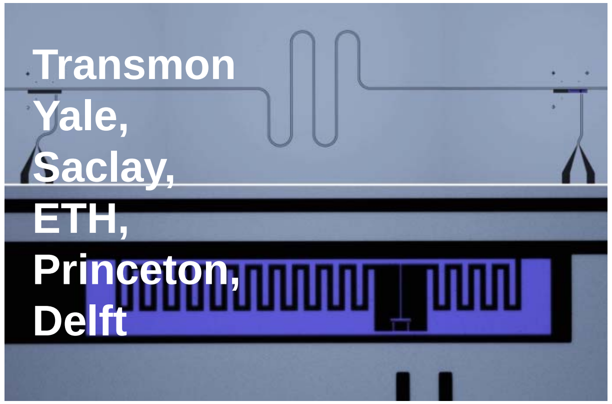
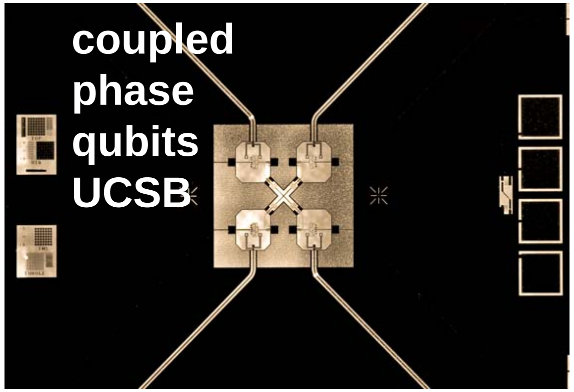
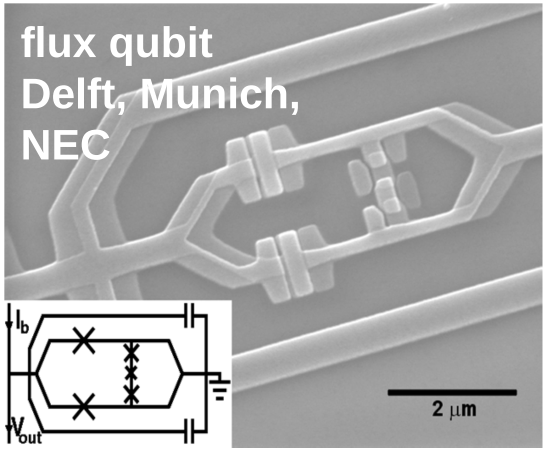
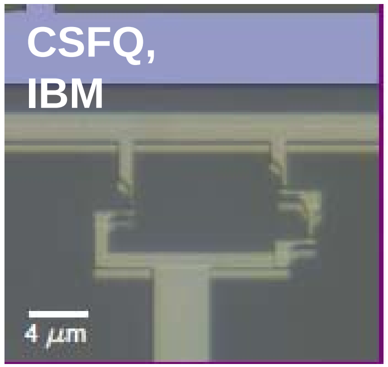
Building a Quantum Information Processor using Superconducting Circuits







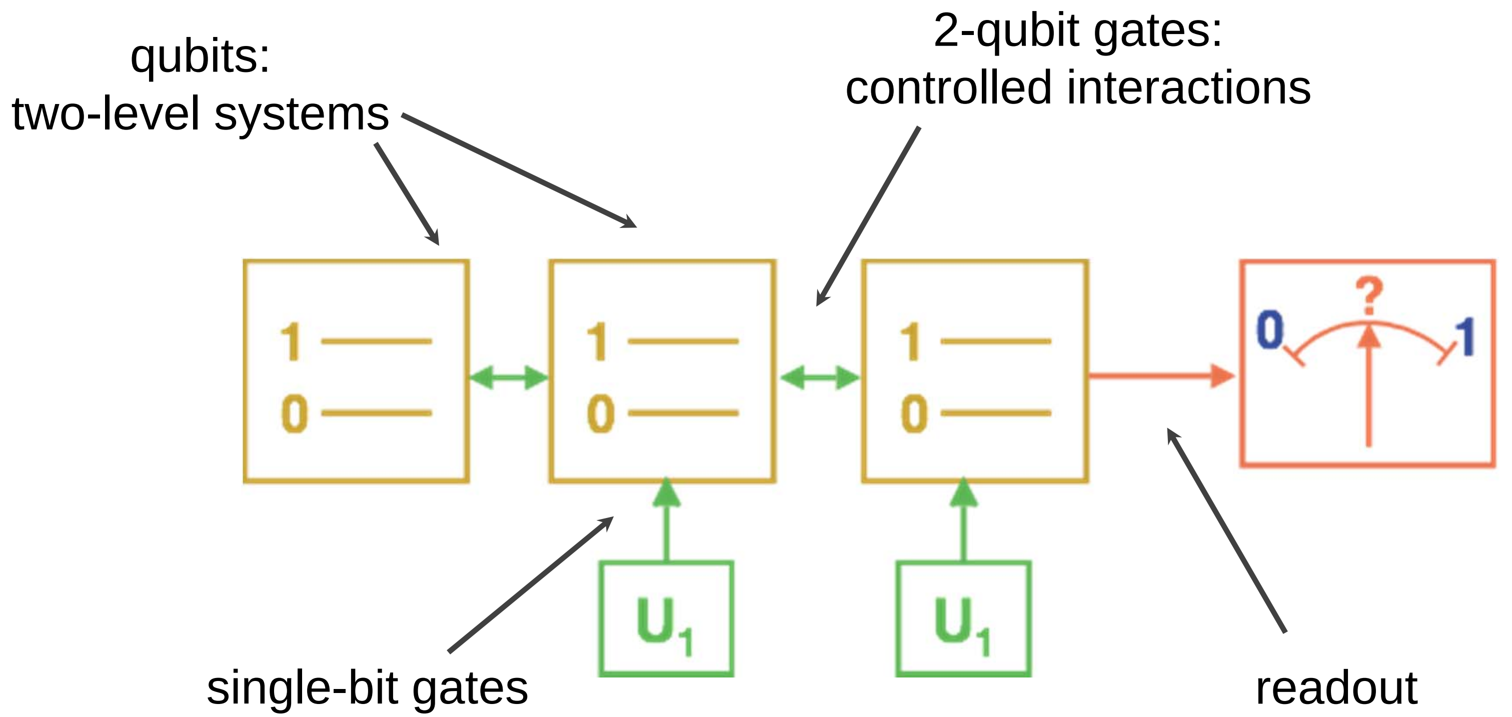


etc…

[review: J. Clarke and F. Wilhelm *Nature* 453, 1031 (2008)]

Generic Quantum Information Processor

The challenge:



- Quantum information processing requires excellent qubits, gates, ...
- Conflicting requirements: good isolation from environment while maintaining good addressability

[M. Nielsen and I. Chuang,
Quantum Computation and Quantum Information (Cambridge, 2000)]

The DiVincenzo Criteria

for Implementing a quantum computer in the standard (circuit approach) to quantum information processing (QIP):

#1. A **scalable** physical system with well-characterized qubits.
#2. The ability to **initialize** the state of the qubits.
#3. **Long (relative) decoherence** times, much longer than the gate-operation time.
#4. A **universal set** of quantum gates.
#5. A qubit-specific **measurement** capability.

plus two criteria requiring the possibility to transmit information:

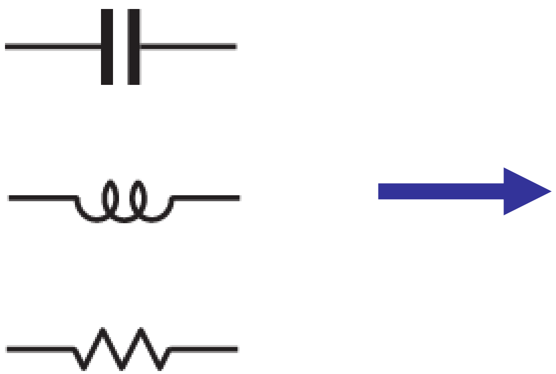
#6. The ability to **interconvert** stationary and mobile (or flying) qubits.
#7. The ability to faithfully **transmit** flying qubits between specified locations.

Topics – superconducting qubits

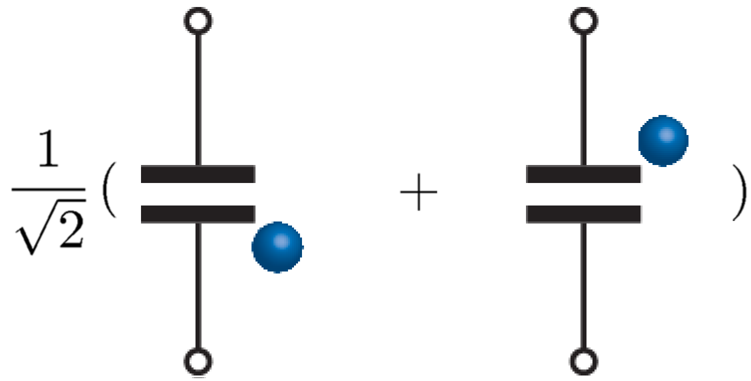
- realization of superconducting quantum electronic circuits
  - harmonic oscillators (photons)
  - non-harmonic oscillators (qubits)
- controlled qubit/photon interactions
  - cavity quantum electrodynamics with circuits
- qubit read-out
- single qubit control
- decoherence
- two-qubit interactions
  - generation of entanglement (C-NOT gate)
  - realization of quantum algorithms (teleportation)

Classical and Quantum Electronic Circuit Elements

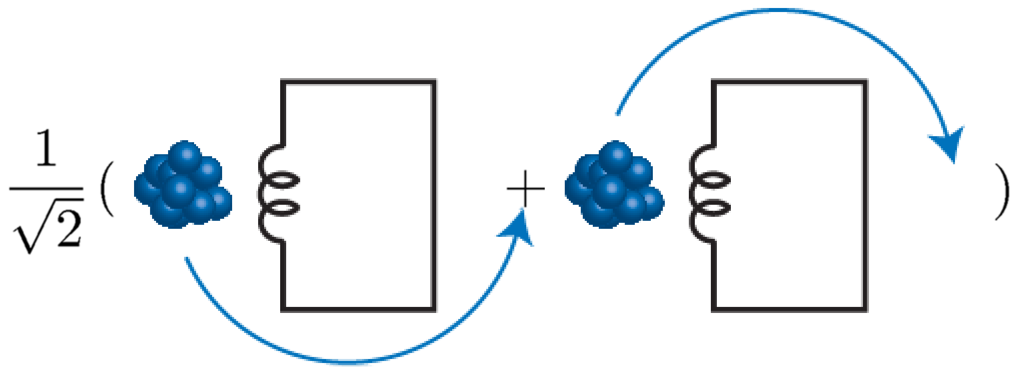
basic circuit elements:



charge on a capacitor:



current or magnetic flux in an inductor:

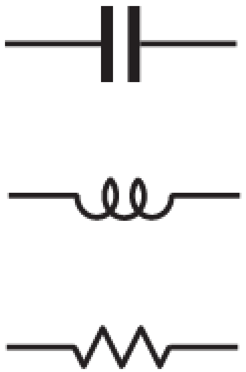


quantum degrees of freedom:

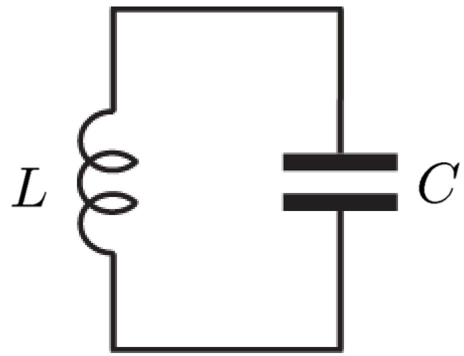
- charge q
- magnetic flux ϕ

Constructing Linear Quantum Electronic Circuits

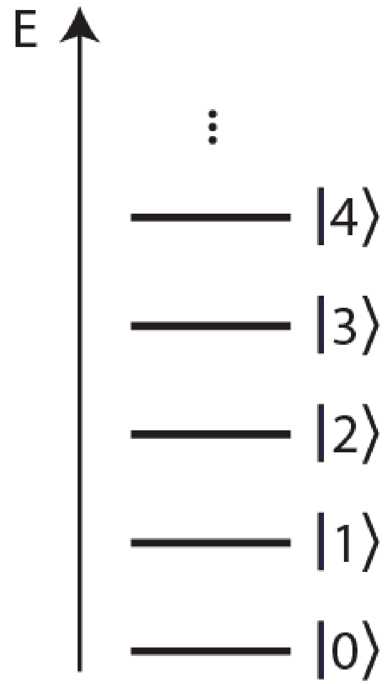
basic circuit elements:



harmonic LC oscillator:



energy:



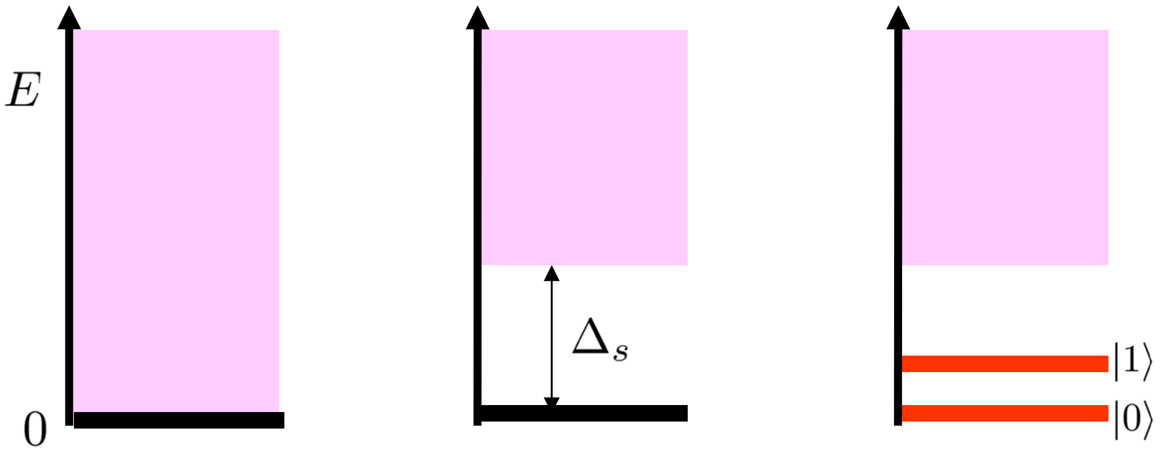
- typical inductor: L = 1 nH
- a wire in vacuum has inductance ~ 1 nH/mm
- typical capacitor: C = 1 pF
- a capacitor with plate size 10 μm x 10 μm and dielectric AlOx (ε = 10) of thickness 10 nm has a capacitance C ~ 1 pF

resonance frequency:

$$\omega = \frac{1}{\sqrt{LC}} \sim 5\,\mathrm{GHz}$$

Review: M. H. Devoret, A. Wallraff and J. M. Martinis, *condmat/0411172* (2004)

Why Superconductors?



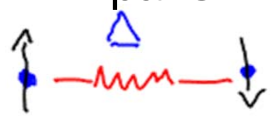
normal metal superconductor How to make qubit?

- single non-degenerate macroscopic ground state
- elimination of low-energy excitations
- no dissipation

Superconducting materials (for electronics):

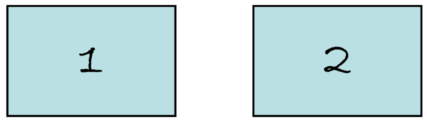
- Niobium (Nb): $2\Delta/h = 725$ GHz, $T_c = 9.2$ K
- Aluminum (Al): $2\Delta/h = 100$ GHz, $T_c = 1.2$ K

Cooper pairs: bound electron pairs



Bosons (S=0, L=0)

2 chunks of superconductors



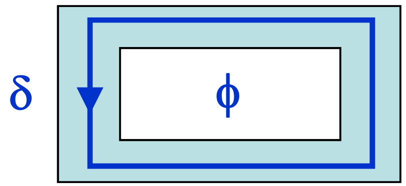
macroscopic wave function

$$\Psi_i = \sqrt{n_i^{CP}}\, e^{i\delta_i}$$

Cooper pair density n_i and global phase δ_i

phase quantization: $\delta = n\, 2\pi$

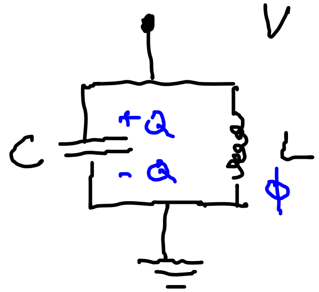
flux quantization: $\phi = n\, \phi_0$



$\phi_0 = h/2e$ … magnetic flux quantum ($2.067\ 10^{-15}$ Wb)

Quantization of the electrical LC harmonic oscillator:

parallel LC oscillator circuit:



voltage across the oscillator:

$$V = \frac{Q}{C} = -L\frac{\partial I}{\partial t}$$

total energy (Hamiltonian): $H = \frac{1}{2} C V^2 + \frac{1}{2} L I^2 = \frac{1}{2}\frac{Q^2}{C} + \frac{1}{2}\frac{\phi^2}{L}$

with the charge Q stored on the capacitor $\quad Q = V C$

a flux ϕ stored in the inductor $\quad \phi = L I$

properties of Hamiltonian written in variables Q and ϕ:

$$\frac{\partial H}{\partial Q} = \frac{Q}{C} = -L\frac{\partial I}{\partial t} = -\dot{\phi}$$

$$\frac{\partial H}{\partial \phi} = \frac{\phi}{L} = I = \dot{Q}$$

Q and ϕ are canonical variables

see e.g.: Goldstein, Classical Mechanics, Chapter 8, Hamilton Equations of Motion

Quantum version of Hamiltonian

$$\hat{H} = \frac{\hat{Q}^2}{2C} + \frac{\hat{\phi}^2}{2L}$$

with commutation relation

$$[\hat{\phi}, \hat{Q}] = i\hbar$$

compare with particle in a harmonic potential:

$$\hat{H} = \frac{\hat{p}^2}{2m} + \frac{1}{2} m \omega^2 \hat{x}^2$$

analogy with electrical oscillator:

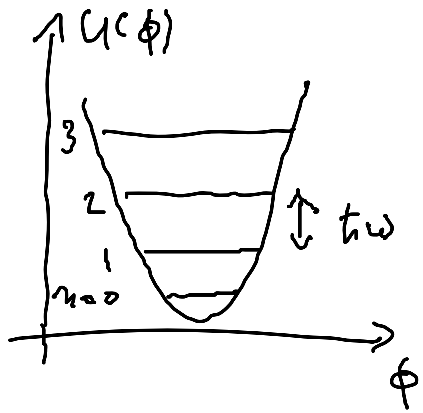
- charge Q corresponds to momentum p
- flux ϕ corresponds to position x

$$[\hat{x}, \hat{p}] = [\hat{x}, -i\hbar \frac{\partial}{\partial x}] = i\hbar$$

$$[\hat{\phi}, \hat{Q}] = [\hat{\phi}, -i\hbar \frac{\partial}{\partial \phi}] = i\hbar$$

Hamiltonian in terms of raising and lowering operators:

$$\hat{H} = \hbar\omega \left(a^{+}a + \frac{1}{2}\right)$$



with oscillator resonance frequency:

$$\omega = \frac{1}{\sqrt{LC}}$$

Raising and lowering operators:

$$a^{+}|n\rangle = \sqrt{n+1}\,|n+1\rangle \; ; \quad \hat{a}|n\rangle = \sqrt{n}\,|n-1\rangle$$

$$a^{+}a\,|n\rangle = n\,|n\rangle$$ number operator

in terms of Q and ϕ:

$$\hat{a} = \frac{1}{\sqrt{2\hbar Z_c}}\left(Z_c\hat{Q} + i\hat{\phi}\right)$$

with Z_c being the characteristic impedance of the oscillator $Z_c = \sqrt{\frac{L}{C}}$

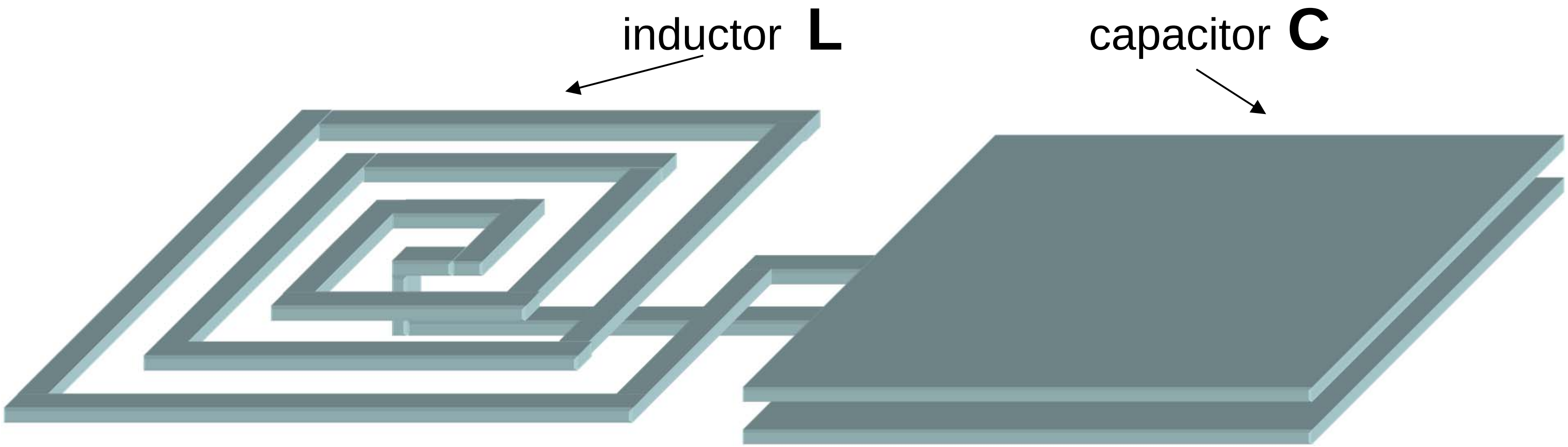
charge Q and flux ϕ operators can be expressed in terms of raising and lowering operators:

$$\hat{Q} = \sqrt{\frac{\hbar}{2Z_c}}\left(a^{+} + a\right)$$

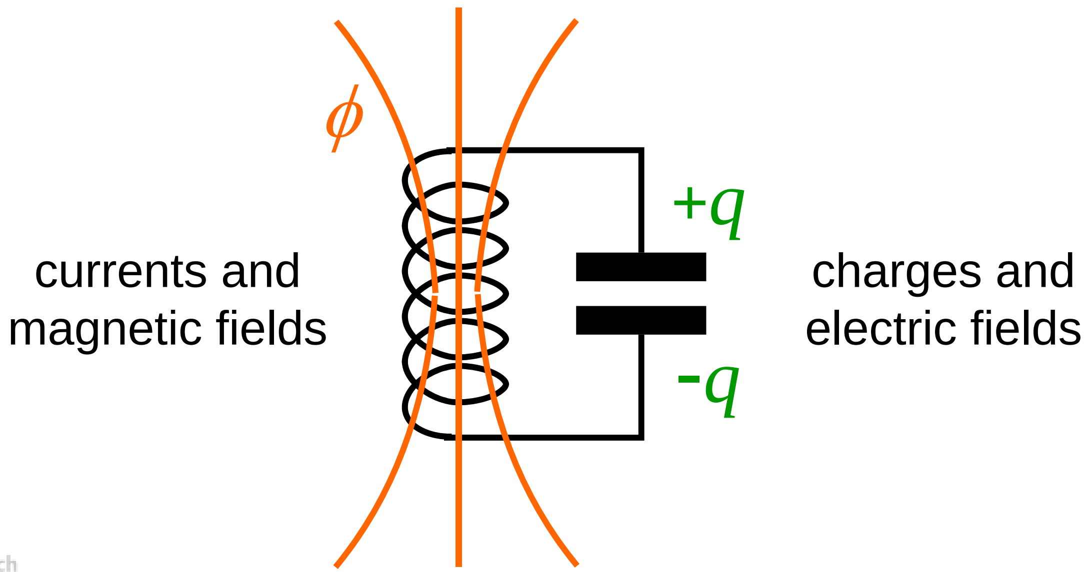
$$\hat{\phi} = i\sqrt{\frac{\hbar Z_c}{2}}\left(a^{+} - a\right)$$

Exercise: Making use of the commutation relations for the charge and flux operators, show that the harmonic oscillator Hamiltonian in terms of the raising and lowering operators is identical to the one in terms of charge and flux operators.

Realization of H.O.: Lumped Element Resonator

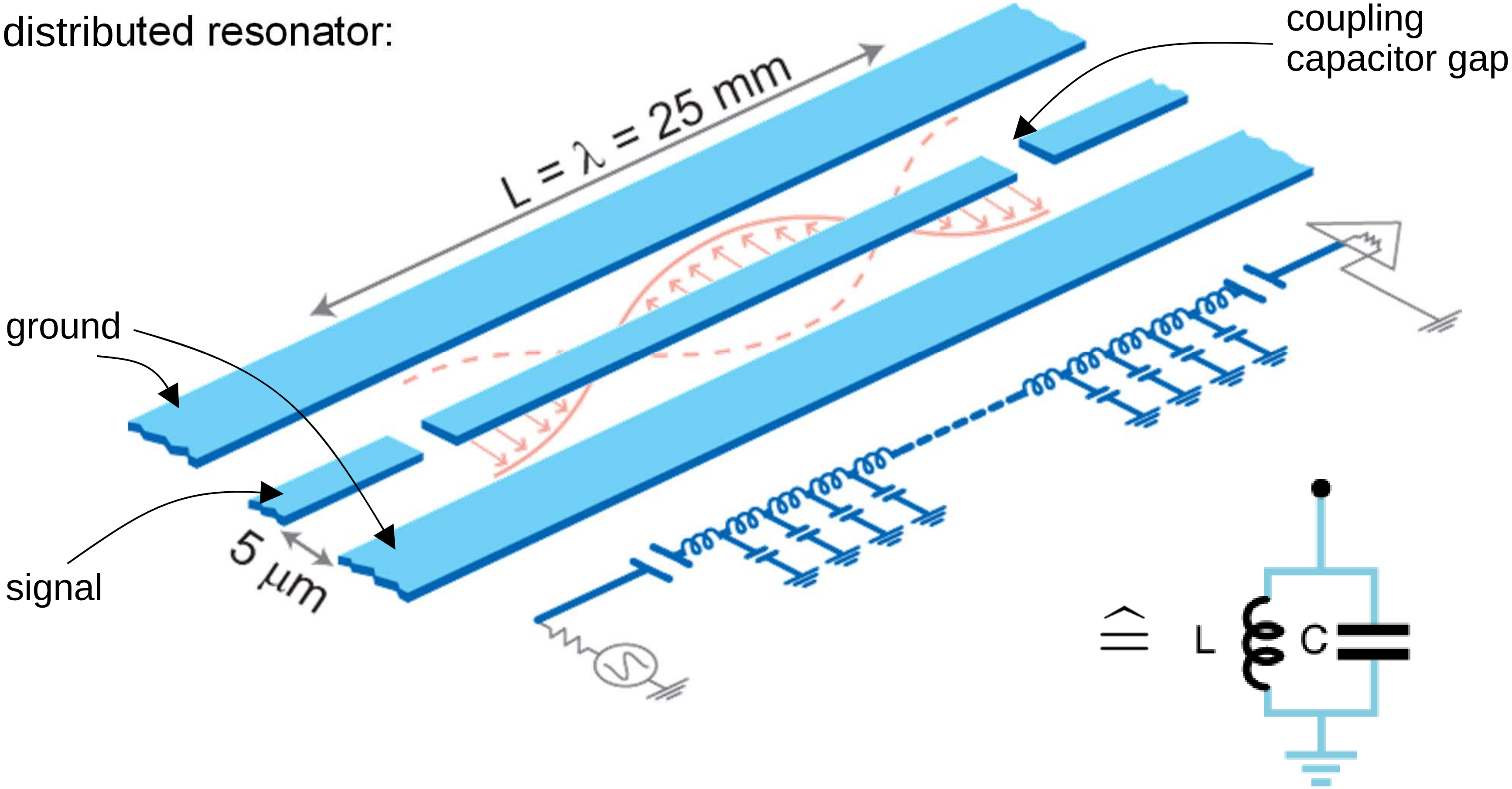


a harmonic oscillator



Realization of H.O.: Transmission Line Resonator

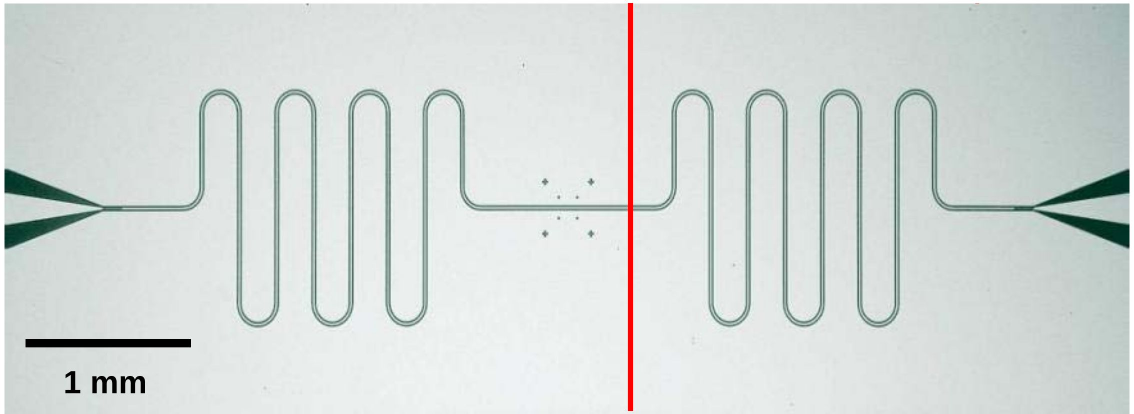
distributed resonator:



- coplanar waveguide resonator
- close to resonance: equivalent to lumped element LC resonator

[M. Goeppl *et al.*, Coplanar Waveguide Resonators for Circuit QED, *Journal of Applied Physics* **104**, 113904 (2008)]

Transmission line resonator

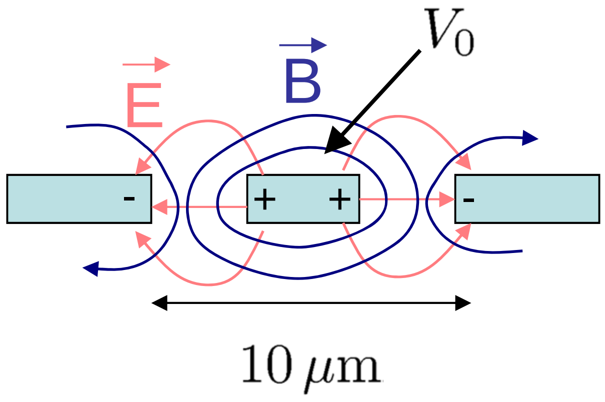


optical microscope image of sample fabricated at FIRST (Nb on sapphire)

electric field across resonator in vacuum state (*n*=0):

$$E_{0,\mathrm{rms}} \approx 0.2\,\mathrm{V/m} \qquad \text{for } \omega_r/2\pi \approx 6\,\mathrm{GHz}$$

cross-section of transm. line (TEM mode):

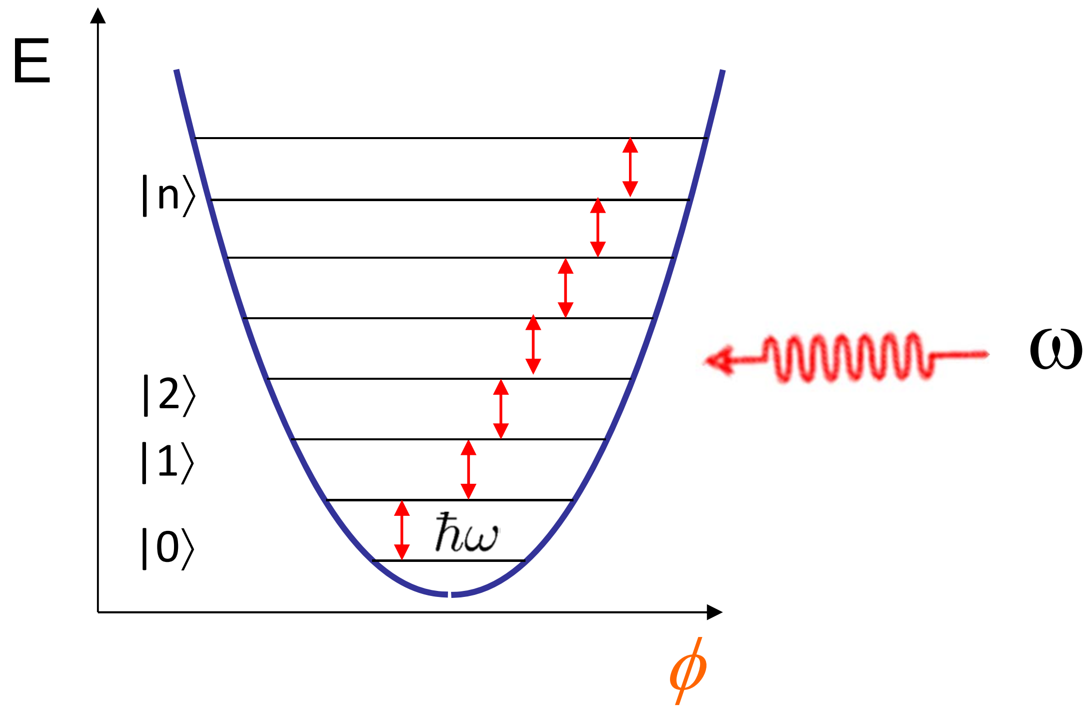


harmonic oscillator

$$H_r = \hbar\omega_r\left(a^\dagger a + \frac{1}{2}\right)$$

How to prepare quantum states?

Question: What happens to the harmonic oscillator (in ground state), if we drive transitions at frequency ω?

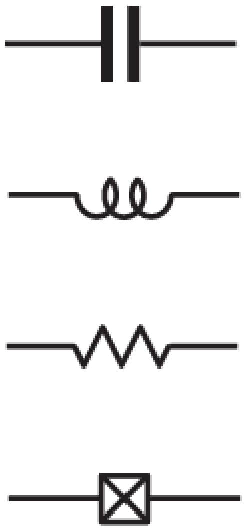


Transitions to higher levels will be driven equally, harmonic oscillator will be in a 'coherent' state, which is the most classical state

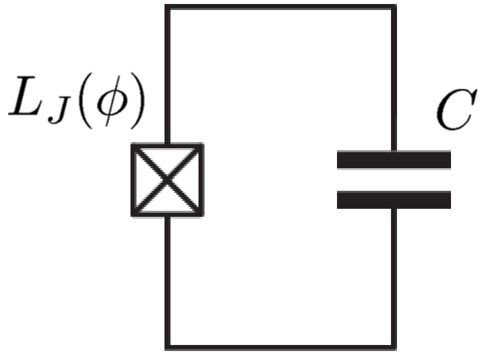
-> no quantum features observables

Constructing Non-Linear Quantum Electronic Circuits

circuit elements:



anharmonic oscillator:



Josephson junction:

a non-dissipative nonlinear element (inductor)

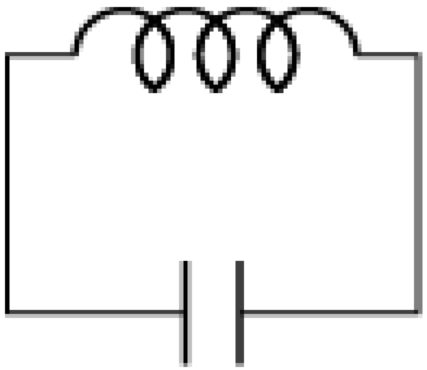
$$L_J(\phi) = \left(\frac{\partial I}{\partial \phi}\right)^{-1}$$

$$= \frac{\phi_0}{2\pi I_c} \frac{1}{\cos(2\pi\phi/\phi_0)}$$

Review: M. H. Devoret, A. Wallraff and J. M. Martinis, *condmat/0411172* (2004)

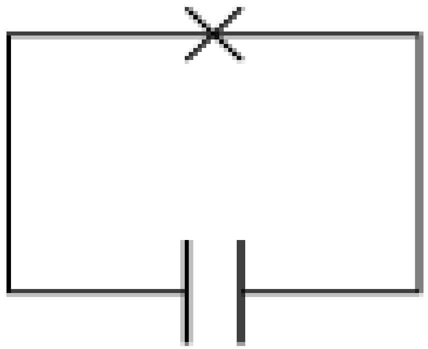
Linear vs. Nonlinear Superconducting Oscillators

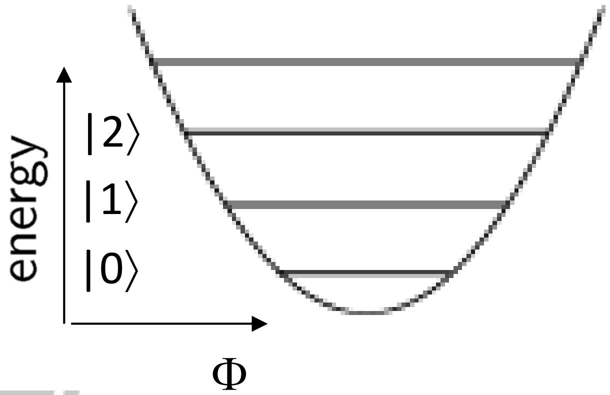
LC resonator



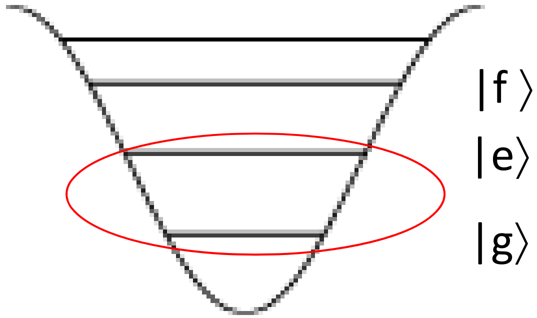
Josephson junction resonator

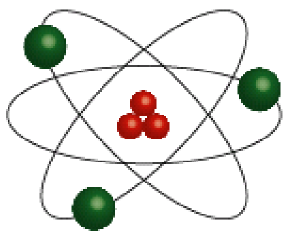
Josephson junction = nonlinear inductor





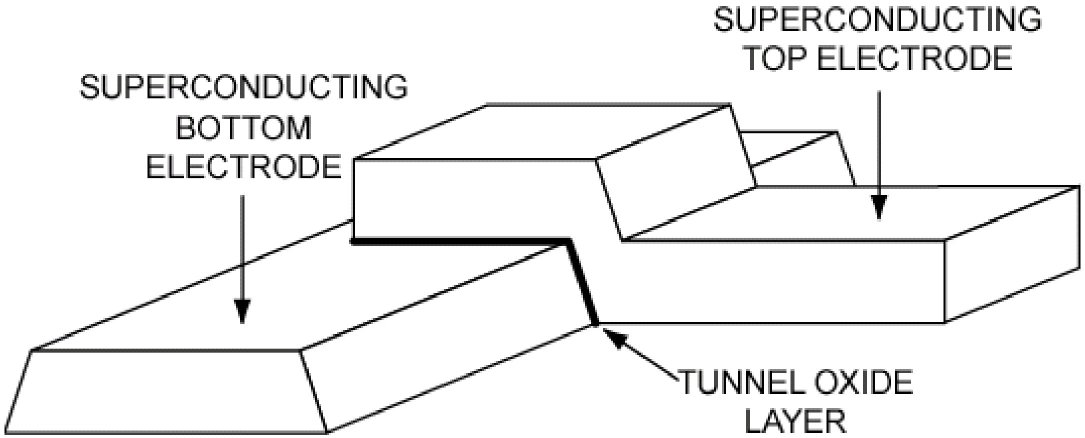
anharmonicity -> artifical atom,
effective two-level system

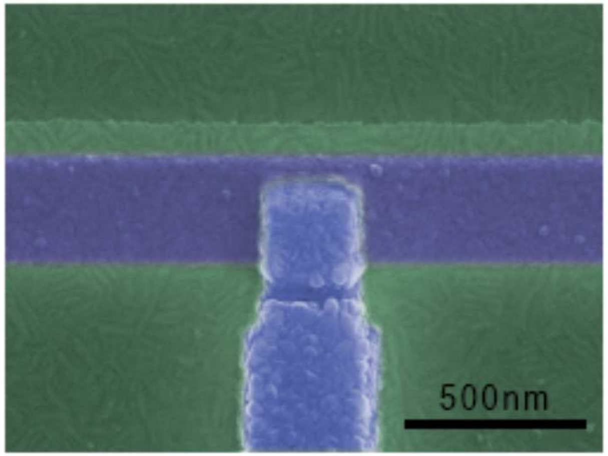




A Low-Loss Nonlinear Element

a (superconducting) Josephson junction



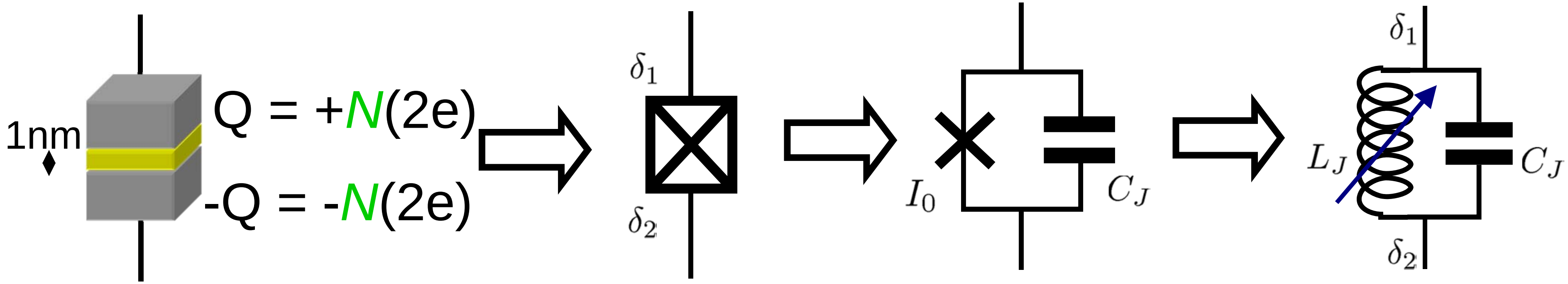


- superconductors: Nb, Al
- tunnel barrier: AlO_x

M. Tinkham, *Introduction to Superconductivity* (Krieger, Malabar, 1985).

Josephson Tunnel Junction

the only non-linear LC resonator with no dissipation (BCS, $k_B T \ll \Delta$)



tunnel junction relations:

- critical current I_c
- junction capacitance C_J
- high internal resistance R_J (insulator)

Josephson relation:
$$I = I_c \sin\delta$$
$$V = \Phi_0 \frac{\partial \delta}{\partial t}$$

(reduced) flux quantum: $\Phi_0 = \frac{\phi_0}{2\pi} = \frac{\hbar}{2e}$

phase difference: $\delta = \delta_2 - \delta_1$

derivation of Josephson effect, see e.g.: chap. 21 in R. A. Feynman: Quantum mechanics, The Feynman Lectures on Physics. Vol. 3 (Addison-Wesley, 1965)

The Josephson junction as a non-linear inductor

induction law: $V = -L \frac{\partial I}{\partial t}$

Josephson effect: dc-Josephson equation $I = I_c \sin\delta$

$$\frac{\partial I}{\partial t} = I_c \cos\delta \frac{\partial \delta}{\partial t}$$

ac-Josephson equation $V = \frac{\phi_0}{2\pi} \frac{\partial \delta}{\partial t} = \underbrace{\frac{\phi_0}{2\pi I_c} \frac{1}{\cos\delta}}_{L_J} \frac{\partial I}{\partial t}$

Josephson inductance $L_J = \underbrace{\frac{\phi_0}{2\pi I_c}}_{\text{specific Josephson inductance}} \underbrace{\frac{1}{\cos\delta}}_{\text{nonlinearity}}$

specific Josephson inductance

nonlinearity

A typical characteristic Josephson inductance for a tunnel junction with $I_c = 100$ nA is $L_{J0} \sim 3$ nH.

review: M. H. Devoret *et al.*,
Quantum tunneling in condensed media, *North-Holland,* (1992)