangle\langle 0| + \frac{1}{4}|1\rangle\langle 1|$$

- (a) The spin polarization of the particles is then analysed by performing a projective measurement along the axis $n = (\sin \phi, 0, \cos \phi)^T$ in the $x - z$ plane. How can you distinguish the pure from the mixed state in this measurement?
- (b) What is observed for a totally mixed state?
- (c) For a full state tomography, three different measurements along different axes have to be performed. Calculate the expectation values for measurements along the (positive) x -, y - and z -axis for the mixed state ρ and reconstruct the state from these measurement outcomes.

2. Thermal state of a harmonic oscillator

The Hamiltonian of a harmonic oscillator is given by

$$\hat{H} = \hbar\omega\left(\hat{n} + \frac{1}{2}\right)$$

with the number operator \hat{n} . The eigenstates, which fulfill the Schrödinger equation $\hat{H}|n\rangle = \hbar\omega(n+1/2)|n\rangle$, are denoted by $|0\rangle, |1\rangle, \dots, |n\rangle$ and correspond to states with n excitation quanta in the system.

In thermal equilibrium with a heat bath at temperature T the probability p_n that the harmonic oscillator is excited to the n th state is given by the Boltzmann distribution

$$p_n = \frac{\exp[-E_n/(k_B T)]}{\sum_n \exp[-E_n/(k_B T)]} = \left(1 - \exp\left[-\frac{\hbar\omega}{k_B T}\right]\right) \exp\left[-\frac{n\hbar\omega}{k_B T}\right].$$

Find the density matrix which describes the equilibrium state of the harmonic oscillator.

3. Quantum search algorithm (Grover's algorithm)

You are given a search space with $N = 16$ elements, and the element you are looking is encoded in the state $|5\rangle$.

- (a) How many qubits do you need to encode the whole search space?
- (b) Construct the needed oracle.
- (c) How many Grover iterations are needed to find the needed element?