Bell Inequality - Question

Which values can S take, if $a_j=\pm1$, $b_j=\pm1$, $c_j=\pm1$, $d_j=\pm1$?

$$\begin{aligned} S_j &= a_j b_j + b_j c_j + c_j d_j - a_j d_j \\ &= b_j(a_j + c_j) + d_j(c_j - a_j) = ? \end{aligned}$$

Answer: (a) 0 (b) 2 (c) 4

Bell-Inequality:

$$S = |\langle ab\rangle + \langle bc\rangle + \langle cd\rangle - \langle ad\rangle| \leq 2$$

This inequality must be fulfilled by any **local, realistic** theory.

Assumptions:

- Values of a_j , b_j, c_j, d_j are predetermined for each measurement
- Measurement 1 cannot influence measurement 2

Locality & Realism

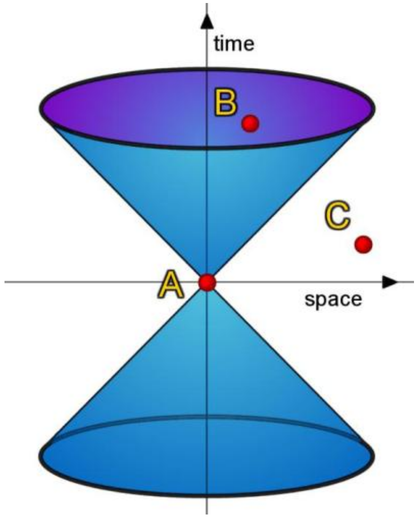
Einstein-Podolsky-Rosen (1932):
Quantum theory is not complete, if we assume that physics is local and realistic!

"While we have thus shown that the wave function does not provide a complete description of the physical reality, we left open the question of whether or not such a description exists. We believe, however, that such a theory is possible."

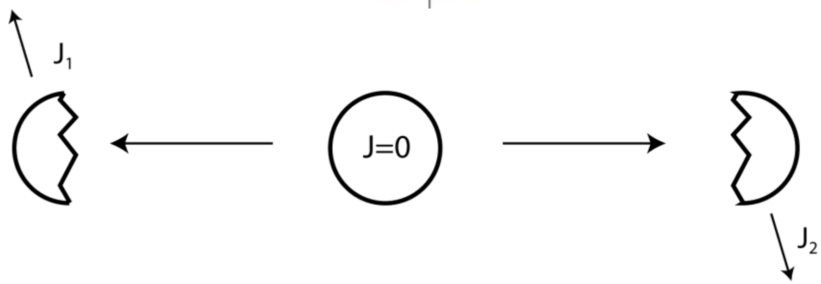
Locality: Two measurements cannot influence each other, if they are far apart (spacelike separation)

$$d \geq c\, t_{meas}$$

d... distance; t_{meas}... duration of measurement; c... speed of light



Reality: Objects possess properties, which are independent of a later measurement.



„Local Realism"

EPR Paradox

With the example advocated by Bohm and Aharonov [6], the EPR argument is the following. Consider a pair of spin one-half particles formed somehow in the singlet spin state and moving freely in opposite directions. Measurements can be made, say by Stern-Gerlach magnets, on selected components of the spins $\vec{\sigma}_1$ and $\vec{\sigma}_2$. If measurement of the component $\vec{\sigma}_1 \cdot \vec{a}$, where $\vec{a}$ is some unit vector, yields the value $+1$ then, according to quantum mechanics, measurement of $\vec{\sigma}_2 \cdot \vec{a}$ must yield the value -1 and vice versa. Now we make the hypothesis [2], and it seems one at least worth considering, that if the two measurements are made at places remote from one another the orientation of one magnet does not influence the result obtained with the other. Since we can predict in advance the result of measuring any chosen component of $\vec{\sigma}_2$, by previously measuring the same component of $\vec{\sigma}_1$, it follows that the result of any such measurement must actually be predetermined. Since the initial quantum mechanical wave function does *not* determine the result of an individual measurement, this predetermination implies the possibility of a more complete specification of the state.

John Bell. On the Einstein-Podolsky-Rosen paradox. Phzsics 1, 165 (1964)

Quantum mechanical description

singlet - state: $$|\psi\rangle = \frac{1}{\sqrt{2}}\left(|01\rangle - |10\rangle\right)$$

correlation function: $$\langle \hat{a}\hat{b}\rangle = \langle\psi|\vec{\alpha}\cdot\vec{\sigma}\otimes\vec{\beta}\cdot\vec{\sigma}|\psi\rangle = -\cos(\alpha-\beta)$$

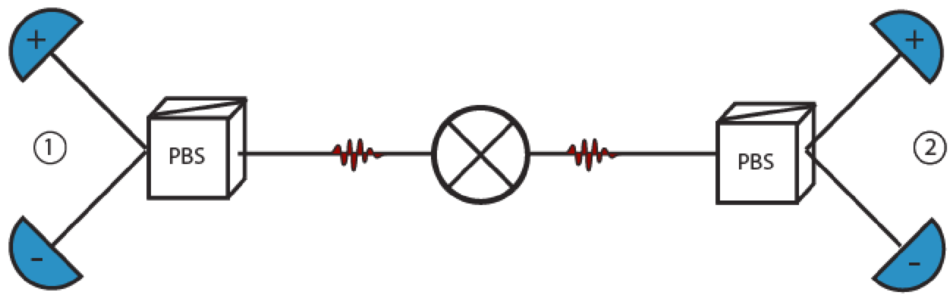
$$\begin{aligned} S =& |\langle \hat{a}\hat{b}\rangle + \langle \hat{b}\hat{c}\rangle + \langle \hat{c}\hat{d}\rangle - \langle \hat{a}\hat{d}\rangle| \\ =& |\cos(\alpha-\beta) + \cos(\beta-\gamma) + \cos(\gamma-\delta) - \cos(\alpha-\delta) \\ =& |1/\sqrt{2} + 1/\sqrt{2} + 1/\sqrt{2} - (-1/\sqrt{2})| \\ =& 2\sqrt{2} \end{aligned}$$

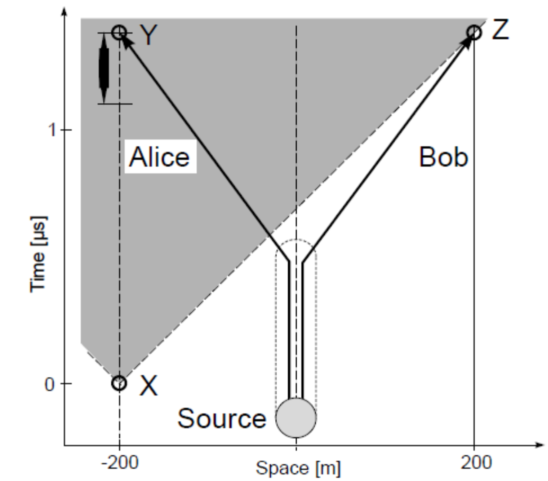
for $\alpha = 0,\ \beta = \pi/4,\ \gamma = \pi/2$ and $\delta = 3\pi/4$

quantum mechanics violates Bell Inequalities!

Implications: (1) 'spooky action at a distance' (non-locality) or
(2) state is not determined before it is measured (non-realistic)

Experimental realization with photons





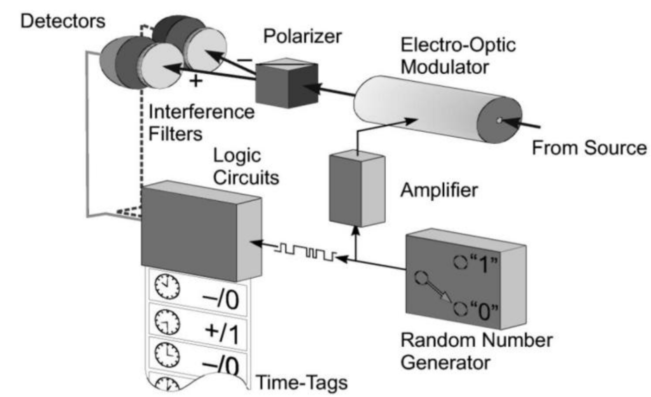
- photon pair in singlet state

  (0... horizontal polarisation,
  1... vertical polarisation)

$$|\psi\rangle = \frac{1}{\sqrt{2}}\left(|01\rangle - |10\rangle\right)$$

- polarising beam splitter after **400m ~ 1.3 µs**
- duration of measurement $\mathbf{t_{meas}}$ **= 100ns**
- random choice of measurement angles **after** generation of photons
- Measured value of Bell-Inequality:

S -= 2.73 +/- 0.02



Conclusion: QM is either **non-local** or **non-realistic**!

Weihs et al., PRL 81 (1998)

Bell Inequality - Question

Is it possible to construct a theory -- different from quantum mechanics -- that agree with experimental results (S>2)?

a) Yes, but such a theory must involve *both super-luminal effects (non-locality)* and states, that are *undetermined before measured* (non-realism).

b) Yes, but such a theory involves *super-luminal (non-local) effects*.

c) No, such a theory is not possible.

d) I don't know.

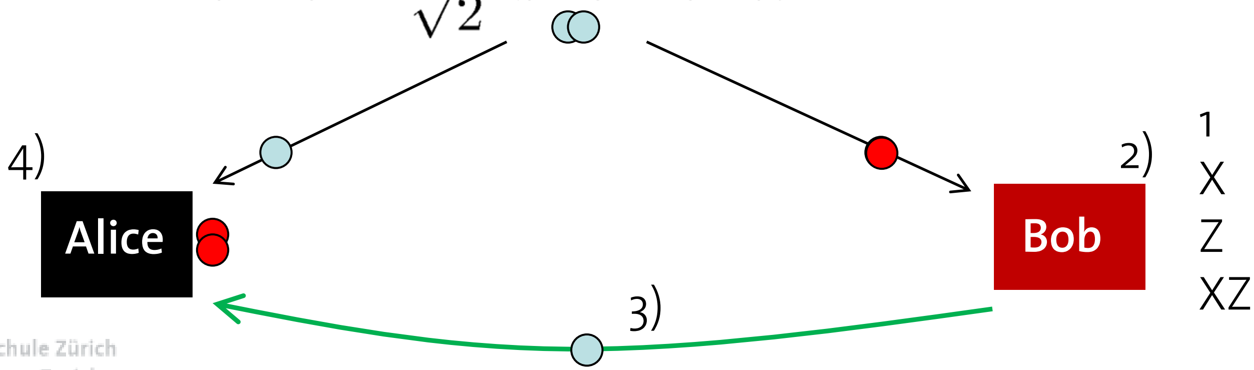
Superdense coding – entanglement as a resource

task: Transmit two bits of classical information between Alice (A) and Bob (B) using only one qubit.
Alice and Bob share an entangled qubit pair prepared ahead of time.

protocol:

1) Alice and Bob each have one qubit of an entangled pair
2) Bob does a quantum operation on his qubit depending on which 2 classical bits he wants to communicate
3) Bob sends his qubit to Alice
4) Alice does one measurement on the entangled pair

1) $|\Psi^+\rangle = \frac{1}{\sqrt{2}}(|01\rangle + |10\rangle)$



Superdense coding

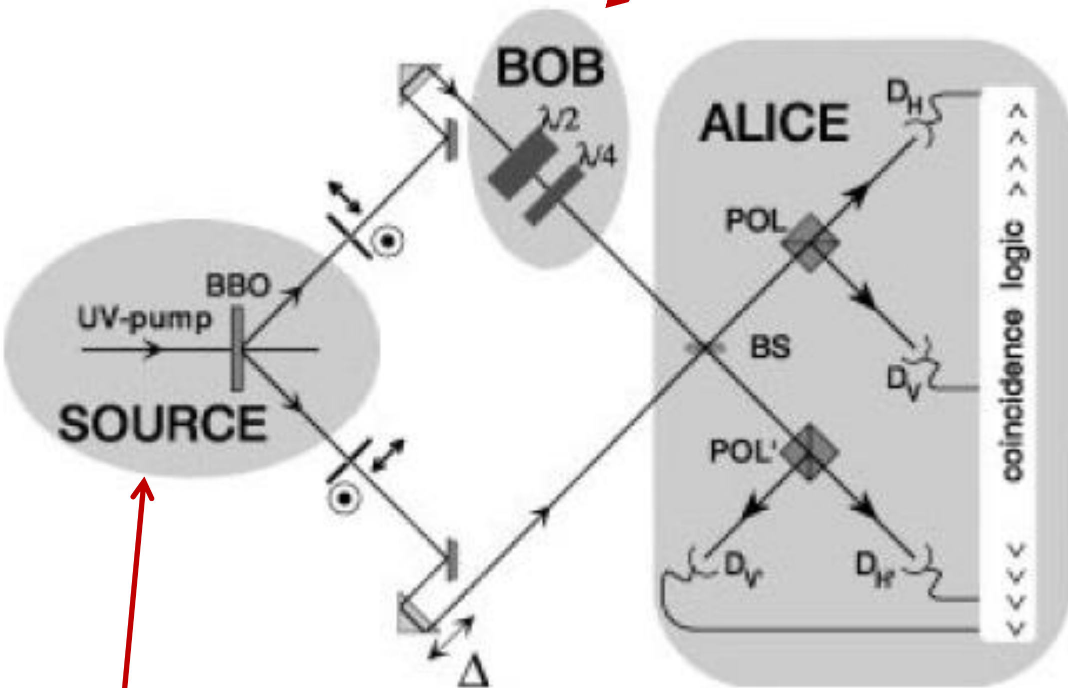
bit to be transferred	Bob's operation	resulting 2-qubit state (Bell states)	Alice's measurement
00	I_2	$I_2\ \lvert\psi\rangle = (\lvert 01\rangle + \lvert 10\rangle)/\sqrt{2} = \lvert\Psi^+\rangle$	$\lvert\Psi^+\rangle$
01	X_2 (HWP)	$X_2\ \lvert\psi\rangle = (\lvert 00\rangle + \lvert 11\rangle)/\sqrt{2} = \lvert\Phi^+\rangle$	$\lvert\Phi^+\rangle$
10	Z_2(QWP)	$Z_2\ \lvert\psi\rangle = (\lvert 01\rangle - \lvert 10\rangle)/\sqrt{2} = \lvert\Psi^-\rangle$	$\lvert\Psi^-\rangle$
11	X_2Z_2 (HWP + QWP)	$X_2Z_2\ \lvert\psi\rangle = (\lvert 00\rangle - \lvert 11\rangle)/\sqrt{2} = \lvert\Phi^-\rangle$	$\lvert\Phi^-\rangle$

- two qubits are involved in protocol BUT Bob only interacts with one and sends only one along his quantum communications channel
- two bits cannot be communicated sending a single classical bit along a classical communications channel

Bennett & Wiesner, Communication via one- and two-particle operators on Einstein-Podolsky-Rosen states, Phys Rev Lett 60, 2881 (1992).

Realization of superdense coding

state manipulation



preparation of entangled pair

Bell state measurement