2.11. SWAP gate (virtual photon mediated)

Dienstag, 08. November 2011
14:57

$$H = \hbar J \left(\sigma_1^+ \otimes \sigma_2^- + \sigma_1^- \otimes \sigma_2^+\right)$$

$$\sigma_1^+ \otimes \sigma_2^- = \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix} \otimes \begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix}$$

$$\sigma_1^- \otimes \sigma_2^+ = \begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix} \otimes \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix}$$

$$H = \hbar J \left[\begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} + \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} \right]$$

$$= \hbar J \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} \quad 0_2 \quad \to \tilde{\sigma}_x$$

Basis: $|gg\rangle = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 1 \end{pmatrix}$ $|ge\rangle = \begin{pmatrix} 0 \\ 0 \\ 1 \\ 0 \end{pmatrix}$

$|eg\rangle = \begin{pmatrix} 0 \\ 1 \\ 0 \\ 0 \end{pmatrix}$ $|ee\rangle = \begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \end{pmatrix}$

time evolution:

$$U = e^{-\frac{i}{\hbar} H t} = e^{-iJt\cdot\tilde{\sigma}_x} \oplus e^{\overbrace{-i 0_2 t}^{\mathbb{1}_2}}$$

$$= \left(\mathbb{1}\cdot \cos Jt - i\tilde{\sigma}_x \sin Jt\right) \oplus \mathbb{1}_2$$

$$= \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos Jt & -i\sin Jt & 0 \\ 0 & -i\sin Jt & \cos Jt & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

$$= |gg\rangle\langle gg| + |ee\rangle\langle ee| + \cos Jt\left[|eg\rangle\langle eg| + |ge\rangle\langle ge|\right]$$
$$- i\sin Jt\left[|eg\rangle\langle ge| + |ge\rangle\langle eg|\right]$$

effect on state $|eg\rangle = \begin{pmatrix} 0 \\ 1 \\ 0 \\ 0 \end{pmatrix}$:

$$U|eg\rangle = \cos Jt \begin{pmatrix} 0 \\ 1 \\ 0 \\ 0 \end{pmatrix} - i \sin Jt \begin{pmatrix} 0 \\ 0 \\ 1 \\ 0 \end{pmatrix} = \cos Jt\, |eg\rangle - i \sin Jt\, |ge\rangle$$

*) SWAP $|eg\rangle \leftrightarrow |ge\rangle$ for $t = \frac{\pi}{2J}$: $|eg\rangle \rightarrow -i|ge\rangle$ (iSWAP)

*) entanglement generation at $t = \frac{\pi}{4J}$: $|eg\rangle \rightarrow \frac{1}{\sqrt{2}}\left(|eg\rangle - i|ge\rangle\right)$ ($\sqrt{\text{iSWAP}}$)

maximally entangled state

*) phase shift for $t = \frac{\pi}{J}$: $|eg\rangle \rightarrow -|eg\rangle$

CPHASE gate from $|fg\rangle \leftrightarrow |ee\rangle$ coupling

$|fg\rangle$ ... state not in computational subspace

coupling: $\tilde{H} = \hbar J_2 \left(|f\rangle\langle e| \otimes |g\rangle\langle e| + |e\rangle\langle f| \otimes |e\rangle\langle g| \right)$

$\hat{=} J_2 \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$ in basis $|fg\rangle = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$, $|ee\rangle = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$

$$\tilde{U} = e^{-\frac{i}{\hbar}\tilde{H}t} = \begin{pmatrix} \cos J_2 t & -i \sin J_2 t \\ -i \sin J_2 t & \cos J_2 t \end{pmatrix}$$

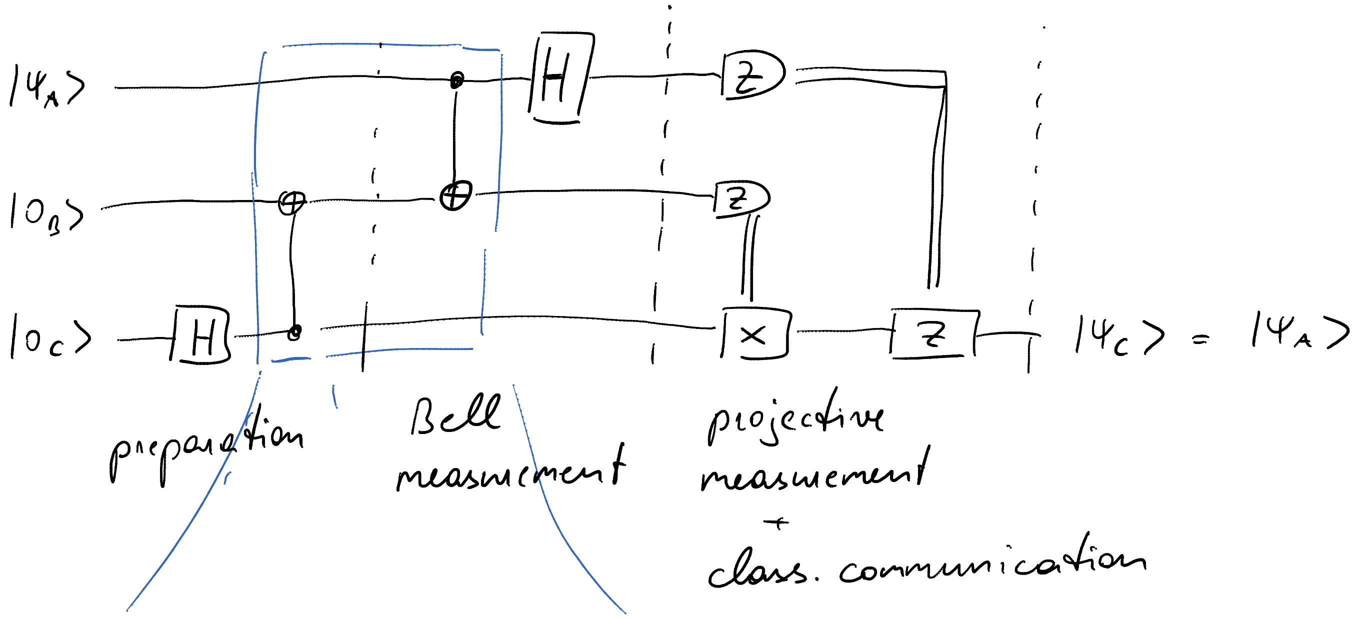
$$U_{phase} = \tilde{U}\left(t = \frac{\pi}{J_2}\right) = \begin{pmatrix} -1 & 0 \\ 0 & -1 \end{pmatrix} \Rightarrow U_{phase}|ee\rangle = -|ee\rangle$$

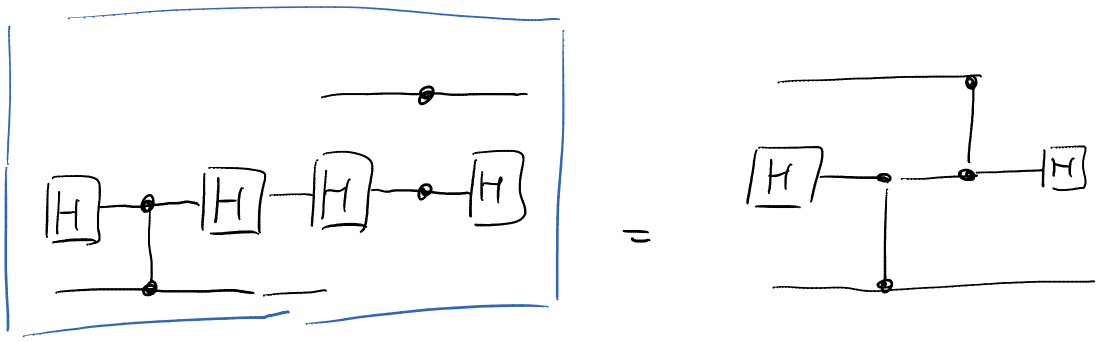
in standard 2-qubit basis: $\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix} = U_{cPhase}$

$\Rightarrow$ only state $|ee\rangle$ obtains phase of π !

2.12. Entanglement protocol

Freitag, 30. März 2012
11:54





since $\boxed{H}-\boxed{H} = \mathbb{1}$ (identity)