5. Photons

Montag, 21. November 2011
09:07

Why have the first quantum information processing experiments been performed with photons?

- Preparation of single photon states by attenuation
- Detection with high efficiency (single photon detectors)
- Manipulation of polarization/path (phase shifter, beam splitter, mirror, ...)

→ well developed for photons!

Also: only weak interaction with environment
(long coherence time, long-distance transmission)

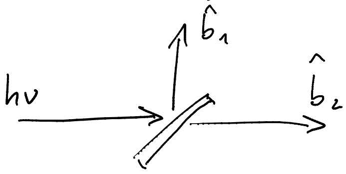
BUT: photon-photon interaction also weak
⇒ impractical for 2-qubit gates

Photon qubits for Quantum Communication:

(e.g. Koh & Lovett – Optical Quantum Inf. Proc.)

e.g. using polarisation + spatial degree of freedom

spatial modes:



beamsplitter

b_1, b_2 .. spatial modes

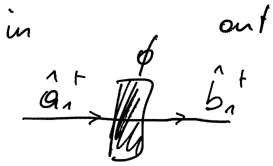
$$|0\rangle = b_1^\dagger |0\rangle_1 |0\rangle_2 = b_1^\dagger |0,0\rangle = |1,0\rangle$$

photons in mode 1

$$|1\rangle = b_2^\dagger |0,0\rangle = |0,1\rangle$$

operations:

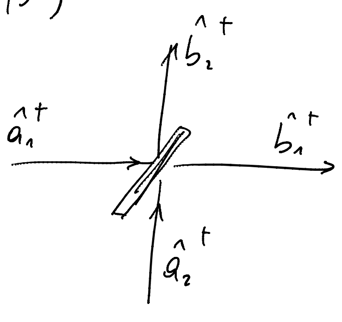
phase shifter



(dielectric with refractive index $n_n \neq 1$)

$$\hat{a}_1^\dagger \longrightarrow \hat{b}_1^\dagger = e^{+i\phi}\,\hat{a}_1^\dagger$$

beam splitter (50/50)
(half silvered mirror)



$$\left.\begin{aligned} \hat{a}_1^\dagger &\xrightarrow{BS} \hat{\tilde{a}}_1^\dagger = \frac{1}{\sqrt{2}}\left(\hat{b}_1^\dagger + i\,\hat{b}_2^\dagger\right) \\ \hat{a}_2^\dagger &\xrightarrow{BS} \hat{\tilde{a}}_2^\dagger \ \frac{1}{\sqrt{2}}\left(+i\,\hat{b}_1^\dagger + \hat{b}_2^\dagger\right) \end{aligned}\right\} \text{unitary transformation}$$

$$\begin{pmatrix} \hat{\tilde{a}}_1^\dagger \\ \hat{\tilde{a}}_2^\dagger \end{pmatrix} = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 & i \\ i & 1 \end{pmatrix}\begin{pmatrix} \hat{a}_1^\dagger \\ \hat{a}_2^\dagger \end{pmatrix}$$

photon at input 1:

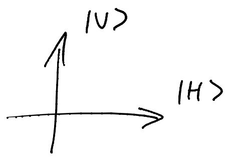
$$|0\rangle = |1,0\rangle = a_1^\dagger |0,0\rangle \longrightarrow \frac{1}{\sqrt{2}}\left(b_1^\dagger + i b_2^\dagger\right)|0,0\rangle$$

$$= \frac{1}{\sqrt{2}}\left(|1,0\rangle + i|0,1\rangle\right) =$$

$$= \frac{1}{\sqrt{2}}\left(|0\rangle + i|1\rangle\right)$$

→ 50/50 beamsplitter: 50% probability for scattering into either output port

polarization basis:

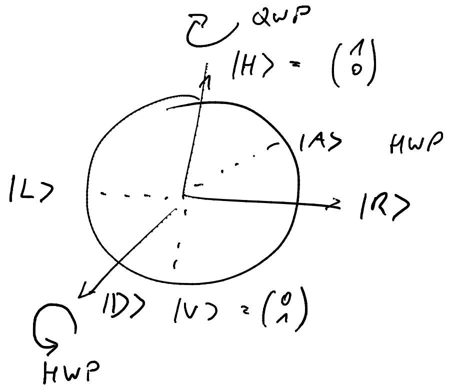


$a_H^\dagger$ ... creation of photon in mode a with horizontal pol.

$a_V^\dagger$ ... ——— " ——— vertical pol.

$$|0\rangle \hat{=} a_H^\dagger |0,0\rangle_{HV} = |1,0\rangle_{HV} = |H\rangle$$

$$|1\rangle \hat{=} a_V^\dagger |0,0\rangle_{HV} = |0,1\rangle_{HV} = |V\rangle$$



$$|L\rangle = \frac{1}{\sqrt{2}}\left(|H\rangle + i|V\rangle\right)$$

$$|R\rangle = \frac{1}{\sqrt{2}}\left(|H\rangle - i|V\rangle\right)$$

$$|D\rangle = \frac{1}{\sqrt{2}}\left(|H\rangle + |V\rangle\right)$$

$$|A\rangle = \frac{1}{\sqrt{2}}\left(|H\rangle - |V\rangle\right)$$

Operations:

half-wave plate:

$$|H\rangle \to \cos 2\theta\,|H\rangle + i\sin 2\theta\,|V\rangle$$
$$|V\rangle \to i\sin 2\theta\,|H\rangle + \cos 2\theta\,|V\rangle$$

$$U_{HWP}(\theta) = \begin{pmatrix} \cos 2\theta & i\sin 2\theta \\ i\,\mathrm{in}\,2\theta & \cos 2\theta \end{pmatrix}$$

$\Rightarrow$ rotation about x-axis

$$\theta = \frac{\pi}{4}: \quad U_{HWP} = e^{i\frac{\pi}{2}} \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \equiv X$$

$$|V\rangle \to |H\rangle \;;\quad |H\rangle \to |V\rangle$$

quarter wave plate: $\quad \phi_f - \phi_s = \frac{\pi}{2} \qquad \theta = \frac{\pi}{4}$

$$U_{QWP} = e^{-i\frac{\pi}{4}} \begin{pmatrix} 1 & 0 \\ 0 & i \end{pmatrix} \equiv Z/2$$

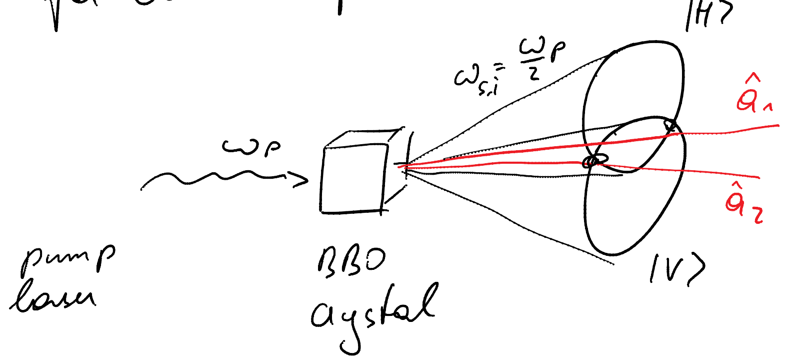
$\Rightarrow$ rotation about z-axis

$$|L\rangle = \frac{1}{\sqrt{2}}\left(|H\rangle + i\,|V\rangle\right) \;\hat{=}\; \frac{1}{\sqrt{2}}\begin{pmatrix} 1 \\ i \end{pmatrix}$$

$$U_{QWP}\,|L\rangle \to e^{-i\frac{\pi}{4}}\left(|H\rangle - |V\rangle\right) \propto |A\rangle$$

Parametric Down Conversion

for creation of entangled photon pairs



conservation of
- energy $(\omega_p = \omega_s + \omega_i)$
- momentum $(\vec{k}_p = \vec{k}_s + \vec{k}_i)$

at crossing points : $|\Psi\rangle = \frac{1}{\sqrt{2}}\left(|H\rangle_1 |V\rangle_2 - |V\rangle_1 |H\rangle_2\right)$

anti-symmetric Bell state

single photons: attenuate laser beam such that $\langle n\rangle \ll 1$

Super Dense Coding:

1) Preparation of initial entangled state (PDC)

$$|\psi^+\rangle = \frac{1}{\sqrt{2}}\left(|HV\rangle + |VH\rangle\right)$$

2) Generation of 4 maximally entangled 2 photon polarization states

$$|\psi^+\rangle \xrightarrow{\mathbb{1}_2} |\psi^+\rangle$$

$$|\psi^+\rangle \xrightarrow{X_2} \frac{1}{\sqrt{2}}\left(|HH\rangle + |VV\rangle\right) = |\phi^+\rangle \quad (HWP)$$

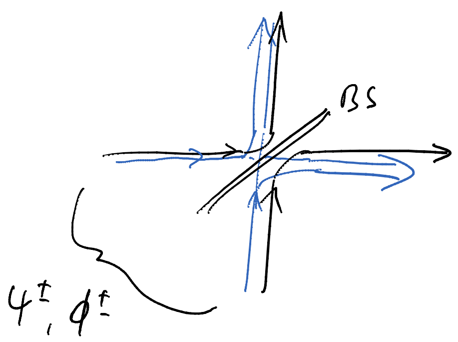
(i) ... omitted

$$|\psi^+\rangle \xrightarrow{Z_2} \frac{1}{\sqrt{2}}\left(|HV\rangle \overset{(i)}{-} |VH\rangle\right) = |\psi^-\rangle \quad (QWP)$$

$$|\psi^+\rangle \xrightarrow{Z_2 X_2} \frac{1}{\sqrt{2}}\left(|HH\rangle \overset{(i)}{-} |VV\rangle\right) = |\phi^-\rangle \quad (HWP + QWP)$$

3) Bell State measurement using beam splitter

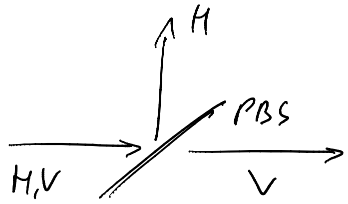
A) distinguish symmetric ($|\psi^+\rangle, |\phi^+\rangle, |\phi^-\rangle$) from antisymmetric ($|\psi^-\rangle$) state using a beamsplitter (BS)



anti-bunching for ψ^-

bunching for symm. states

B) distinguish polarization states using polarizing beam splitter



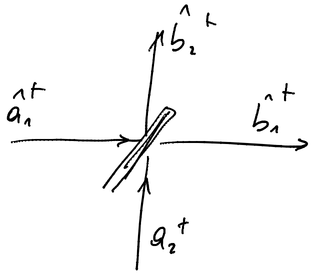
outcomes: $|\psi^+\rangle$: coincidence D_H & D_V or $D_{H'}$ & $D_{V'}$

$|\psi^-\rangle$: coincidence $D_{H'}$ & D_V or D_H & $D_{V'}$

$|\phi^+\rangle, |\phi^-\rangle$: 2 photons in $D_H, D_V, D_{H'}, D_{V'}$

(cannot be distinguished)

Bell State measurement using 50/50 beamsplitter

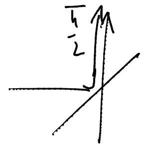


$$\hat{a}_1 \xrightarrow{BS} \frac{1}{\sqrt{2}}\left(\hat{b}_1 - i\hat{b}_2\right) \qquad a_1^+ \longrightarrow \frac{1}{\sqrt{2}}\left(\hat{b}_1^+ + i\,\hat{b}_2^+\right)$$

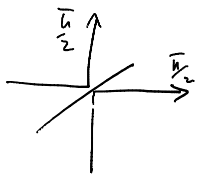
$$\hat{a}_2 \xrightarrow{BS} \frac{1}{\sqrt{2}}\left(-i\hat{b}_1 + \hat{b}_2\right) \qquad a_2^+ \longrightarrow \frac{1}{\sqrt{2}}\left(i\hat{b}_1^+ + \hat{b}_2^+\right)$$

two photons incident on the beamsplitter:

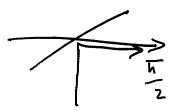
4 possibilities:

① 

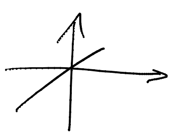
bunching

② 

anti-bunching

③ 

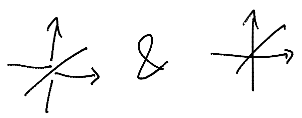
bunching

④ 
anti-bunching

Hong-Ou Mandel effect: for identical polarization $\left(\Phi^\pm = \frac{1}{\sqrt{2}}\left(|HH\rangle \pm |VV\rangle\right)\right)$

$\Rightarrow$ destructive interference due to phase shift 0 or π

$\Rightarrow$ bunching



cannot be distinguished

BUT: anti-bunching for anti-symmetric state
$|\psi^-\rangle = \frac{1}{\sqrt{2}}\left(|HV\rangle - |VH\rangle\right)$!

input state: $\frac{1}{\sqrt{2}}\left(h_1^\dagger v_2^\dagger \overset{\text{anti-sym}}{\underset{\text{symm.}}{\mp}} v_1^\dagger h_2^\dagger\right) \xrightarrow{BS}$
$(\hat{v}_i = \hat{a}_{iV},\ \hat{h}_i = \hat{a}_{iH})$

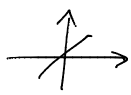
"apply beamsplitter transformation to each mode"

$$\frac{1}{2}\frac{1}{\sqrt{2}}\left(h_1^\dagger + i h_2^\dagger\right)\left(v_2^\dagger + i v_1^\dagger\right) \mp \left(v_1^\dagger + i v_2^\dagger\right)\left(h_2^\dagger + i h_1^\dagger\right)\Big] =$$

$$= \frac{1}{2\sqrt{2}}\Big[h_1^\dagger v_2^\dagger + i h_2^\dagger v_2^\dagger + i h_1^\dagger v_1^\dagger - h_2^\dagger v_1^\dagger \mp v_1^\dagger h_2^\dagger \mp i v_2^\dagger h_2^\dagger \mp i v_1^\dagger h_1^\dagger \pm v_2^\dagger h_1^\dagger\Big]$$

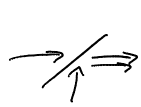
for anti-symmetric spatial wavefunction (−) : " "

$$= \frac{1}{\sqrt{2}}\left(h_1^\dagger v_2^\dagger - v_1^\dagger h_2^\dagger\right) \Rightarrow \text{anti-bunching}$$



for symmetric spatial wavefunction (+) : " "

$$= \frac{1}{\sqrt{2}}\left(h_1^\dagger v_1^\dagger + i h_2^\dagger v_2^\dagger\right) \Rightarrow \text{bunching}$$

 oder 

bunching also for other symmetric spatial wavefunctions $\left[\frac{1}{\sqrt{2}}\left(h_1^\dagger h_2^\dagger \pm v_1^\dagger v_2^\dagger\right)\right]$
$\left[\frac{1}{\sqrt{2}}\left(h_1^\dagger h_2^\dagger \pm v_2^\dagger v_2^\dagger\right)\right] \Rightarrow$ bunching