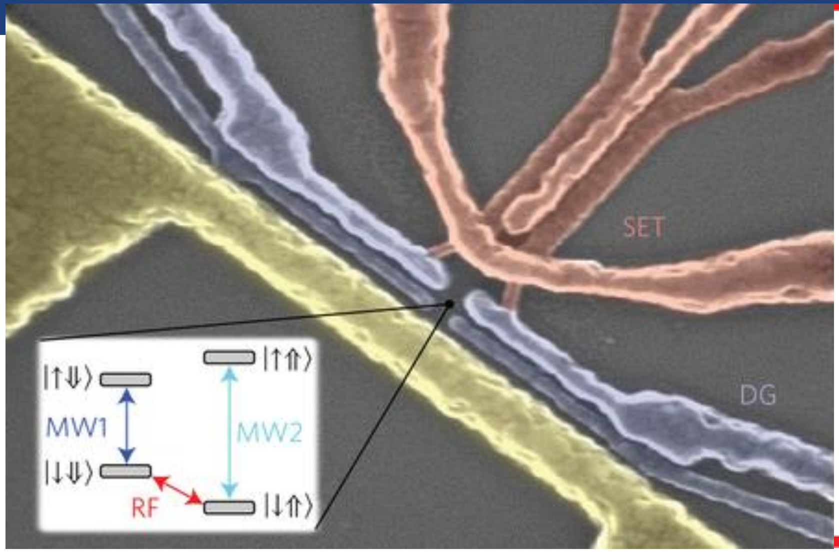


nature nanotechnology

Bell’s inequality violation with spins in silicon

Dehollain et al., Nature Nanotechnology (2015)

Some history: from EPR to CHSH, and beyond

Einstein, Podolsky, Rosen (1935):
Quantum theory not complete,
Local realism

↓

Bohm (1951,57):
"Hidden variable" theory,
Gedankenexperiment

Bell (1964):
Inequality which bounds correlations in local realistic theories

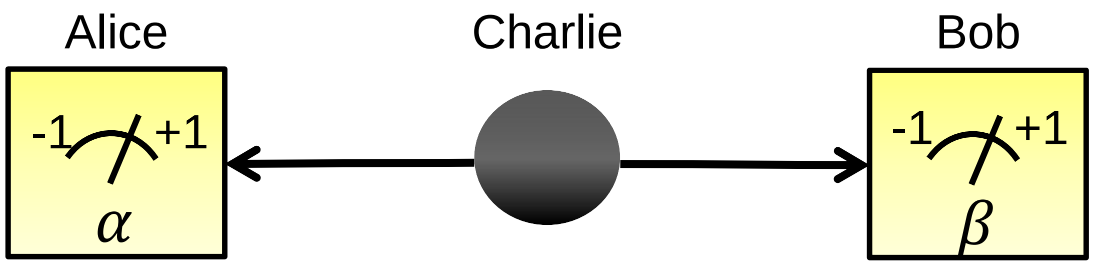
↓

Clauser, Horne, Shimony, Holt (1970):
Reformulation of Bell’s theorem,
tailored to specific experiment

From the 1970’s up to now:
- Increasing evidence that QM is correct
- Local realism still not completely ruled out

Bell’s Theorem and the CHSH inequality

- Bell: Correlations in local realistic theories are bound by certain values
- CHSH inequality: valid for a specific setup with two "bits"



P_{ij} : Probability of outcome ij
α, β : Settings used for measurement
S : “Bell signal”

- Define correlations $E(\alpha,\beta) = P_{++} + P_{--} - P_{+-} - P_{+-}$

CHSH: $\boldsymbol{S = E(\alpha,\beta) + E(\alpha',\beta) + E(\alpha,\beta') - E(\alpha',\beta') \leq 2}$

With proper choice of $\alpha, \alpha', \beta, \beta'$, QM allows up to $\boldsymbol{S_{\max} = 2\sqrt{2}}$!

- Fundamental vs. practical aspect of violating the inequality

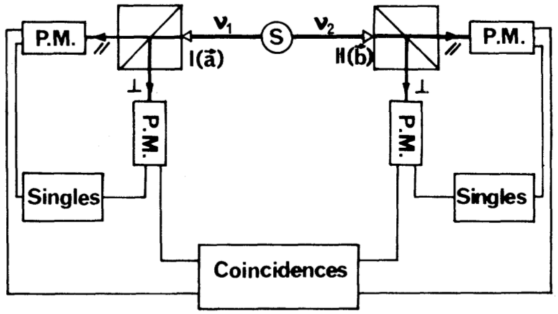
On loopholes – and how to close them

Experiments: need additional assumptions about your system in order to show violation of Bell inequality

- Locality loophole and communication

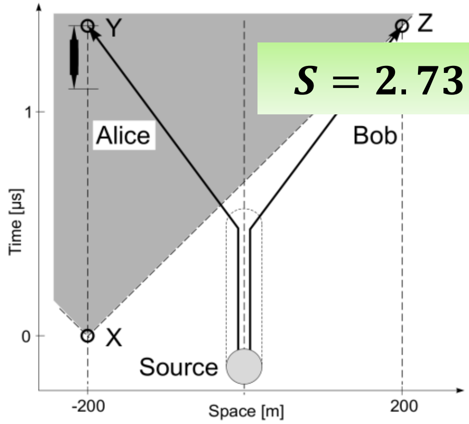
Photons

$S = 2.697 \pm 0.015$



Aspect et al. (1982)

$S = 2.73 \pm 0.02$

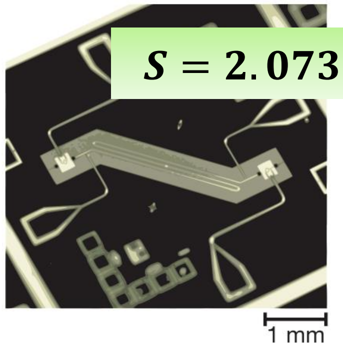


Weihs et al. (1998)

- Detection loophole and fair sampling

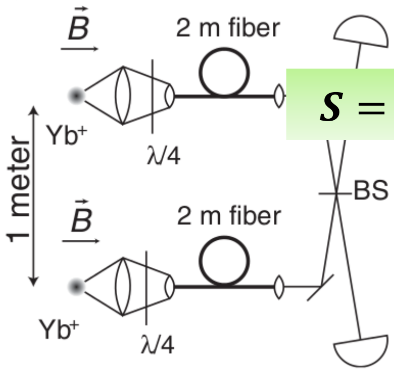
Other systems

$S = 2.0732 \pm 0.0003$



Ansmann et al. (2009)

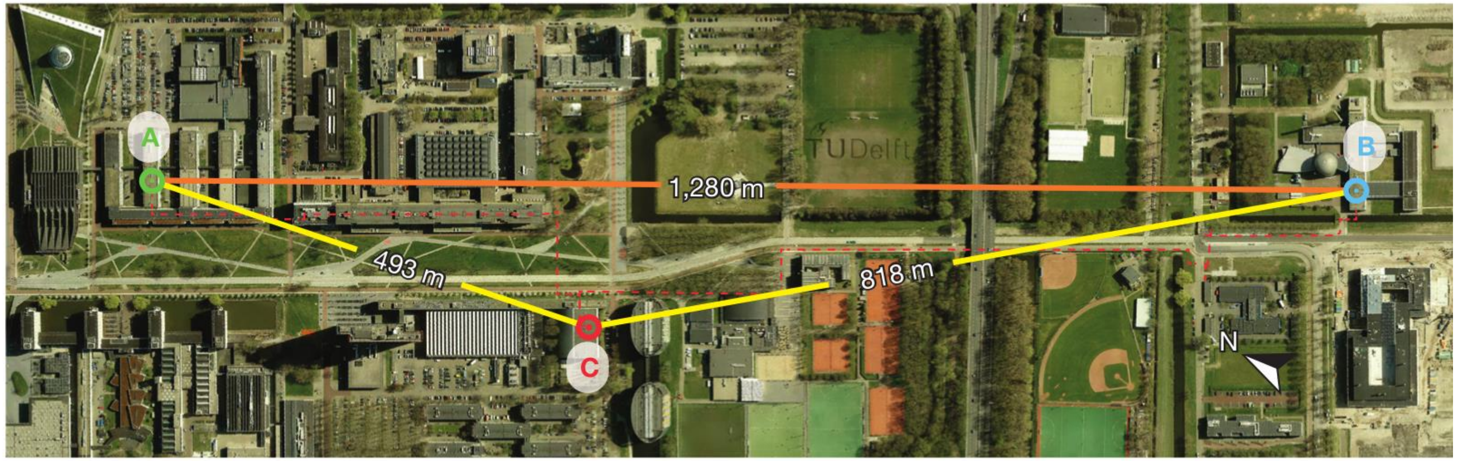
$S = 2.22 \pm 0.07$

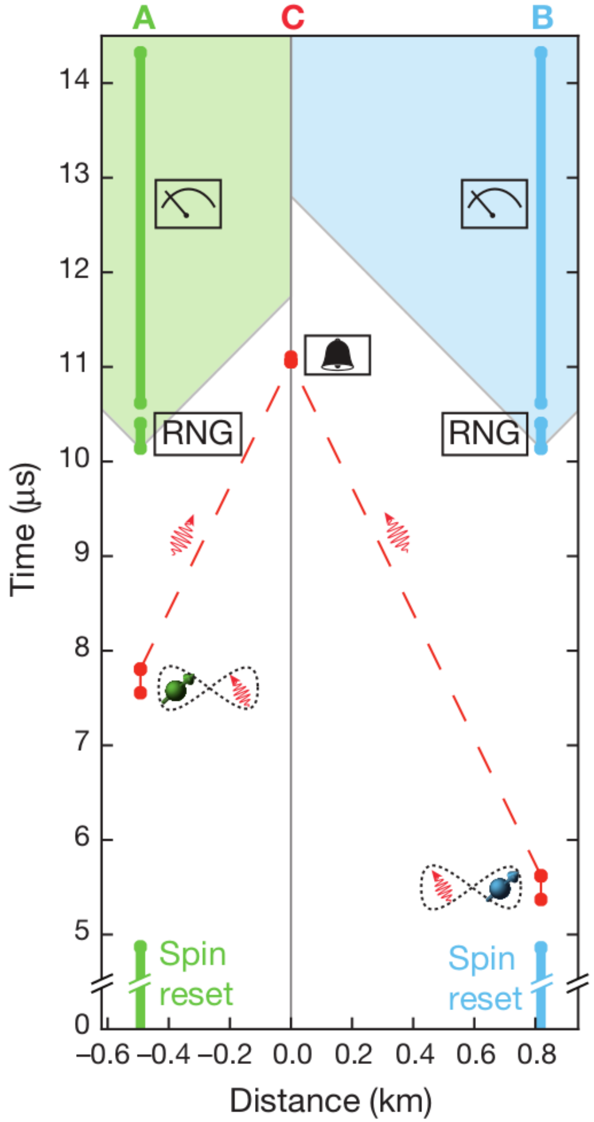


Matsukevich et al. (2008)

One step further: NV centers @ Delft

Using NV centers, both detection and locality loopholes were closed in this experiment

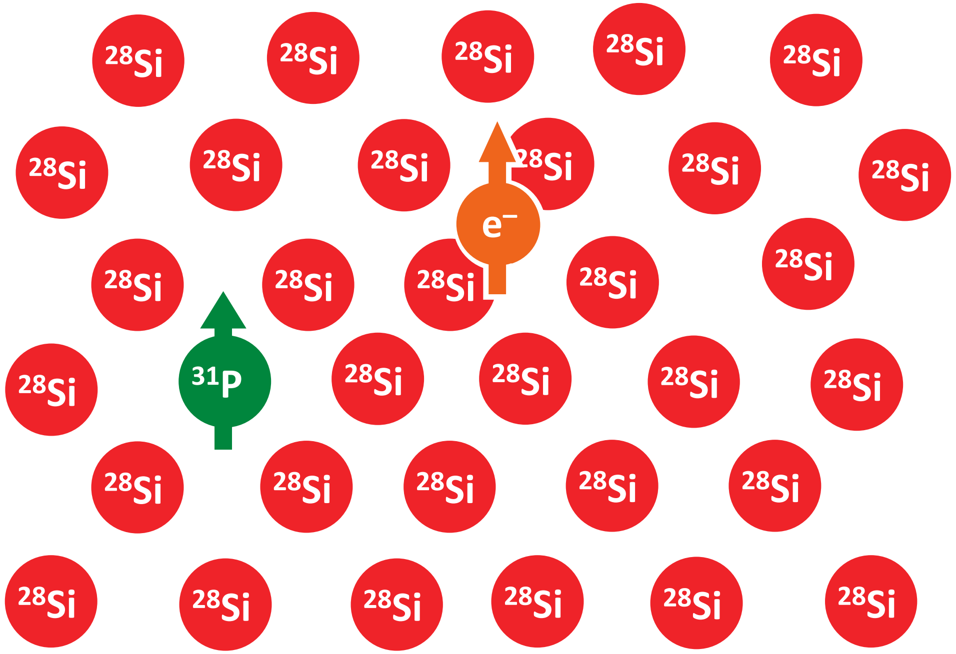




Hensen et al. (2015)

$$S = 2.42 \pm 0.20$$

^{31}P Nuclear Spin – Electron Spin System



^{31}P: Spin = ½

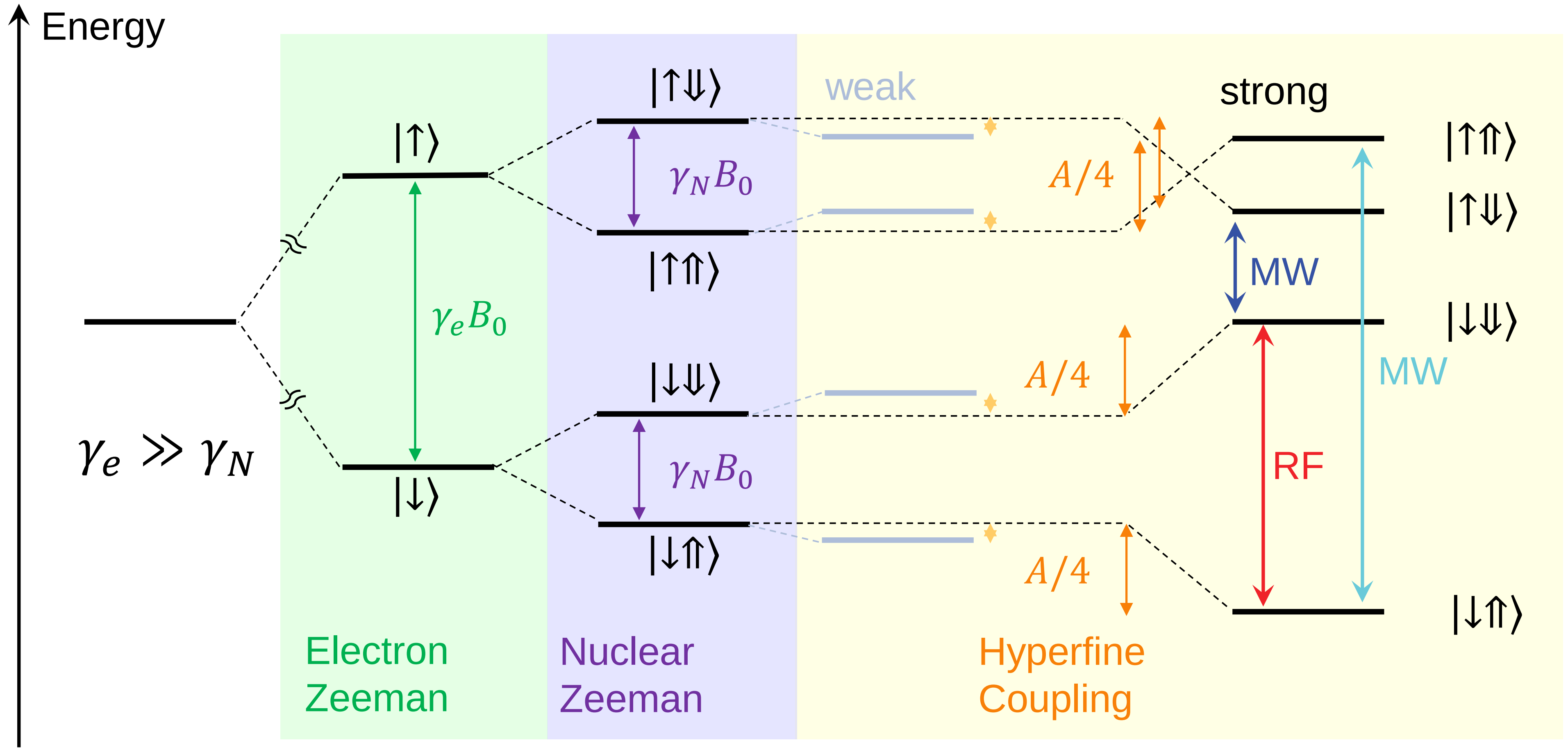
e^- : Spin = ½

^{28}Si: Spin = 0

→ No spin-spin interaction with Si

→ Noise reduction

^{31}P Nuclear Spin – Electron Spin System

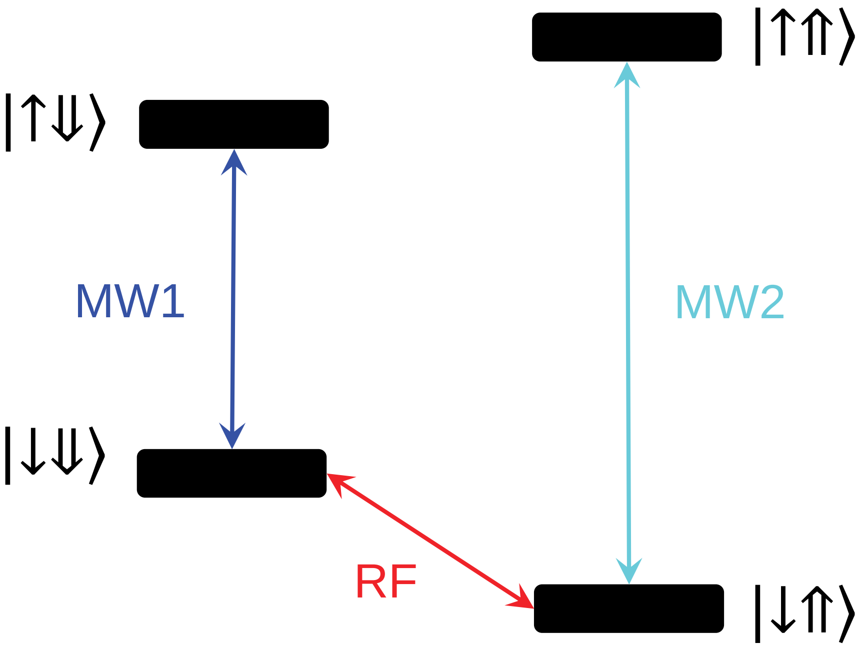


Relevant Energy-Level Scheme

$$\nu_{MW1} = \gamma_e B_0 - \frac{A}{2}$$

$$\nu_{MW2} = \gamma_e B_0 + \frac{A}{2}$$

$$\nu_{RF} = \gamma_N B_0 + \frac{A}{2}$$



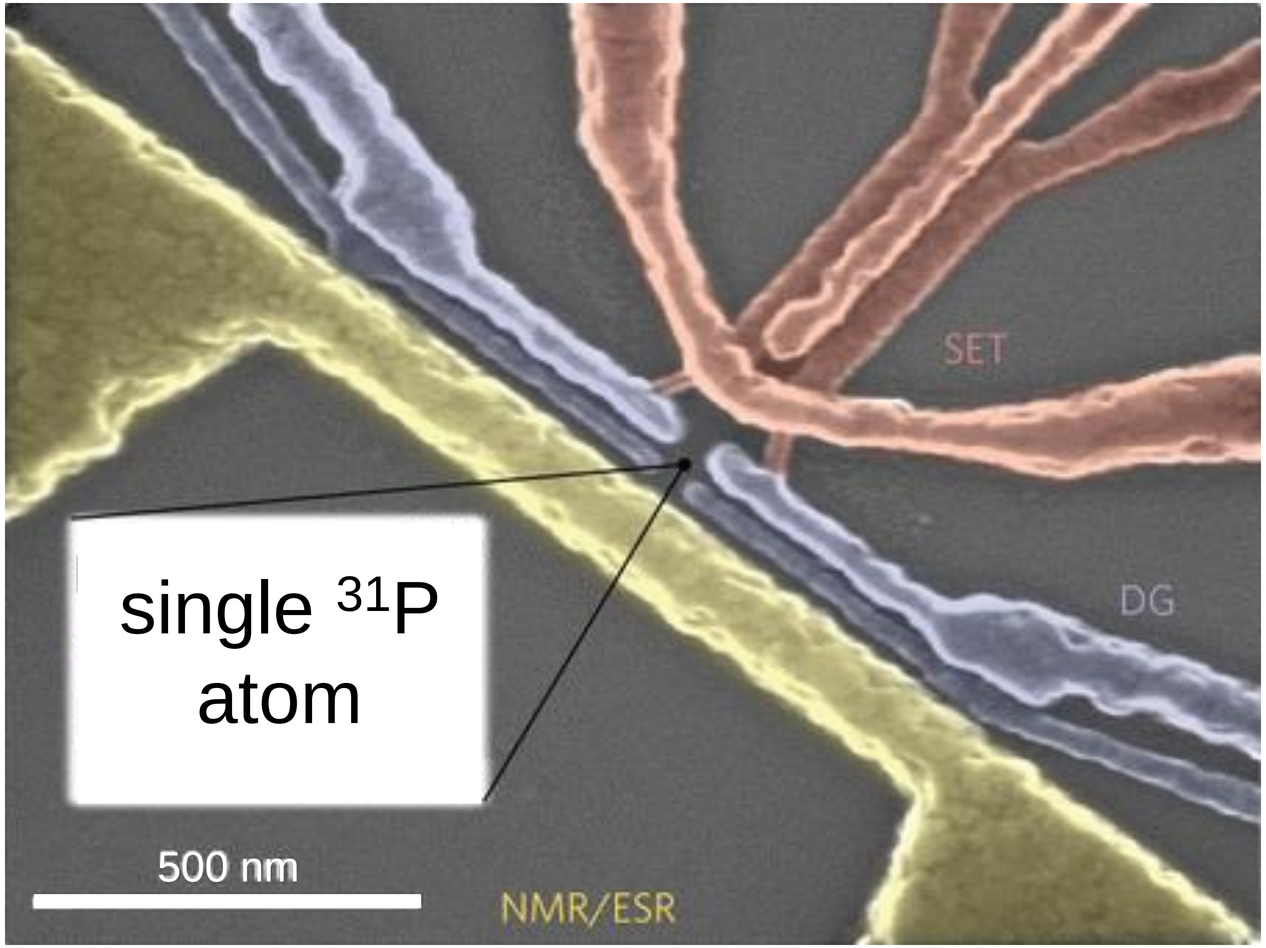
Qubit Operations:

- π pulse: population inversion (CNOT)
- $\frac{\pi}{2}$ pulse: create superposition states (Hadamard)

Experimental Setup

NMR/ESR antenna
→ induces qubit operations in system

Experimental Parameters	
Temperature	≈ 100 mK
B_0-Field	≈ 1.55 T
Distance ^{31}P – SET	~ 25 nm

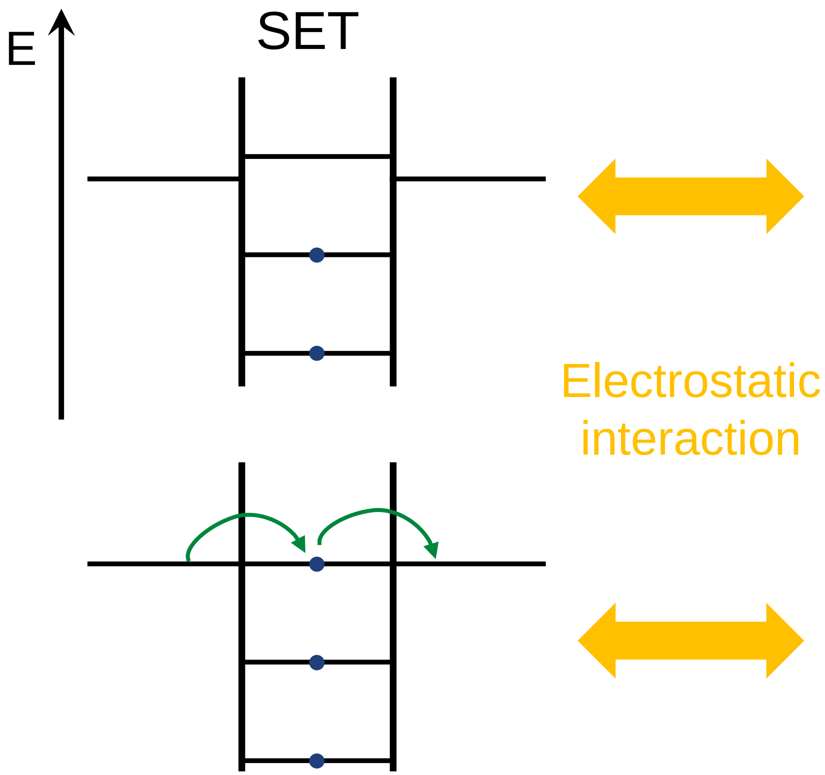


Single electron transistor (SET)
→ spin-selective readout

Donor gate (DG)
→ manipulation of electrochemical potential μ of donor

Electron Spin Readout via SET

Electrostatic Coupling



^{31}P

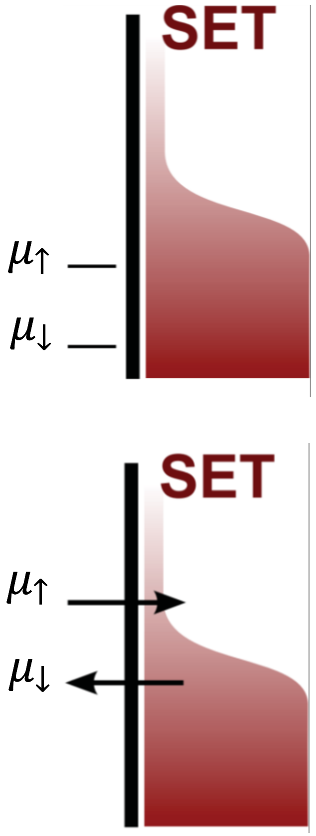
e^- on donor

no e^- on donor

→ positively charged

→ lowered energy levels in SET

Tunnel Coupling



High voltage V_{nt} on donor gate:

$\mu_\uparrow, \mu_\downarrow \ll \mu_{SET}$ → no tunneling

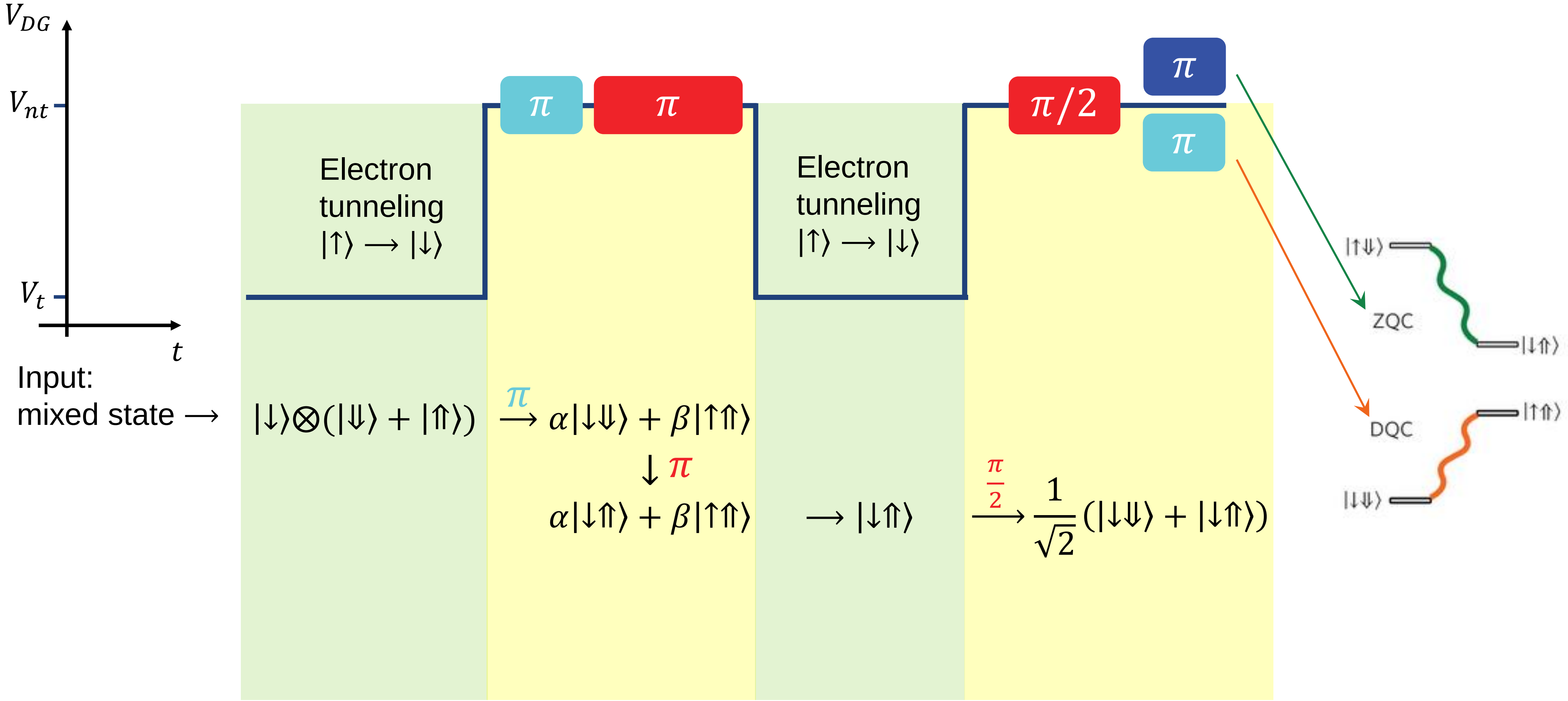
Low voltage V_t on donor gate:

$\mu_\downarrow < \mu_{SET} < \mu_\uparrow$ → tunneling:

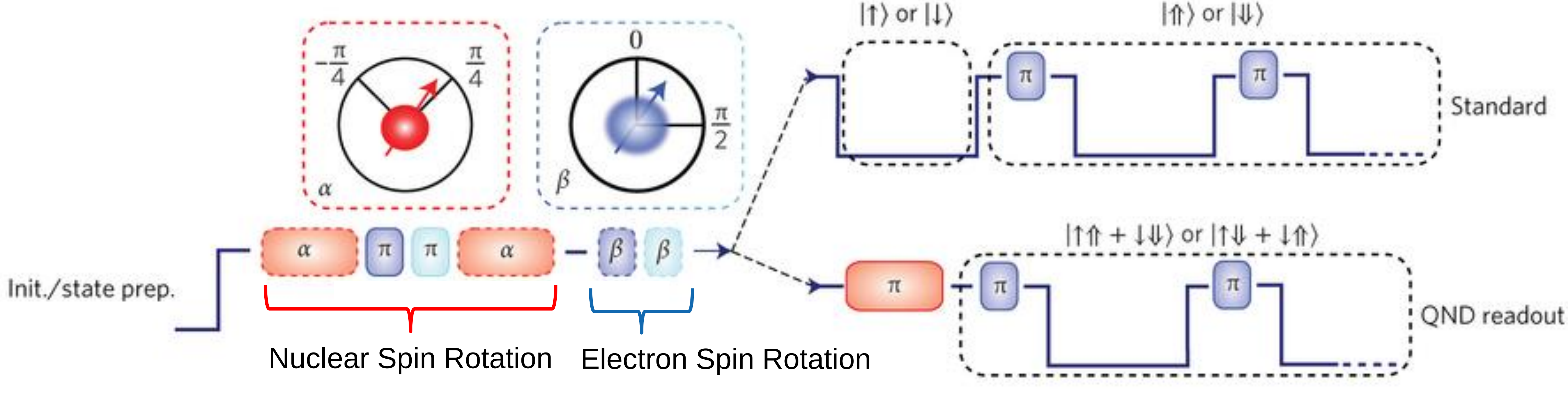
$|\uparrow\rangle$ out of and $|\downarrow\rangle$ into donor

$(\gamma_e B_0 \gg k_B T)$

Bell-State Initialization

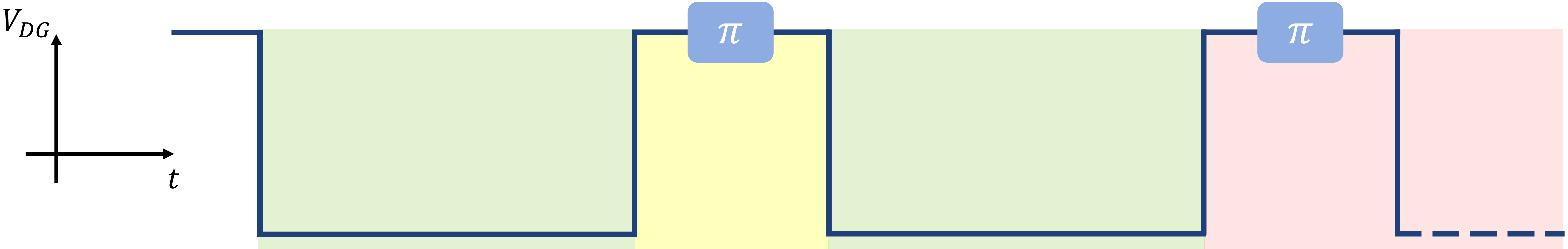


Bell State Measurement



Initialisation	Axis Projections	Bell State Measurement

Standard Readout - destructive



Projection Measurement on $|\uparrow\rangle$ Tunneling of a $|\downarrow\rangle$ electron

Induce MW1 transition

Another Projection Measurement on $|\uparrow\rangle$ followed by electron tunneling

Repeat

e.g. Input State

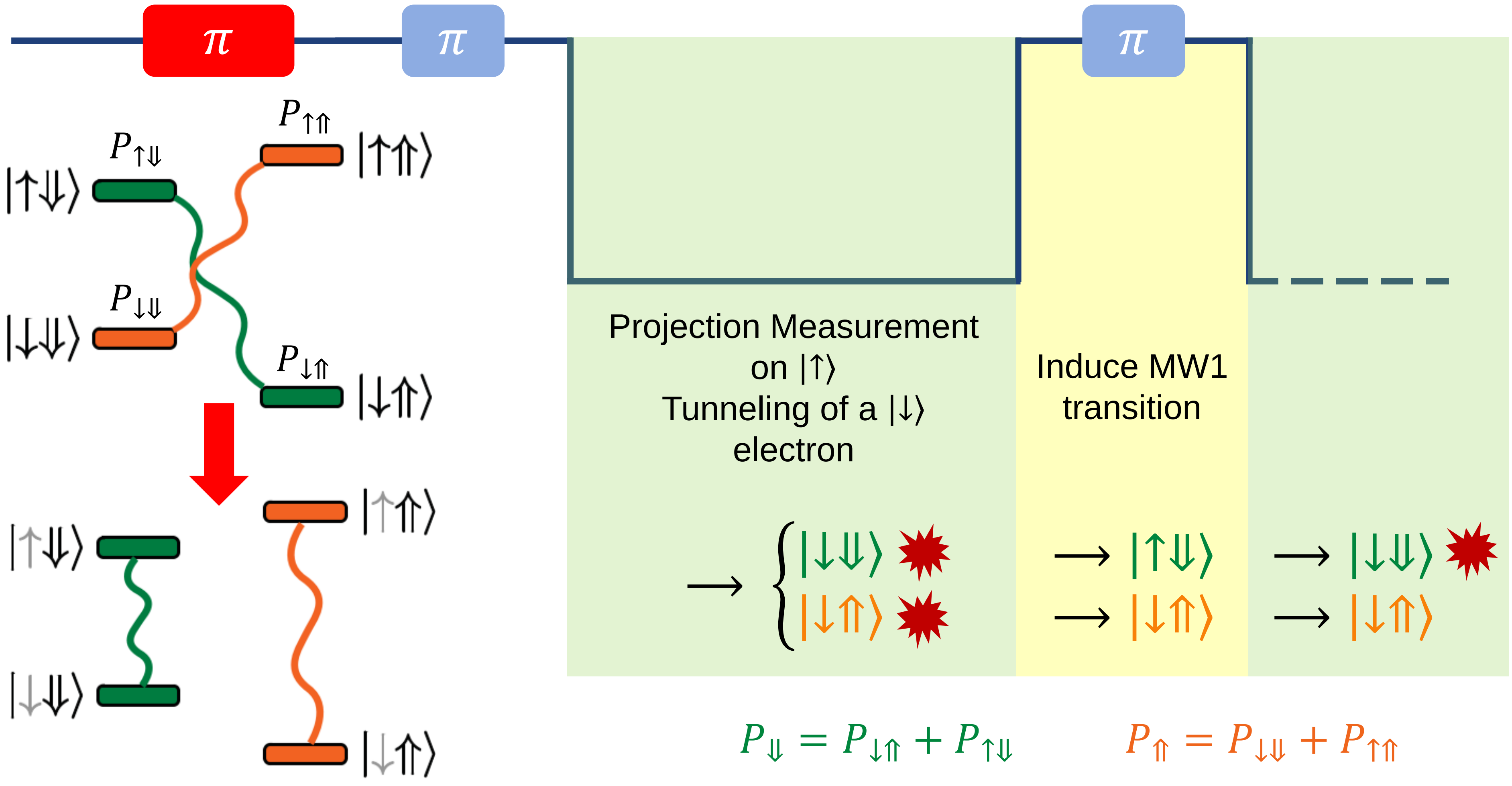
$$\frac{1}{\sqrt{2}}(|\uparrow\Downarrow\rangle + |\downarrow\Uparrow\rangle) \longrightarrow \begin{cases} |\downarrow\rangle|\Downarrow\rangle \; ✸ & \longrightarrow |\uparrow\Downarrow\rangle \longrightarrow |\downarrow\Downarrow\rangle \; ✸ \\ |\downarrow\rangle|\Uparrow\rangle & \longrightarrow |\downarrow\Uparrow\rangle \longrightarrow |\downarrow\Uparrow\rangle \end{cases}$$

$$\frac{1}{\sqrt{2}}(|\uparrow\Uparrow\rangle + |\downarrow\Downarrow\rangle) \longrightarrow \begin{cases} |\downarrow\rangle|\Uparrow\rangle \; ✸ & \longrightarrow |\downarrow\Uparrow\rangle \longrightarrow |\downarrow\Uparrow\rangle \\ |\downarrow\rangle|\Downarrow\rangle & \longrightarrow |\uparrow\Downarrow\rangle \longrightarrow |\downarrow\Downarrow\rangle \; ✸ \end{cases}$$

Process measures $P_{\uparrow\Uparrow}, P_{\uparrow\Downarrow}, P_{\downarrow\Uparrow}, P_{\downarrow\Downarrow}$

Fidelity limited by thermal broadening of the reservoir

QND Readout



Process measures
$E = 1 - 2P_{\Downarrow}$

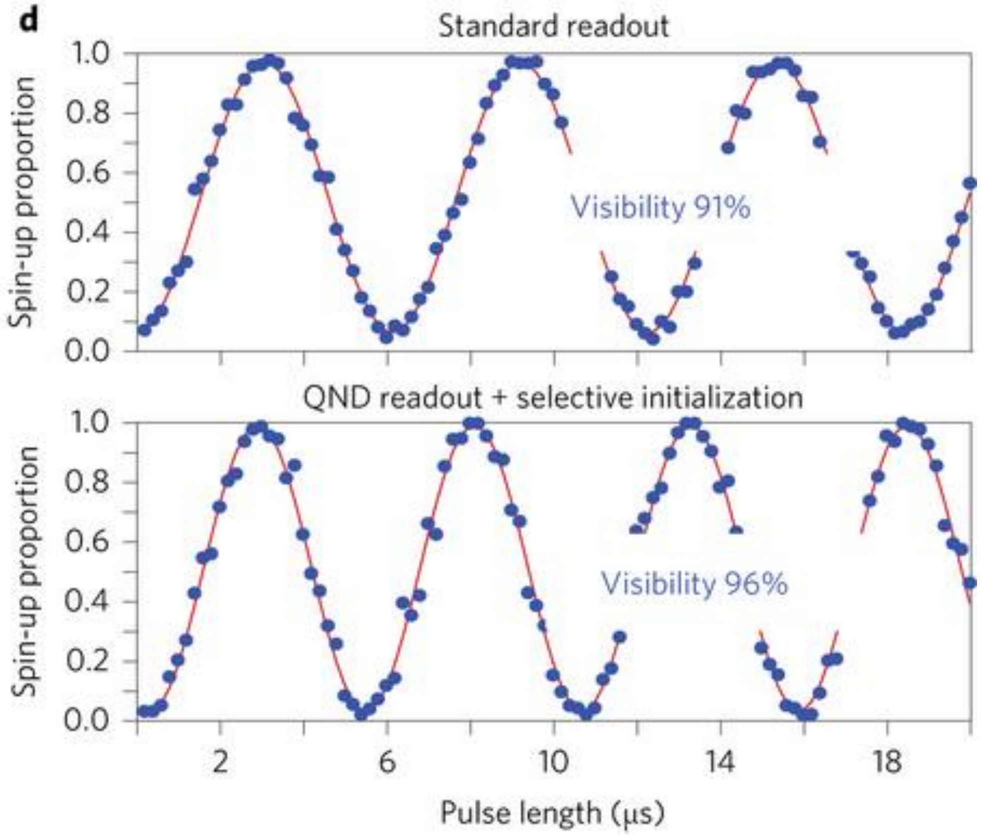
Single Shot Readout, Fidelity not affected by thermal broadening.

$P_{\Downarrow} = P_{\downarrow\Uparrow} + P_{\uparrow\Downarrow}$ $\quad$ $P_{\Uparrow} = P_{\downarrow\Downarrow} + P_{\uparrow\Uparrow}$

$$E = P_{\downarrow\Downarrow} + P_{\uparrow\Uparrow} - P_{\downarrow\Uparrow} - P_{\uparrow\Downarrow} = 1 - 2P_{\Downarrow}$$

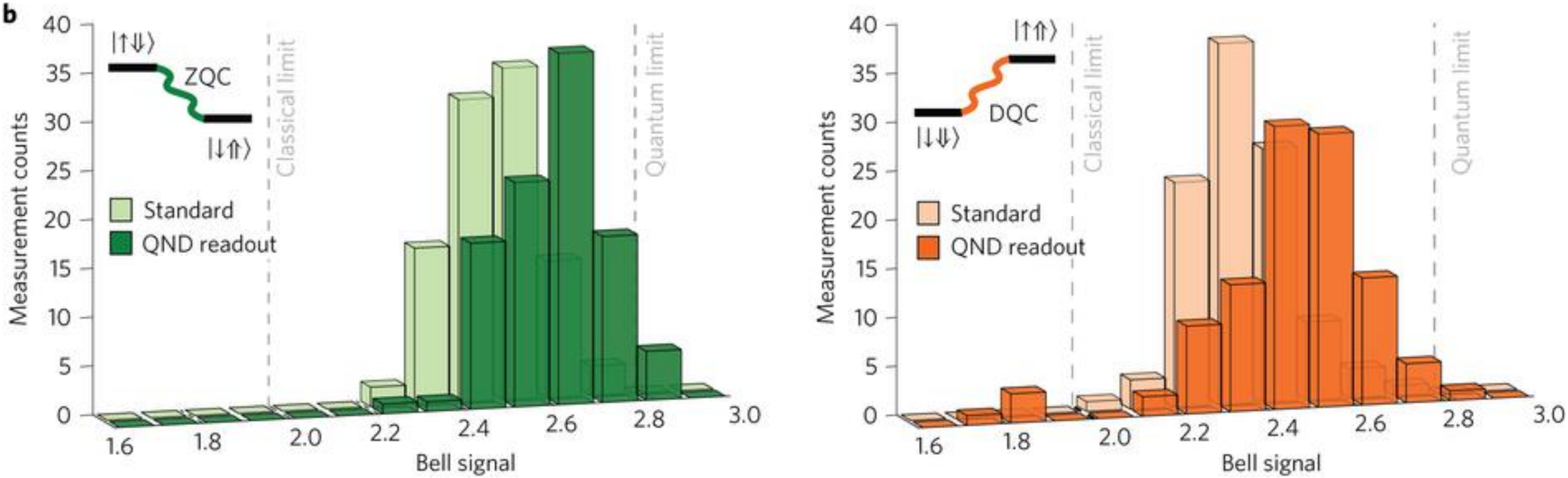
Comparison of Readout Techniques

	Standard	QND
Measurement Variable	$\{P_{\uparrow\Uparrow}, P_{\uparrow\Downarrow}, P_{\downarrow\Uparrow}, P_{\downarrow\Downarrow}\}$	$E = 1 - 2P_{\Downarrow}$
Number of Measurements	300	Single-Shot
Limitations	Thermal Broadening	
Fidelity	Up to 97%	> 99.9%



ETH zürich

Bell State Measurement Results



CHSH: $S = E(\alpha,\beta) + E(\alpha',\beta) + E(\alpha,\beta') - E(\alpha',\beta') \leq 2$

State Tomography

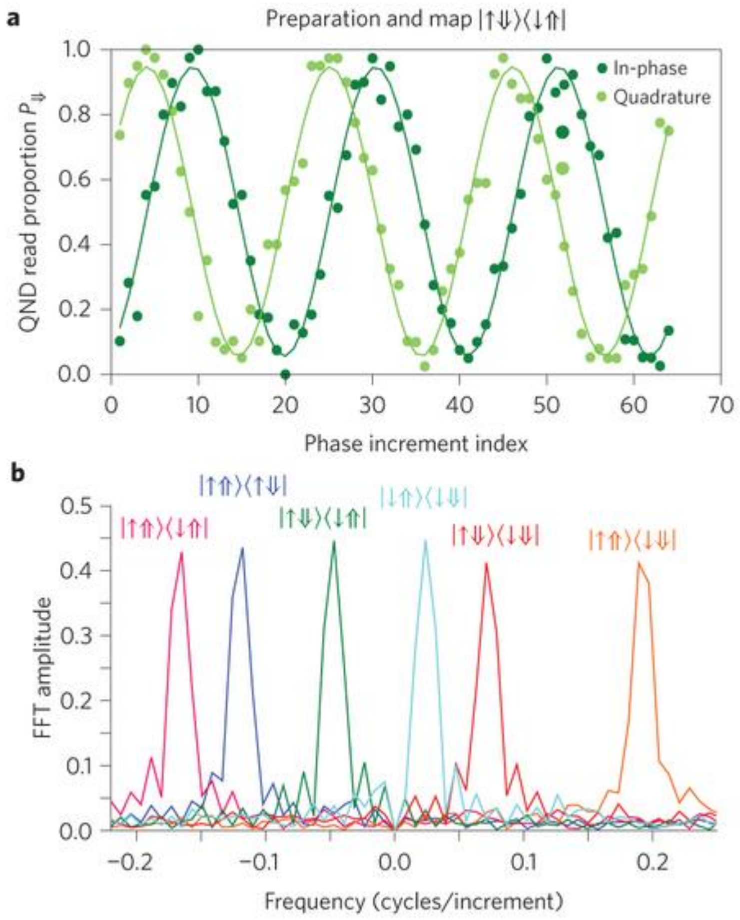
- Principle: nuclear and electron spins pick up different phases (ϕ_S, ϕ_I) under rotation about quantisation axis

$$R_{S_z} R_{I_z} |m_S m_I\rangle = e^{-i\phi_S m_s} e^{-i\phi_I m_I} |m_S m_I\rangle$$

State	$\|\phi^{\pm}\rangle$	$\|\Psi^{\pm}\rangle$
Phase	$e^{-i(\phi_S+\phi_I)} \|\uparrow\Uparrow\rangle \pm e^{i(\phi_S+\phi_I)} \|\downarrow\Downarrow\rangle$	$e^{-i(\phi_S-\phi_I)} \|\uparrow\Downarrow\rangle \pm e^{i(\phi_S-\phi_I)} \|\downarrow\Uparrow\rangle$

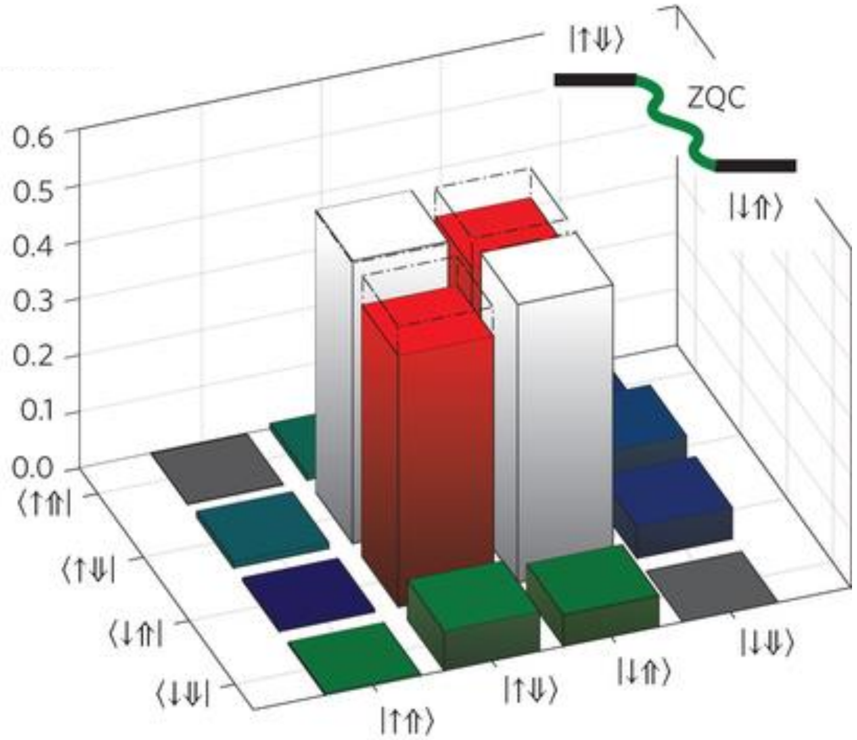
- Phase-shifted pulses ($\Delta\phi_{S,I}$ → Rotations) are applied
  - encode the coherence as phase shifts
  - transfer undetectable coherences into the detectable $|\downarrow\Downarrow\rangle$ state
- We therefore expect oscillations in $P_{\Downarrow}$ as a function of phase increments $\Delta\phi_{S,I}$

State Tomography



- Phase increments $\Delta\phi_{S,I}$ chosen such that oscillation freq of matrix elements are separable
- Oscillation amplitude proportional to off-diagonal density matrix element
- Offsets of tomography signals yield populations

State Tomography



State Fidelity
$> 96\%$

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