Physics IV - Exam - Winter 2007/08

HPV G4, 09:00 – 11:00, Thursday, 31st January 2008

Please note:

- There are a total of 7 questions printed on **FOUR PAGES**.
- There are a total of 60 points. Points for each part of each question are shown in square brackets in the right margin.
- A table of values of physical constants is printed on the back of this cover sheet.
- You are allowed 10 sides of handwritten notes, and a non-programmable calculator in the exam.
- Please write CLEARLY, as if we cannot read your handwriting we cannot award you marks.
- Please WRITE YOUR NAME BELOW. This sheet will be stapled to your answers at the end of the exam.
- Please put your name on all of your answer sheets.
- Throughout the exam the exam overseers are available to answer your questions, do not hesitate to ask for clarification if needed.

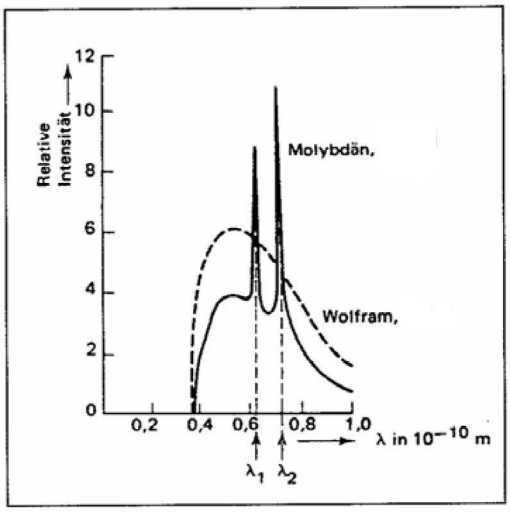
NAME:	FIRST NAME:

Question:	1	2	3	4	5	6	7	TOTAL
Marks:								
Max:	12	11	9	7	9	6	6	60

Table of physical constants

speed of light, c	3.00×10^{8} $\mathrm{ms^{-1}}$
Planck constant, h	6.63×10^{-34} Js
$\hbar = h/(2\pi)$	1.05×10^{-34} Js
electron charge, e	1.60×10^{-19} C
electron volt, eV	1.60×10^{-19} J
electron mass, m_e	9.11×10^{-31} kg
proton or neutron mass, m_u,	1.67×10^{-27} kg
Boltzmann constant, k_B	1.38×10^{-23} $\mathrm{JK^{-1}}$
Stefan-Boltzmann constant, σ	5.67×10^{-8} $\mathrm{Wm^{-2}K^{-4}}$

1. Electrons incident on a metal surface can cause emission of X-ray light. Typical intensity spectra of the metals W^{74} and Mo^{42} are shown in the graph below.



(a) Explain the following features of the spectra:

i. Sharp peaks [1]

ii. Broad background [1]

iii. Lower cutoff [1]

(b) Suggest a reason for there being no sharp peaks in the spectrum of W. [1]

(c) Give an estimate of the energy of the incident electrons. [2]

(d) Explain why the frequency of the K_α spectral line is proportional to $(Z-1)^2$, where Z is the atomic number of the atom. [2]

(e) X-rays are often used to analyze crystal structures, by Bragg scattering. Derive the *Bragg condition* for constructive interference. [2]

(f) Diffraction experiments can also be conducted with particles. Calculate the required *velocity* of neutrons in order to observe the first order (n=1) Bragg diffraction peak at $\theta = 30°$, when scattering from a lattice with a spacing 0.5 nm. [2]

2. A particle is confined to a 1D potential well, with the potential $V(x)$ as follows:

$$\begin{array}{ll} V(x) = 0 & 0 < x < L \\ V(x) = \infty & \text{elsewhere} \end{array}$$

(a) Derive the normalized wavefunctions ϕ_n and energies E_n of the eigenstates of the system, in terms of a quantum number n, working from the time independent Schrödinger equation. [5]

(b) The wavefunction Ψ of a general state of the system can be written in terms of the eigenstates ϕ_n as follows:

$$\Psi = \sum_n a_n \phi_n$$

If we change to a different set of basis states χ_m, we can write:

$$\Psi = \sum_m b_m \chi_m$$

where $b_m = \sum_n a_n \int \chi_m^* \phi_n dx$. With the particle initially in the ground state in the potential from part (a) ($\Psi = \phi_1$), the 1D potential well instantaneously expands to twice it's original size, with $V(x)$ now given by:

$$\begin{array}{ll} V(x) = 0 & 0 < x < 2L \\ V(x) = \infty & \text{elsewhere} \end{array}$$

Work out the probability, immediately after this change takes place, of measuring the system in (i) its new ground state and (ii) its new first excited state. [6]

3. (a) List the possible electronic states for an $n = 4$ hydrogen atom (neglecting the spin quantum numbers) labeling them by the angular momentum quantum numbers l and m_l. [2]

(b) i. Calculate the electron angular momentum $|\vec{L}|$ for $l = 3$ and sketch the possible magnetic quantum numbers for these states in a vector diagram. [2]

ii. Why can $\vec{L}$ not be aligned with the quantization axis? [2]

(c) Consider now the electron wave function $\Psi(r, \theta, \phi)$. Sketch in 2 dimensions the characteristic shape of the probability distribution for the different electron orbitals in the $n = 2$ shell, labeling them with the quantum numbers l and m_l. [3]

4. In a system of discrete energy levels E_i at a finite temperature T, the probability of occupation of level i is $p(E_i) = Ag(E_i)e^{-E_i/k_BT}$.

   (a) What is the meaning of the degeneracy $g(E_i)$? [1]

   (b) Consider a molecule with three energy levels. The lowest state has energy $E_0 = 0$. The two excited states have energies E_1 and E_2 respectively ($E_2 > E_1$). All states have $g(E_i) = 1$.

   (c) A population of n molecules are in thermal equilibrium at temperature T. What fraction f_1 of the molecules are in the first excited state? [2]

   (d) Assuming that $E_2 \gg E_1$ and $E_2 \gg k_BT$, show that the fraction of molecules in the first excited state is now given by [2]

$$f_1 \approx \frac{1}{1 + \exp(E_1/k_BT)}$$

   (e) With the same assumptions as in part (d), what is the average energy of the molecules? [2]

5. Carbon monoxide (CO) has a bond length of 0.113 nm. The mass of ^{12}C and ^{16}O atoms are 1.99×10^{-26} kg and 2.66×10^{-26} kg.

   (a) What is the energy E_1 and the rotation rate ω_1 for the first excited rotational state of CO (the ground state does not rotate)? [3]

   (b) Sketch a diagram of the rotational energy levels of CO, showing the energies in terms of E_1. [1]

   (c) What are the selection rules for transitions between the rotational levels? Sketch the absorption spectrum of CO, as a function of the frequency of incident microwave radiation. [3]

   (d) A cold sample of CO is prepared in the ground state. Instead of one spectral line for the lowest energy transition, *two* close spectral lines are observed, at 110 GHz and 115 GHz. Which of the two lines is the expected line? Show that the second line is due to the presence of ^{13}C in the sample. [2]

6. Sunlight arrives at the earth at 1.4 kWm^{-2} when the sun is directly overhead. The distance between the earth and the sun is 1.5×10^{11} m, and the sun's radius is 7×10^{8} m.

   (a) Assuming that the sun is a perfect black body, show that it's surface temperature is $T_S = 5800$ K. Sketch the spectrum of radiation emitted by the sun. [3]

   (b) 'Sunspots' are areas on the sun with a lower temperature ($T_{ss} = 5000$ K). What proportion of the sun's surface would need to be covered by sunspots, to reduce its radiation intensity by 10%? [3]

7. Free electrons in a metal travel at the Fermi velocity, v_F, and undergo a random walk, characterized by the mean free path λ (the average distance between collisions).

   (a) In an electric field E, an electron experiences a force $F = -eE$, where e is the electron charge. Assuming that the time τ between collisions of free electrons is unaffected by the accelerating electric field, show that the average, or 'drift' velocity v_d of the electrons is given by [3]

$$v_d = (\frac{eE}{m})(\frac{\lambda}{v_F})$$

   (b) By relating the current I in a conductor to the drift velocity v_d, and combining with the above expression, show that the current I through a conductor is proportional to the voltage V applied across it (Ohm's law). [3]