**Josephson Effect**

- The Josephson effect describes tunneling of Cooper pairs through a barrier.

- A Josephson junction is a contact between two superconductors separated from each other by a thin (< 2 nm) dielectric tunnel barrier.

- The tunnel current through a Josephson junction is given by:
  \[ I = I_c \sin \phi \]

- This is called the \textit{dc} Josephson effect.

- The critical current \( I_c \) is the maximum Josephson tunnel current that can flow through the barrier determined by the Cooper pair density \( \rho \), the thickness of the tunnel barrier and the area of the tunnel junction.

- The current depends non-linearly on the phase difference \( \phi \):
  \[ \phi = \phi_u - \phi_L \]

- The \textit{ac} Josephson effect relates the voltage \( V \) across a junction to the temporal change of the phase difference:
  \[ V = \frac{h}{2e} \frac{\partial \phi}{\partial t} = \phi_o \dot{\phi} \]

- Thus a voltage across a Josephson junction leads to a current oscillating at a rate:
  \[ \dot{\phi} = \frac{2eV}{h} = 4\pi^2 \frac{MHz}{\mu V} \cdot V \]

- The \textit{ac} Josephson effect relates voltage to frequency only through fundamental constants, therefore it is used as a standard of voltage by irradiating a large number of Josephson junctions with microwaves.

- Another useful circuit is the superconducting quantum interference device (SQUID) that can be used as a very sensitive magnetometer to precisely measure magnetic field (down to \( 10^{-21} T \)), applications include MEG (magneto encephalography).

\[ I_{\text{max}} \propto \cos\left(\pi \frac{\phi}{\phi_0}\right) \]

\[ I_{\text{max}} \propto \sin\left(\pi \frac{\phi}{4\phi_0}\right) \]
The Nobel Prize in Physics 1973

Brian David Josephson (Cambridge University)

for his theoretical predictions of the properties of a supercurrent through a tunnel barrier, in particular those phenomena which are generally known as the Josephson effects

Leo Esaki (IBM Thomas J. Watson Research Center Yorktown Heights, NY, USA)

Ivar Giaever (General Electric Company, Schenectady, NY, USA)

for their experimental discoveries regarding tunneling phenomena in semiconductors and superconductors, respectively

Conventional Information Processing

- information is stored in bits (Binary digit)

- a classical bit (Binary digit) can take values either 0 or 1

- a bit is physically realized as a voltage level in a digital circuit (CMOS, TTL)
  - 5 V = 1
  - 0 V = 0

- information is processed using classical logical gates

- the logic is realized using transistors

- every two years the number of transistors on a chip doubles

What will happen when circuit components reach atomic scales? Will quantum mechanics get in the way or will it be useful?
Quantum Information Science

Is it interesting to store information in quantum mechanical systems?

Can we accurately control the dynamics of quantum systems rather than just observe their properties?

What can we gain by manipulating information stored in quantum systems?

Can we build machines that can process information quantum mechanically?

Can we use quantum systems for communication?

reading material: Quantum Computation and Quantum Information, Nielsen and Chuang, Cambridge University Press

Storage of Quantum Information

register with n bits:

\[
\begin{array}{c}
\text{classically:} \\
\text{can store 1 number only}
\end{array}\]

\[
\begin{array}{c}
\text{quantum mechanically:} \\
\text{can store all numbers simultaneously!}
\end{array}\]

- the amount of information that can be stored in a quantum system with N qubits is exponential in the number of bits

- even small collections of quantum systems, say 100 bits, can store more information than any classical computer can handle

\[
2^{100} > 10^{20} \approx 10^{10} \times 10^9
\]
Processing of Quantum Information

generic quantum information processor:

applications:
- realize fast quantum algorithms
- prime number factorization (Shor algorithm, exponential speed up)
- database search (Grover algorithm, quadratic speed up)
- simulation of quantum mechanical systems

Quantum Computation and Quantum Information, Nielsen and Chuang, Cambridge University Press

Quantum Bits

- quantum bits (qubits) are quantum mechanical systems with two distinct quantum mechanical states.

- qubits can be realized in a wide variety of physical systems displaying quantum mechanical properties. These include atoms, ions, electronic and nuclear magnetic moments, charges in quantum dots, charges and fluxes in superconducting circuits and many more.

- a useful qubit must fulfill the DiVincenzo criteria

a quantum bit can take values (quantum mechanical states)

\[ |0\rangle, |1\rangle \]

in Dirac notation

or both of them at the same time, i.e. it can be in a superposition of states
A qubit can be in a superposition of states

$$|\psi\rangle = \alpha |0\rangle + \beta |1\rangle \quad \text{where} \quad \alpha, \beta \in \mathbb{C}$$

When the state of a qubit is measured one will find

- \( |0\rangle \) with probability \( |\alpha|^2 = \alpha \alpha^* \)
- \( |1\rangle \) with probability \( |\beta|^2 = \beta \beta^* \)

Where the normalization condition is

$$|\alpha|^2 + |\beta|^2 = 1$$

This just means that the sum over the probabilities of finding the qubit in any state must be unity.

Example:

$$|\psi\rangle = \frac{1}{\sqrt{2}} |0\rangle + \frac{1}{\sqrt{2}} |1\rangle$$

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**DiVincenzo Criteria for Implementations of a Quantum Computer:**

#1. A scalable physical system with well-characterized qubits.

#2. The ability to initialize the state of the qubits to a simple fiducial state.

#3. Long (relative) decoherence times, much longer than the gate-operation time.

#4. A universal set of quantum gates.

#5. A qubit-specific measurement capability.

#6. The ability to interconvert stationary and flying qubits.

#7. The ability to faithfully transmit flying qubits between specified locations.
A qubit: The spin of an electron

\[ \begin{pmatrix} 0 \downarrow \mu \leftrightarrow \mu \downarrow \leftrightarrow \mu \downarrow \ \\
0 \leftrightarrow 0 \leftrightarrow 0 \ \\
\end{pmatrix} \]

\[ E = -\hat{\mu} \cdot \mathbf{B} = \pm \frac{1}{2} \hbar \omega_{01} + \Delta E \]

\[ 0\downarrow \leftrightarrow 0\downarrow \leftrightarrow 0\downarrow \]

Time-independent Schrödinger equation

\[ \hat{H} \psi - E \psi = \hat{H} \psi = E \psi \]

\[ \begin{align*}
H(0) &= -\frac{1}{2} \hbar \omega_{01}(0) \\
H(1) &= \frac{1}{2} \hbar \omega_{01}(1)
\end{align*} \]

Matrix representation of Hamiltonian and eigen functions

\[ \hat{H} = -\mu_b \mathbf{B} \begin{pmatrix} 1 & 0 \\
0 & -1
\end{pmatrix} \]

\[ 0\downarrow = \begin{pmatrix} 1 \\
0
\end{pmatrix} ; 1\downarrow = \begin{pmatrix} 0 \\
1
\end{pmatrix} \]

QM postulate: The quantum state of an isolated physical system is completely described by its state vector in a complex vector space with an inner product (a Hilbert space that is). The state vector is a unit vector in that space.