Two- Qubit State Tomography Using a Joint Dispersive Readout

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What is State Tomography?

- Tomography originates from the Greek word tomos which means part and graphein which can be translated as to write.
- From a set of images that contain reduced information about the object, the complete object is reconstructed.
- For example the 3D content of a picture can be extracted from several 2D pictures taken from different directions.
- This is also the basic idea for quantum state tomography where the state is projected on a set of different bases, called the tomographic set.
- State tomography aims to extract all possible information about the state that are contained in the density operator.

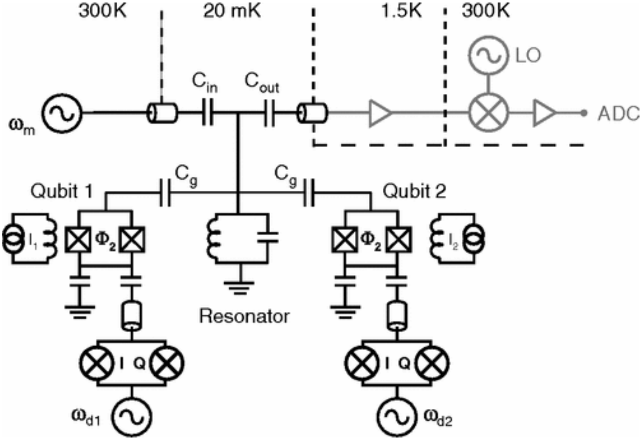
What is a Density Matrix/Density Operator?

- The density operator is often written as $\hat{\rho}$.
- It can be written in terms of a given set of basis elements $|i\rangle$ as: $\sum_{i,j} \rho_{i,j} |i\rangle \langle j|$, where the $\rho_{i,j}$ form a Hermitian matrix.
- The elements on the diagonal $\rho_{i,i}$ give the probabilities of finding the system in each basis element if such a measurement is made.
- The trace of the density matrix 1. This is obvious from the previous statement, because it must be the sum of all probabilities.
- The density matrix is linear.
- The eigenvalues of the density operator are positive.
- The off-diagonal elements $\rho_{i,j}$ of the density matrix provide information about interference between the amplitudes of states $|i\rangle$ and $|j\rangle$, i.e., they represent the coherences in the mixed state.

Description

- State Tomographic methods have been used to reconstruct density matrices of single qubits. In the paper "Two-Qubit State Tomography Using a Joint Dispersive Readout" the reconstruction of two qubit states using only a single apparatus is described.
- Using a superconducting circuit implementation of cavity quantum electrodynamics this readout is accomplished by measuring the transmission of a microwave frequency resonator strongly coupled to both qubits.
- Two quit correlations are extracted from an averaged measurement that acts simultaneously on the qubits.

In the figure below two superconducting qubits are coupled to a transmission line resonator operating in the microwave regime



Hamiltonian

The Hamiltonian for the two- qubit state reads:

$$H = \hbar(\Delta_{rm} + \underbrace{\chi_1 \hat{\sigma}_{z1} + \chi_2 \hat{\sigma}_{z2})\hat{a}\hat{a}^\dagger} + \frac{\hbar}{2} \sum_{j=1,2} (\omega_{aj} + \chi_j)\hat{\sigma}_{zj} + \hbar\epsilon(t)(\hat{a} + \hat{a}^\dagger) \quad (1)$$

where

- $\hat{a}^\dagger$ and $\hat{a}$ are the annihilation and creation operator respectively.
- $\Delta_{rm} = \omega_r - \omega_m$ is the detuning of the measurement drive from the resonator frequency.
- $\chi_1 \neq \chi_2$ are the cavity pulls.
- $\epsilon(t)$ denotes the amplitude.
- The σ's denote the Pauli matrices.
- ω_{a1} and ω_{a2} are the transition frequencies of the individual qubits.
- The operator $\hat{\chi} = (\chi_1 \hat{\sigma}_{z1} + \chi_2 \hat{\sigma}_{z2})\hat{a}\hat{a}^\dagger$ describes the dispersive shift of the resonator frequency. It is linear in both qubit states.

- In circuit QED one measures $\langle \hat{I}(t) \rangle$ and $\langle \hat{Q}(t) \rangle$
- One assumes an initially separable state $\hat{\rho}(0) = |0\rangle \langle 0| \otimes \hat{\rho}_q(0)$ for the qubits $[\hat{\rho}_q(0)]$ and the resonator $[|0\rangle \langle 0|]$
-

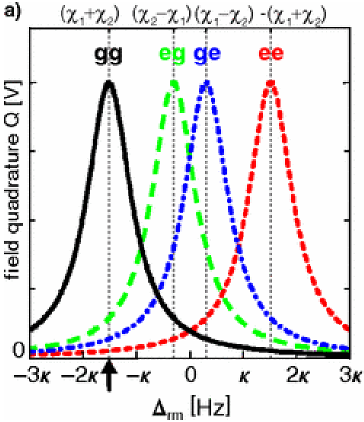
$$\hat{\rho}_q(0) = \sum_{\sigma,\sigma'} p_{\sigma,\sigma'}(0) |\sigma\rangle \langle \sigma'| \tag{2}$$

$$\rightarrow \hat{\rho}(t) = \sum_{\sigma,\sigma'} p_{\sigma,\sigma'}(t) |\sigma \alpha_\sigma\rangle \langle \sigma' \alpha_{\sigma'}|$$

$$\sigma = \{ee, eg, ge, gg\}, \ \alpha_\sigma \text{ the coherent state amplitude}$$

Measurement Outcomes

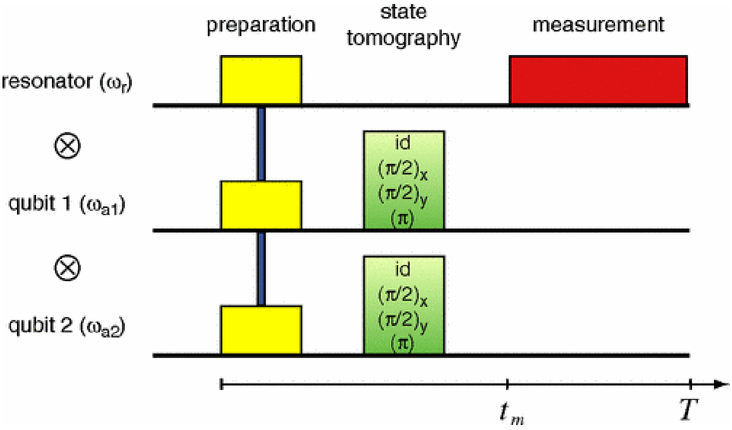
- $\langle \hat{I}(t) \rangle = Tr_q[\hat{\rho}_q(0)\hat{M}_I(t)]$
- $\hat{M}_I(t) = \sum_\sigma \langle \alpha_\sigma(t)| \hat{I} |\alpha_\sigma(t)\rangle |\sigma\rangle \langle\sigma|$
- $\hat{M}_I = -\epsilon \frac{2(\Delta_{rm}+\hat{\chi})}{(\Delta_{rm}+\hat{\chi})^2+(\kappa/2)^2}$



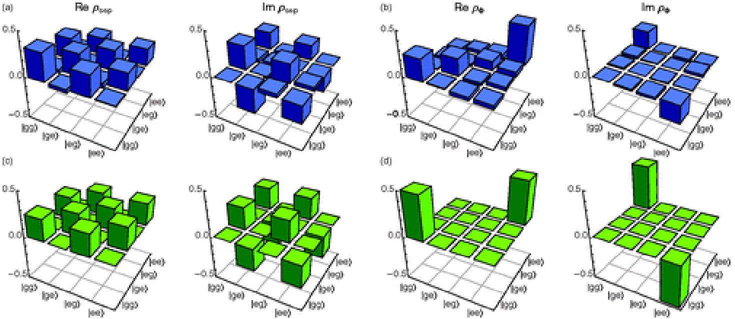
Measurement Process

- Qubit relaxation cannot be neglected
- readout time limited to $1/\gamma_1$, where γ_1 is the qubit decay rate
- initially states are prepared in the states $|ee\rangle$, $|eg\rangle$, $|ge\rangle$, $|gg\rangle$
- the combined state $\hat{\rho}_q$ of qubits is reconstructed by usint a suitable set of measurements $\rightarrow$ get the 16 coefficients r_{ij} of the density matrix $\hat{\rho}_q = \sum_{i,j=0}^{3} r_{i,j} \hat{\sigma}_i \otimes \hat{\sigma}_j$

- start with a 2-qubit state
- apply the combination of $\{(\pi/2)_x, (\pi/2)_y, (\pi), id\}$ pulses to both qubits



Real and imaginary parts of reconstructed density matrices



- product state on the left:
  $|\Psi_{sep}\rangle = 1/\sqrt{2}(|g\rangle + |e\rangle) \otimes 1/\sqrt{2}(|g\rangle + i\,|e\rangle)$
- bell state on the right: $|\Phi\rangle = 1/\sqrt{2}(|g\rangle \otimes |g\rangle - i\,|e\rangle \otimes |e\rangle)$

- To avoid unphysical, non-positive-semidefinite, density matrices originating from statistical uncertainties, all tomography data have been processed by a maximum likelihood method.
- In statistics, maximum likelihood estimation (MLE) is a method of estimating the parameters of a statistical model given observations, by finding the parameter values that maximize the likelihood of making the observations given the parameters.

- Two-Qubit State Tomography Using a Joint Dispersive Readout
- Lecture script QIP, Professor Home
- https://en.wikipedia.org/wiki/Maximum_likelihood_estimation

The End