Quantum Error Correction with Trapped Ions

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Overview

1. Principles of Error Correction
   - Classical Error Correction
   - Basics of QEC

2. Experimental Realisation
   - QEC with Ions
   - 3 Qubit Repetition Code
   - High Fidelity Entanglement (Mølmer-Sørensen Gates)

3. The Lab Point of View
   - The Experiment
   - Gates and Friends (Implementation)
   - Errors

4. Results
   - Definition of the Process Fidelity
   - Experimental Results
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Classical Error Correction

Majority vote

Input bit: \[
\begin{bmatrix}
0 \\
0 \\
0
\end{bmatrix}
\]

bit flip error \[\rightarrow\]

\[
\begin{bmatrix}
0 \\
0 \\
1
\end{bmatrix}
\]

Understanding majority vote as parity check:

\[
\begin{bmatrix}
0 \\
0 \\
1
\end{bmatrix}
\]

parity check \[\rightarrow\]

\[
\begin{bmatrix}
0 \\
1
\end{bmatrix}
\]

\Rightarrow determines error

Parity-check matrix

For 3 bit majority vote: \( H = \begin{bmatrix}
1 & 1 & 0 \\
0 & 1 & 1
\end{bmatrix} \)

For any single bit flip error, \( H \) maps to the correct syndrome.

Knowing the syndrome immediately yields the erroneous bit.
Problems with QEC

- No cloning theorem!

- More than one type of error! E.g:
  - Bit flips
  - Phase flips
  - Superposition between different errors

- Also errors are continuous (unwanted rotations on Bloch sphere)
Solutions

- Information spreading: Embedding qubit state to protect in three qubit states

Closer look at Error types:

Bit flips:
- In computational basis: Described by the Pauli-X gate: \( X = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \)
- \( X (\alpha |0\rangle + \beta |1\rangle) = \alpha |1\rangle + \beta |0\rangle \)

Phase flips:
- In computational basis:
  - Pauli-Z gate \( Z = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} \)
  - \( \Rightarrow Z (\alpha |0\rangle + \beta |1\rangle) = \alpha |0\rangle - \beta |1\rangle \)
- Phase flips look like bit flips in X-basis: \( |\pm\rangle = \frac{1}{\sqrt{2}} (|0\rangle \pm |1\rangle) \)
Majority Vote in QEC

Initial state:
- $\alpha |000\rangle + \beta |111\rangle \xrightarrow{\text{single bit flip on 2nd qubit}} \alpha |010\rangle + \beta |101\rangle$

Measuring the syndrome:
- Operators $Z_1 \otimes Z_2 \otimes I_3$ and $I_1 \otimes Z_2 \otimes Z_3$ perform the parity check

Note: Any single bit flip $X_{1,2,3}$ rotates the initial state to an eigenstate with eigenvalue $\pm 1$ of both these operators.
Experimental Realisation

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QEC with Ions

Features when using multiple ions as qubits:

- Internal (atomic) as well as external (motional) states.
- Ions provide a well isolated and stable quantum system.
- Easy preparation of states by optical pumping.
- Simple to trap (e.g. Paul traps).
- Multiple ions can be simultaneously entangled (Mølmer-Sørensen gates) with high fidelity ($\sim 99\%$).
The Experiment in Theory

Key aspects:

- Measurement free - 3 cycle repetition code
- 3 $^{40}$Ca$^+$ ions as qubits
- One protected qubit state $|\Psi\rangle$, two ancillas.
- Initial states:
  $|\Psi\rangle = \alpha |+\rangle + \beta |-\rangle$
  Ancillas: $|1\rangle (4S_{1/2})$
- Ancilla reset by optical pumping.

The code:
Single Cycle

Encoding:
- Change $|\Psi\rangle$ to computational basis.
- Entangle $|\Psi\rangle$ with ancillas.
- Change to $X$ basis.
- "Simplify" by adding an operation $D$ which commutes with any phase error.

Decoding:
- Change back to computational basis.
- Perform measurement free correction and de-entanglement.
- Change basis of protected qubit back to $X$ basis.
- Applying U improves reset process.
Realizing the Code

- For better fidelity, change encoding gates to single Mølmer-Sørensen gate operation.
- Mølmer-Sørensen: Entangling multiple ion qubits simultaneously, making use of vibrational states
- The simultaneous entanglement can be described as a two qubit process for each qubit pair.
Mølmer-Sørensen Gate Transitions

MS 2-ion transitions:

\[ \Delta \sim c \]
\[ \omega_b \]
\[ \omega_r \]
\[ 2(\delta + \nu) \]

Red sidebands:

Blue sidebands:
Mølmer-Sørensen Gate Transitions

MS 2-ion transitions:

Red sidebands:

Blue sidebands:
The Lab Point of View

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Experimental Setup

- Macroscopic linear Paul trap. → Size $\sim$ mm.
- String of three $^{40}\text{Ca}^+$ ions. → Distance $\sim \mu$m.
- Collective operations: → Wide beam.
- Single-ion operations: → Tightly focused beam.
Experimental Cycle

- Cool the ion string to motional ground state.
- Apply operation cycles:
  - Initialize qubits.
  - Encode.
  - Apply error.
  - Decode and correct.
- Measure the population of the qubit states.
  → Up to 1000 repetitions.
Energy Level Structure

Internal degrees of freedom:
- $4^2S_{1/2}(m_J = -\frac{1}{2}) \equiv |g\rangle \equiv |1\rangle$
- $3^2D_{5/2}(m_J = -\frac{1}{2}) \equiv |e\rangle \equiv |0\rangle$.
- $|1\rangle \leftrightarrow |0\rangle$: unitary gates.
- $|1\rangle \leftrightarrow 4^2P_{1/2}(m_J = -\frac{1}{2})$: measurements.

With motional degrees of freedom:
- Entanglement between ions.
- Red sideband transitions: cooling to motional ground.
Reset the Qubits

Optical pumping technique:
- Stimulate $|0\rangle \rightarrow |S'\rangle \equiv 4^2S_{1/2}(m_J = +\frac{1}{2})$
- Pump $|S'\rangle \rightarrow 4^2P_{1/2}(m_J = -\frac{1}{2})$
- Spontaneous decay $4^2P_{1/2}(m_J = -\frac{1}{2}) \rightarrow |1\rangle$
- Re-pump losses $3^2D_{3/2} \rightarrow 4^2P_{1/2}$

Caused errors:
- Heating rate: 0.015 phonons per reset.
- MS gates are insensitive to motion in first order.
  $\Rightarrow$ QEC unaffected.
Phase Flip Errors

Single-qubit phase flip $|\pm\rangle \rightarrow |\mp\rangle$: 
- E.g. $|+++angle \rightarrow |++-\rangle$.
- Correctable by means of three-qubit repetition code.
- Implemented by single-qubit MS gate $Z_i(\pi)$. 
De-phasing:

- Phase-flip probability $p$.
- Total dephasing for $p = 0.5$.
- Irreversible process.
- Reduce off-diagonal elements of the density matrix by a factor $1 - 2p$.

→ Destroys coherence.
Phase Noise Errors

Correlated noise:
- Conditional phase-flip probabilities.
  → Sources: noise in magnetic, trap field, or laser frequency.
  → Implementation: waiting time between encoding and decoding.
→ Most important in the experiment: correlated noise.

Uncorrelated noise:
- Independent phase-flip probabilities.
  → Implementation: short measurement pulses on single qubits (weak projection) with probability $p$. 
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Quantum process tomography of the process $\mathcal{E} \rightarrow$ Process matrix $\chi$, defined as:

$$\mathcal{E}(\rho) = \sum_{m,n=0}^{3} \chi_{mn} \sigma_m \rho \sigma_n^\dagger$$

The perfect correction process has (final state unchanged):

$$\chi_{\text{perfect}} = \chi_{\text{id}} \equiv \chi(\mathcal{E}(\rho) = \rho)$$

⇒ Process fidelity:

$$F_{\text{proc}} = \langle \chi, \chi_{\text{id}} \rangle_{H.S.} = \text{Tr}(\chi \cdot \chi_{\text{id}})$$

Optimized process fidelity $F_{\text{opt}}$: without constant operational errors (reversible by means of unitary operations), just with decoherence (irreversible).
Phase Flip Correction
### Phase Flip Correction

<table>
<thead>
<tr>
<th>Number of QEC cycles</th>
<th>No error $F^0_{proc}$</th>
<th>Opt. no error $F^0_{opt}$</th>
<th>Single-qubit errors $F_{proc}$</th>
<th>Opt. single-qubit errors $F_{opt}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>97(2)</td>
<td>97(2)</td>
<td>–</td>
<td>–</td>
</tr>
<tr>
<td>1</td>
<td>87.5(2)</td>
<td>90.1(2)</td>
<td>89.1(2)</td>
<td>90.1(2)</td>
</tr>
<tr>
<td>2</td>
<td>77.7(4)</td>
<td>79.8(4)</td>
<td>76.3(2)</td>
<td>80.1(2)</td>
</tr>
<tr>
<td>3</td>
<td>68.3(5)</td>
<td>72.9(5)</td>
<td>68.3(3)</td>
<td>70.2(3)</td>
</tr>
</tbody>
</table>

Within the statistical uncertainty:

$$F_{proc} = F_{opt},$$

⇒ single-qubit errors are corrected perfectly within the statistical uncertainty.
Phase Noise Correction

- Total dephasing kills fidelity.
- Many-qubit errors are not correctable.
  ⇒ Correlated noise harder to correct.
- Decoherence-free subspaces (DFS) can protect against correlated noise, but do not exist for uncorrelated one.

(green: correlated, red: uncorrelated)
Conclusions

- Repeatable error correction is achievable.
- Short and efficient QEC cycles.
- For uncorrelated errors the correction improve the fidelity.
- In conjunction with DFS, this may lead toward quantum error correction.
- Just one error type.
- There is still space for improvement...