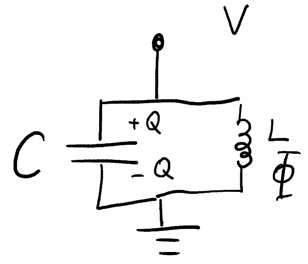
2. Superconducting circuits for QIP

Freitag, 21. Oktober 2011
09:34

2.1. Parallel LC circuit - harmonic oscillator

Mittwoch, 21. März 2012
12:50

parallel LC oscillating circuit:



Voltage across oscillator:

$$V = \frac{Q}{C} = -\frac{d\Phi}{dt} = -L\frac{dI}{dt}$$

energy stored in the circuit: (Hamiltonian)

$$H = \underbrace{\frac{1}{2}\cdot CV^2}_{\substack{\text{capacitive}\\ \text{(electrostatic)}\\ \text{energy}}} + \underbrace{\frac{1}{2}LI^2}_{\substack{\text{inductive}\\ \text{(magnetic)}\\ \text{energy}}} = \frac{1}{2}\frac{Q^2}{C} + \frac{1}{2}\frac{\Phi^2}{L}$$

Φ ... magnetic flux stored in the inductor ($\Phi = LI$)

Q ... charge stored on the capacitor ($Q = CV$)

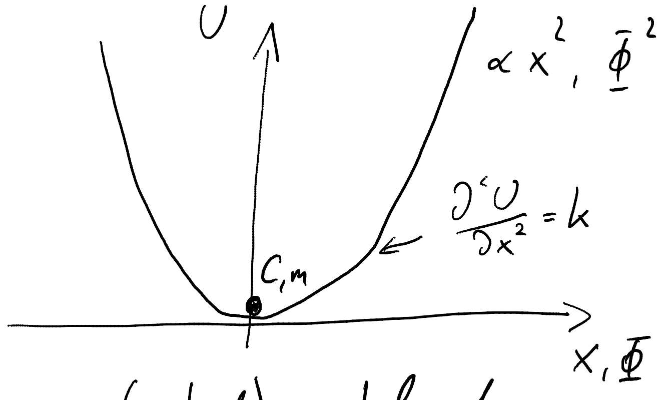
canonical variables

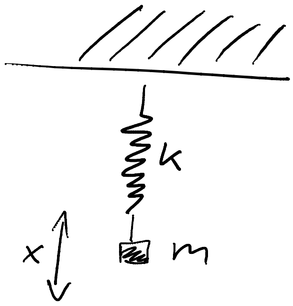
$$\left[\begin{aligned} \frac{\partial H}{\partial Q} &= \frac{Q}{C} = -L\frac{\partial I}{\partial t} = -\dot{\Phi} \\ \frac{\partial H}{\partial \Phi} &= \frac{\Phi}{L} = I = \dot{Q} \end{aligned}\right]$$

mechanical harmonic oscillator:

$$H = \underbrace{\frac{p^2}{2m}}_{\text{kinetic}} + \underbrace{\frac{k}{2}x^2}_{\text{potential}}$$

energy





(virtual) particle of mass m (C) moving in potential

Characteristic quantities:

mechanical	electronic	HO
position x	flux Φ	canonical variables
momentum p	charge Q	
mass m	capacitance C	
spring constant k	inverse inductance $\frac{1}{L}$	
frequency $\omega = \sqrt{\frac{k}{m}}$	$\omega = \frac{1}{\sqrt{LC}}$	

2.2. Quantum harmonic oscillator

Mittwoch, 21. März 2012
12:54

$$x \rightarrow \hat{x} \qquad \Phi \rightarrow \hat{\Phi}$$

$$p \rightarrow \hat{p} = -i\hbar \frac{\partial}{\partial x} \qquad Q \rightarrow \hat{Q} = -i\hbar \frac{\partial}{\partial \Phi}$$

Commutation relations:

$$[\hat{x}, \hat{p}] = i\hbar \qquad [\hat{\Phi}, \hat{Q}] = i\hbar \quad \text{flux-charge} \quad \Big| \cdot \frac{1}{\hbar}$$

$$\Leftrightarrow \left[2\pi \frac{\hat{\Phi}}{\phi_0}, \frac{\hat{Q}}{2e}\right] = [\hat{\delta}, \hat{N}] = i \quad \text{phase-number}$$

ϕ_0 .. magnetic flux quantum $\frac{h}{2e}$

Hamilton operator

$$\boxed{H = \frac{\hat{Q}^2}{2C} + \frac{\hat{\Phi}^2}{2L}}$$

creation and annihilation operators

$$\hat{a}^+ = \frac{1}{\sqrt{2\hbar Z_c}} \left(Z_c \hat{Q}^+ - i \hat{\Phi}^+\right) \quad \text{creation}$$

operator

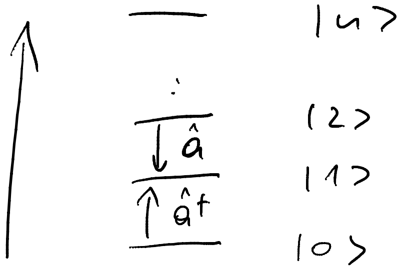
$$\hat{a} = \frac{1}{\sqrt{2\hbar Z_c}} \left(Z_c \hat{Q} + i \hat{\Phi}\right) \quad \text{annihilation}$$

$$Z_c = \sqrt{L/C}, \quad \omega = 1/\sqrt{LC} \quad \text{impedance, resonance freq.}$$

$$\Rightarrow \hat{H} = \hbar\omega\left(\hat{a}^+\hat{a} - \frac{1}{2}\right) \quad \hat{a}^+\hat{a} \text{ ... number operator}$$

energy level spectrum

equidistant level spacing



properties

$$\hat{a}^\dagger |n\rangle = \sqrt{n+1}\, |n+1\rangle$$

$$\hat{a}\, |n\rangle = \sqrt{n}\, |n-1\rangle$$

$|n\rangle$ ... Fock state with n excitations

$$\hat{a}^\dagger \hat{a} |n\rangle \equiv \hat{n} |n\rangle = n |n\rangle$$

relation to $\hat{Q}$, $\hat{\Phi}$ & $\hat{V}$, $\hat{I}$ (inverse relations)

$$\hat{Q} = \sqrt{\frac{\hbar}{2Z_c}} \left(\hat{a}^\dagger + \hat{a}\right) \qquad \hat{\Phi} = i\sqrt{\frac{\hbar Z_c}{2}} \left(\hat{a}^\dagger - \hat{a}\right)$$

a $$\hat{V} = \frac{\hat{Q}}{C} = \sqrt{\frac{\hbar \sqrt{C}}{2C^2\sqrt{L}}} \left(\hat{a}^\dagger + \hat{a}\right) = \sqrt{\frac{\hbar\omega}{2C}} \left(\hat{a}^\dagger + \hat{a}\right)$$

$$\hat{I} = \frac{\hat{\Phi}}{L} = i\sqrt{\frac{\hbar\omega}{2L}} \left(\hat{a}^\dagger - \hat{a}\right) \qquad \left[\omega = \frac{1}{\sqrt{LC}}\right]$$

2.3. Transmission line resonator

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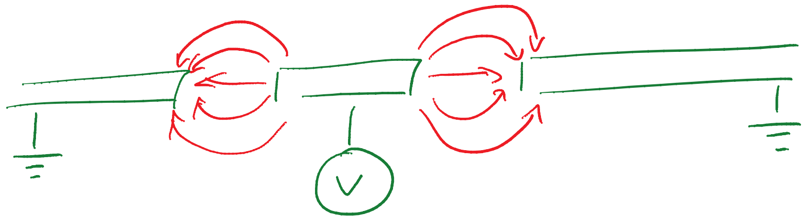
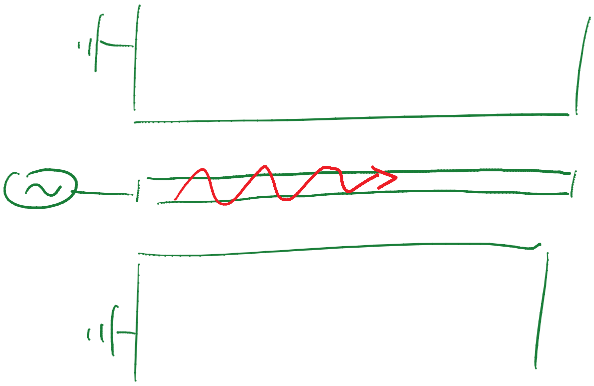
12:56

Field distribution of the transmission line?



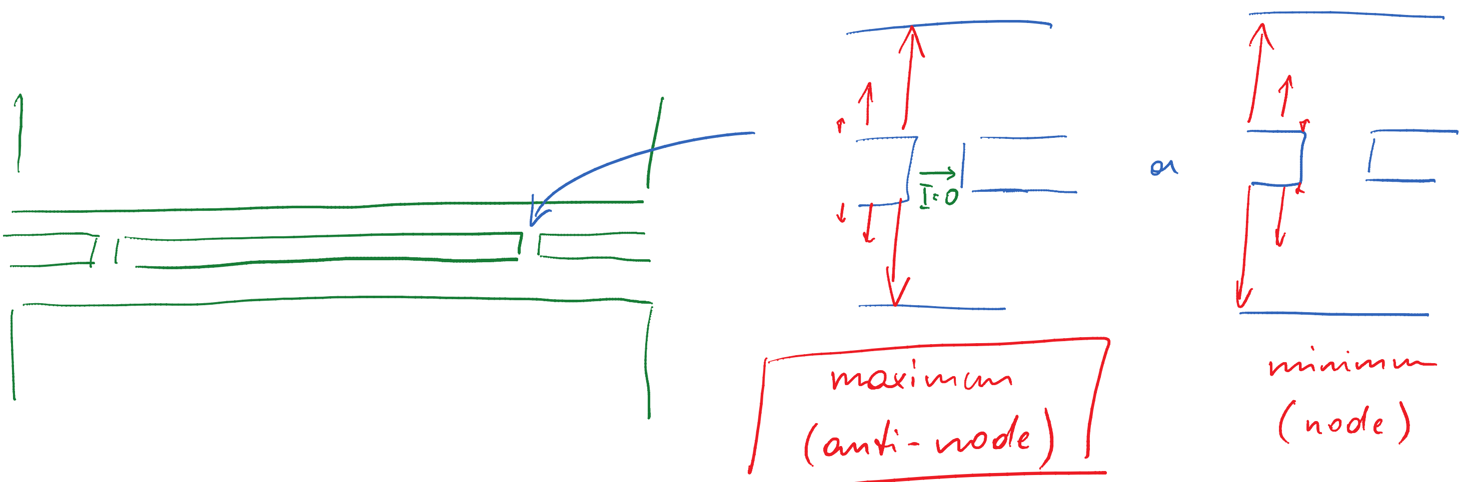
Wave propagation in a transmission line?



$$E = E_0 \cdot e^{i(kx - \omega t)} + h.c.$$

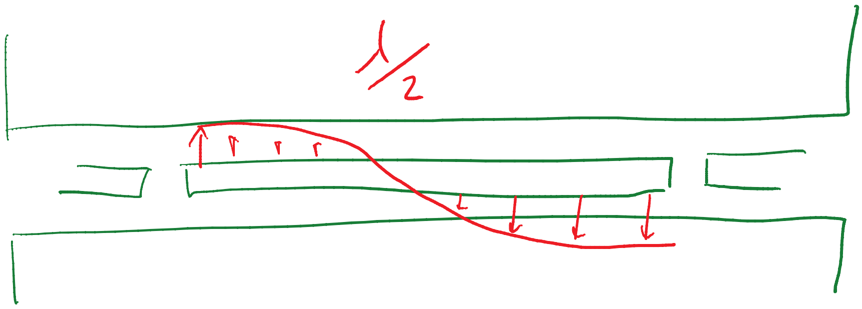
$$\omega = c_{eff} \cdot k = \frac{c}{\sqrt{\varepsilon}} \cdot \frac{2\pi}{\lambda}$$

Electric field at the boundary?



no current => voltage maximum => field maximum

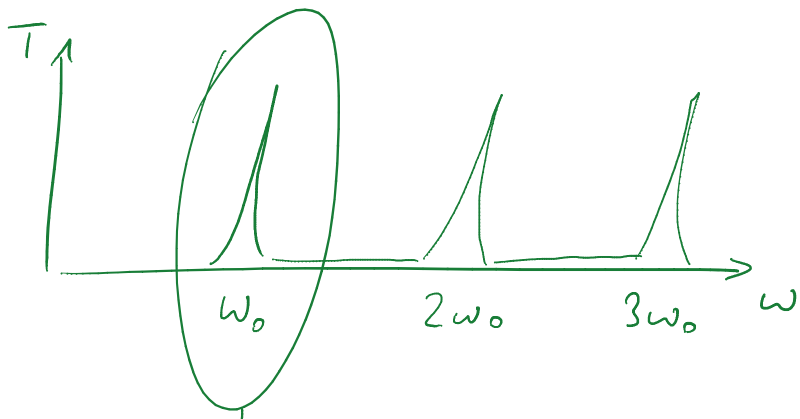
Fundamental mode?



Resonance frequency?

$$\left.\begin{array}{l} \omega = c_{eff} \dfrac{2\pi}{\lambda} \\[2ex] \ell = \dfrac{\lambda}{2} \cdot n \end{array}\right\} \quad \omega = c_{eff} \frac{2\pi}{2\ell} = \frac{\pi c_{eff}}{\ell} \cdot n$$

Excitation spectrum:



→ harmonic oscillator

photons $\hat{=}$ excitation quanta

2.4. Josephson Junction as non-linear inductor

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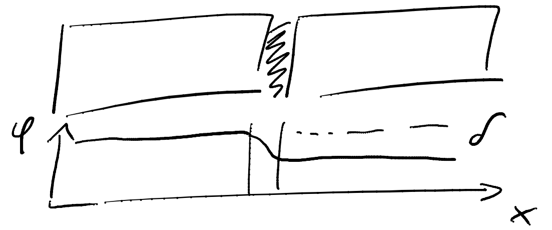
12:58

induction law: $V = -L\dot{I}$

Josephson relation: $\boxed{I = I_0 \cdot \sin\delta}$ [dc]

(supercurrent across junction)



$$\boxed{V = \frac{\phi_0}{2\pi}\dot{\delta}}$$ [ac]

(time-dependent phase if voltage is applied)

$$\dot{I} = I_0 \cdot \cos\delta\,\dot{\delta} \quad \Rightarrow \quad \boxed{V = \frac{\phi_0}{2\pi I_0}\frac{1}{\cos\delta}\dot{I} \equiv L_J \dot{I}}$$

Josephson Inductance:

$$L_J = L_{J0}\frac{1}{\cos\delta} \rightarrow \text{non-linearity}$$

$$L_{J0} = \frac{\phi_0}{2\pi I_0} \quad \text{specific Josephson inductance}$$

Josephson Energy: $E_J = \int V I \, dt' = \int \frac{\phi_0 I_0}{2\pi}\dot{\delta}\sin\delta\, dt$

$$= \int \frac{\phi_0 I_0}{2\pi}\sin\delta\, d\delta = \frac{\phi_0 I_0}{2\pi}\cos\delta$$

$$\boxed{= E_{J0}\cos\delta \quad \text{with} \quad E_{J0} = \frac{\phi_0 I_0}{2\pi}}$$

typical parameters:

$$I_0 = 100\,nA$$

$$\Rightarrow L_{J0} = \frac{\phi_0}{2\pi I_0} \approx 3\,nH \quad (\sim 3\,mm \text{ of wire})$$

$$\Rightarrow E_{J0} = \frac{\phi_0 I_0}{2\pi} \approx 50\,GHz$$