Nuclear Composition

- the forces binding protons and neutrons in the nucleus are much stronger (binding energy of MeV) than the forces binding electrons to the atom (binding energy of eV)

- the constituents of a nucleus are called **nucleons**

- a nucleus is characterized by its **atomic number** Z (the number of protons) and the **nucleon number** A (the total number of protons and neutrons in the nucleus)

- nuclei with the same Z but different A are called **isotopes**

- the naming convention for **nuclides** is $\quad {}^{A}_{Z}X$

- the **atomic mass** includes the mass of the electrons and is measured in atomic mass unit u that is

$$1\,u = \frac{1}{12}\, m({}^{12}_{6}C) = 1.66\ 10^{-27}\ kg$$

- energy equivalent

$$1u\, c^2 = 931.49\ MeV$$

Masses in Atomic Units

- proton	1.007 u	938.28 MeV
- neutron	1.008 u	939.57 MeV
- electron	0.00054 u	0.511 MeV

Size of Nucleus

- the size of nuclei can be determined from scattering experiments (Rutherford), the size resolution is best for particles with small deBroglie wavelength (highly energetic electrons) and/or small charge (neutrons)

- volume of nucleus

$$V = \frac{4\pi}{3} R^3 \propto A$$

- nuclear radius

$$R = R_0\, A^{1/3} \qquad \text{with } R_0 \approx 1.2\ fm$$

Nuclear Spin and Magnetic Moment

- similar to electrons, **protons and neutrons are fermions** with spin $s = 1/2$

$$S = \sqrt{s(s+1)}\,\hbar = \frac{\sqrt{3}}{2}\hbar \qquad s = \frac{1}{2}$$

$$S_z = m_s \hbar \qquad m_s = \pm\frac{1}{2}$$

- the magnetic moment of a nucleus is expressed in **nuclear magnetons**

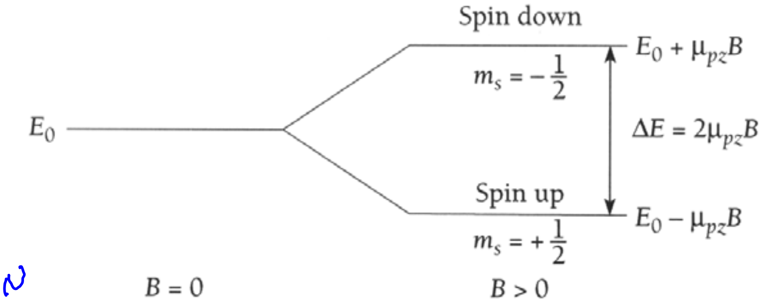
$$\mu_N = \frac{e\hbar}{2m_p} = 3.152\ 10^{-8}\ eV/T$$

$$\frac{\mu_B}{\mu_N} = \frac{m_p}{m_e} = 1836$$

- magnetic energy

$$E = -\vec{\mu}\cdot\vec{B} = -\mu_z B$$

$$\mu_{pz} = \pm 2.8\ \mu_N\ ;\ \mu_{Nz} = \mp 1.9\ \mu_N$$



Stability of Nuclei and Binding Energy

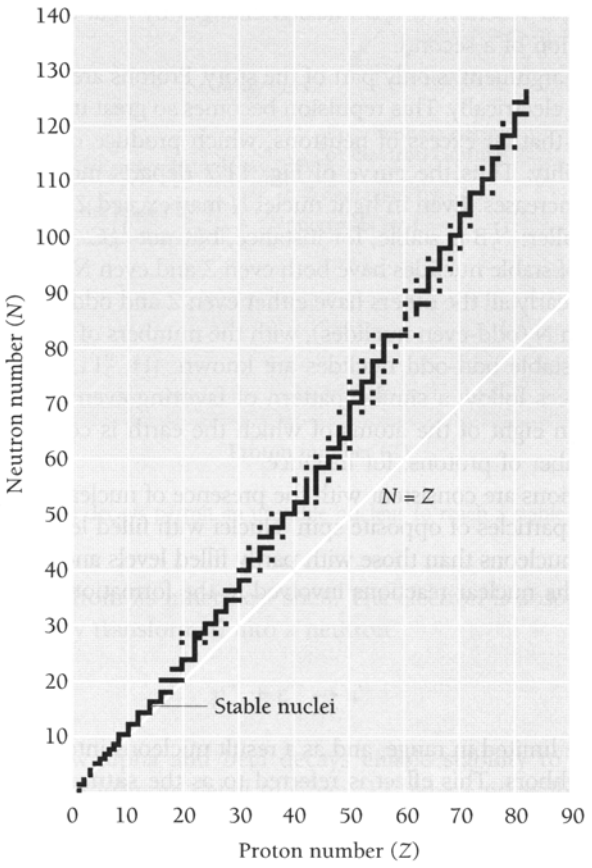
- stability diagram of nuclei

- no stable nuclei with $Z > 83$, $N > 126$, $A > 209$

- larger nuclei radioactively decay into smaller fragments

- for large Z there is an excess of neutrons contributing to nuclear binding counteracting the Coulomb repulsion of the protons

- the binding energy is given by the difference between the mass of the nucleus and the sum of the masses of the individual neutrons and protons making up the nucleus



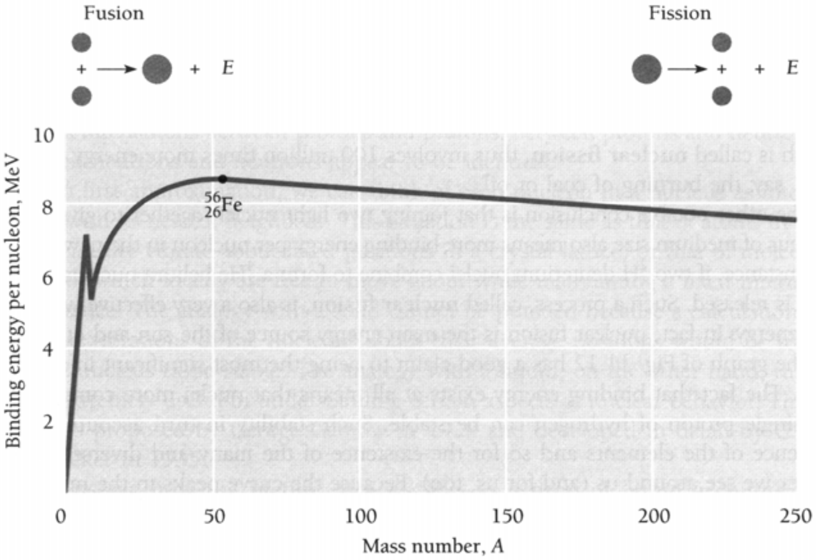
$$E_B = c^2\left(m_{^A_Z X} - (A-Z)m_n - Z m_p - Z m_e\right)$$

Binding Energy per Nucleon

- small nuclei can lower their energy by forming larger nuclei in a process called **fusion**

- large nuclei can lower their energy by breaking up into smaller nuclei in a process called **fission**

- the most stable nucleus is an isotope of iron (Fe)



- the binding energy of nucleons is extremely large compared to other atomic energy scales

Liquid Drop Model for Nuclear Binding Energy

- consider densely packed nuclei, then each nucleus has 12 nearest neighbors to each of which it is bound with an binding energy 1/2 u resulting in a **volume energy**

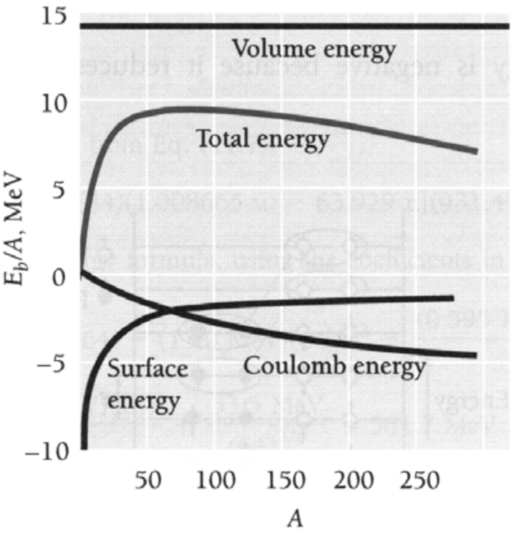
$$E_v = 6uA = a_1 A$$

- nucleons on the surface of the nucleus do not bind to neighbors resulting in a negative **surface energy**

$$E_s = -a_2 A^{2/3}$$

$$S = 4\pi R^2$$

$$R \propto A^{1/3}$$



- the **Coulomb energy** between all of the Z protons in the nucleus is

$$E_c = -\underbrace{\frac{Z(Z-1)}{2}}_{\#\text{ pairs of protons}} \frac{1}{4\pi\varepsilon_0}\frac{1}{r} = -a_3 \frac{Z(Z-1)}{A^{1/3}}$$

- sum of energies approximates the observed binding energy per nucleon

Radioactive Decay Processes

atomic nuclei can undergo a variety of radioactive decays

alpha decay $$ {}^{A}_{Z}X \longrightarrow {}^{A-4}_{Z-2}Y + {}^{4}_{2}He $$

beta decay $$ {}^{A}_{Z}X \longrightarrow {}^{A}_{Z+1}Y + e^{-} + \bar{\nu}_e $$

positron emission $$ {}^{A}_{Z}X \longrightarrow {}^{A}_{Z-1}Y + e^{+} + \nu_e $$

electron capture $$ {}^{A}_{Z}X + e^{-} \longrightarrow {}^{A}_{Z-1}Y + \bar{\nu}_e $$

gamma decay $$ {}^{A}_{Z}X^{*} \longrightarrow {}^{A}_{Z}X + \gamma $$

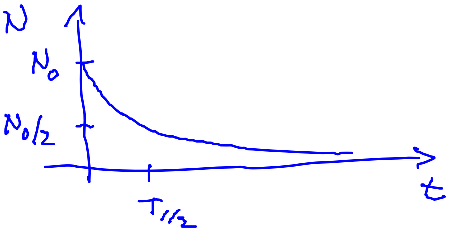
Radioactive Decay Law

the number of nuclei dN decaying per unit time dt a radioactive material containing N nuclei is proportional to a decay constant λ

$$ dN = -N\lambda\, dt \quad \Leftrightarrow \quad \int_{N_0}^{N} \frac{dN'}{N'} = \int_{0}^{t} -\lambda\, dt' $$

therefore the number of nuclei N remaining after a time t when the initial number of nuclei is N_0 is given by

$$ N = N_0\, e^{-\lambda t} $$



which is known as the **radioactive decay law**

the **half-life** ($T_{1/2}$) of a radioactive material is known as the time at which half of the material ($N = N_0/2$) has undergone nuclear decay

$$ \frac{N_0}{2} = N_0\, e^{-\lambda T_{1/2}} \quad \Rightarrow \quad T_{1/2} = \frac{\ln 2}{\lambda} $$

Activity Law

the activity R of a radioactive material is given by the number of decays occurring per unit time

$$R = \frac{dN}{dt} = -\lambda N_0 e^{-\lambda t} = R_0 e^{-\lambda t}$$

in contrast to the half-life $T_{1/2}$ the mean life-time $<T>$ of a nucleus s given by

$$\langle T \rangle = \frac{1}{\lambda}$$

therefore $$\langle T \rangle = \frac{T_{1/2}}{\ln 2} = 1.44\ T_{1/2}$$

The time dependence of activity of radioactive materials can be used for **dating** purposes in cases that one can assume to know the initial concentration of a radioactive material in the sample the age of which is to be determined. This works for organic materials by looking at the activity of ^{13}C that is naturally incorporated at a certain concentration into plants and bodies of living objects.

Alpha Decay

- the strong force mediates the binding of nucleons in the nucleus

- for nuclei with $A > 210$ the binding energy can hardly overcome the Coulomb repulsion between the protons, in this case the stability can be increased by decreasing the size of the nucleus

- this reduction in size is realized by an alpha decay

$$^{A}_{Z}X \longrightarrow {}^{A-4}_{Z-2}Y + {}^{4}_{2}He + KE$$

- the energy Q released in an alpha decay is

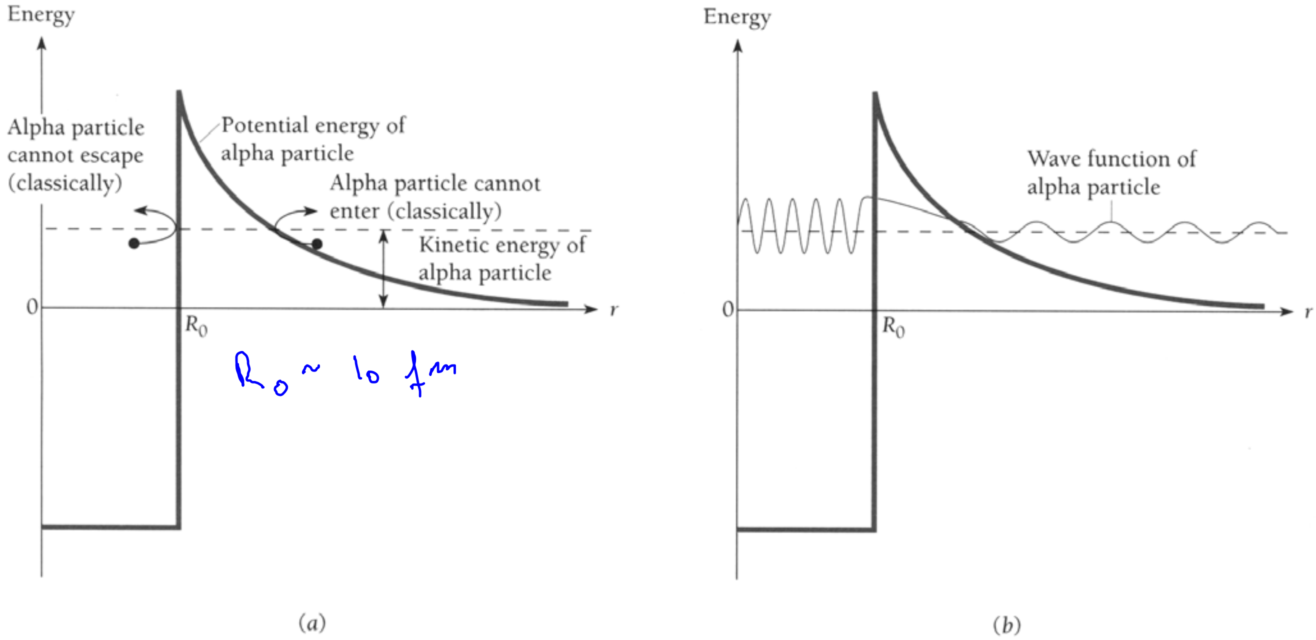
$$Q = (m_X - m_Y - m_{^4_2He})\, c^2$$

- the released energy is stored in the kinetic energy of the α particle and the final nucleus Y

- all nucleons with $A > 210$ decay in processes generating alpha particles

Tunneling of an α Particle out of the Nucleus

- the alpha particle has to overcome a barrier due to the Coulomb repulsions of about 25 MeV



(a) (b)

$R_0 \sim 10 \text{ fm}$

- the alpha particles tunnel through Coulomb barrier

Beta$^-$ Decay

- decay process

$$n \longrightarrow p^+ + e^- + \bar{\nu}_e$$

- in the beta decay an **electron antineutrino ν_e** is generated along with the electron, it has a mass smaller than a few eV/c^2 and interacts extremely weakly with anything

- without it energy, momentum and angular momentum conservation would be violated

- the half life of a free neutron is 616 s

Positron emission (Beta$^+$ decay)

- decay process $p \longrightarrow n + e^+ + \nu_e$

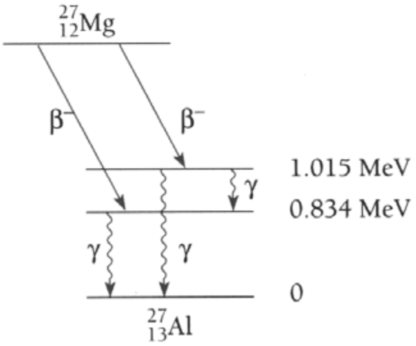
- positron emission can only occur in a nucleus, a free proton is stable and does not show positron emission

Electron Capture

- decay process: $p^+ + e^- \longrightarrow n + \nu_e$

- a K shell electron gets captured in the nucleus and transforms a proton to a neutron, this process is accompanied by the emission of an X-ray photon

Gamma Decay



- relaxation of an excited state nucleus (indicated by a *) to a lower energy nucleus by emission of a high energy photon

- photons with energies up to MeV are called **gamma rays**

- excited state nuclei have usually a short life time against gamma decay, typically much less than a few hours

---

Fission

- the process of a large nucleus decaying into two smaller nuclei while releasing energy is called fission

- a well known process is the neutron induced fission of ^{235}Ur

$$^{235}_{92}U + {}^{1}_{0}n \longrightarrow {}^{140}_{54}Xe + {}^{94}_{38}Sr + 2\,{}^{1}_{0}n + 200\ MeV$$

- most of the released energy is converted to kinetic energy of the fragments

- the additional neutrons released upon the fission process of one Ur nucleus can induce fission of other nuclei in a **chain reaction** process

- the controlled chain reaction can be used for generation of energy in **nuclear reactors**

- one gram of Ur generated 1 MW of power for a day, equivalently 2.6 tons of coal would be needed to generate that amount of energy conventionally

Fusion

- when lighter nuclei are combined into heavier ones energy can be released in a process called **fusion**

- the fusion of hydrogen to helium powers the sun

- the proton-proton cycle

$$p + p \xrightarrow{\beta^+} p + n + e^+$$

$$p + n + p \rightarrow {}^{3}_{2}He$$

$$2\,{}^{3}_{2}He \rightarrow {}^{4}_{2}He + 2p$$

$$+ 24.7\ MeV$$

- power of the sun is 4 10^{26} W, thus 10^{38} fusion processes per second take place in the sun

- there is enough hydrogen in the sun to keep the fusion process going for a few billion years

Interactions and Particles

there are four fundamental interactions governing the physical world around us

Interaction	particles affected	strength	exchange particle
strong	quarks	1	gluons/mesons
electromagnetic	charges	10^{-2}	photons
weak	quarks/leptons	10^{-5}	intermediate bosons
gravitational	all	10^{-39}	gravitons

Leptons

lepton		antiparticle	mass, MeV/c^2	lifetime, s	spin
electron	e^-	e^+	0.511	stable	1/2
e-neutrino	ν_e	$\bar{\nu}_e$	small	stable	1/2
muon	μ^-	μ^+	106	2.2 10^{-6}	1/2
m-neutrino	ν_μ	$\bar{\nu}_\mu$	small	stable	1/2
tau	τ^-	τ^+	1777	2.9 10^{-23}	1/2
t-neutrino	ν_τ	ν_τ	small	stable	1/2

Quarks

the constituents of hadrons

3 generations of quarks

quark/symbol		mass, GeV/c2	charge	strangeness
up	u	0.3	+2/3	0
down	d	0.3	- 1/3	0
strange	s	0.5	- 1/3	-1
charm	c	1.5	+2/3	0
top	t	174	+2/3	0
bottom	b	4.3	- 1/3	0

Protons, Neutrons and other Hadrons

composition of the proton:	quarks	$u u d$
	charge	$+2/3 + 2/3 - 1/3 = +1$
	spin	$+1/2 \ +1/2 \ -1/2 = +1/2$

composition of the neutron:	quarks	$d d u$
	charge	$-1/3 - 1/3 + 2/3 = 0$
	spin	$-1/2 - 1/2 + 1/2 = -1/2$

Hadrons composed of 3 quarks are called **baryons**

Hadrons composed of 2 quarks are called **mesons**

composition of the π^+-meson:	quarks	$u \bar{d}$
	charge	$+2/3 + 1/3 = +1$
	spin	$+1/2 - 1/2 = 0$

composition of the K^+-meson:	quarks	$u \bar{s}$
	charge	$+2/3 + 1/3 = +1$
	spin	$+1/2 - 1/2 = 0$