Experimental Repetitive Quantum Error Correction

May 22, 2017

Problem

Physically stored information is susceptible to errors:

Classical error correction:

- **redundancy**: copying the information on additional bits and doing a majority vote afterwards
- **repetition**: readout and recharging of transistors $\rightarrow$ (DRAM)

Quantum error correction:

- **redundancy**: No-cloning theorem
- **repetition**: No one-shot readout

Solution

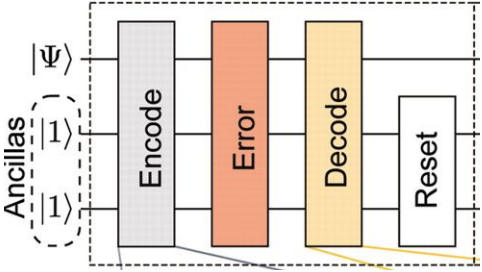
Redundancy:

- one logical qubit $\rightarrow$ multible physical qubits
- entangle the system qubit with the ancilla qubits

Repetition:

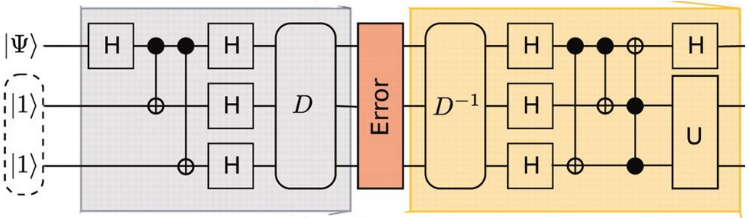
- repeat the error-correction to enhance the life time

Error correction cycle



- **Encoding**: Logical qubit $|\psi\rangle$ entangled with ancillas
- **Error**: Potential error occurs (phase flip)
- **Decoding**: Restore by error correction the system qubit $|\psi\rangle$
- **Reset**: reset the ancillas to $|1\rangle$

Error correction cycle - no error



- $|\psi\rangle\,|1\rangle\,|1\rangle$

$= (\alpha\,|+\rangle + \beta\,|-\rangle)\,|1\rangle\,|1\rangle$

- $(\alpha\,|0\rangle + \beta\,|1\rangle)\,|1\rangle\,|1\rangle$
- $\alpha\,|0,0,0\rangle + \beta\,|1,1,1\rangle$
- $\alpha\,|+,+,+\rangle + \beta\,|-,-,-\rangle$

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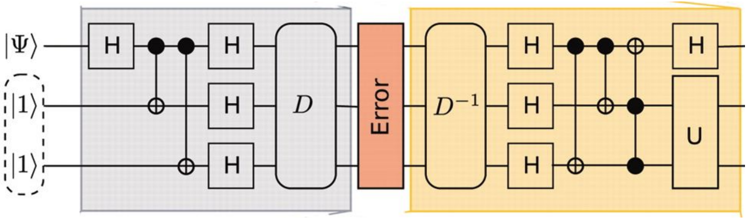
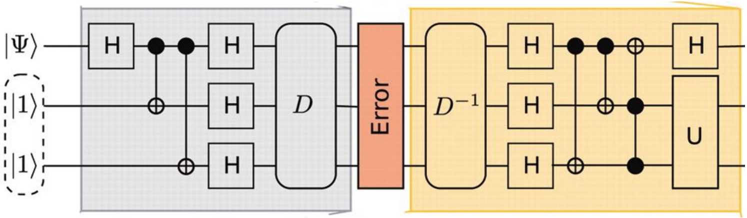
- $\alpha\,|0,0,0\rangle + \beta\,|1,1,1\rangle$
- $\alpha\,|0,1,1\rangle + \beta\,|1,1,1\rangle$
- $(\alpha\,|0\rangle + \beta\,|1\rangle)\,|1\rangle\,|1\rangle$
- $(\alpha\,|+\rangle + \beta\,|-\rangle)\,|1\rangle\,|1\rangle$

Error correction cycle - one error



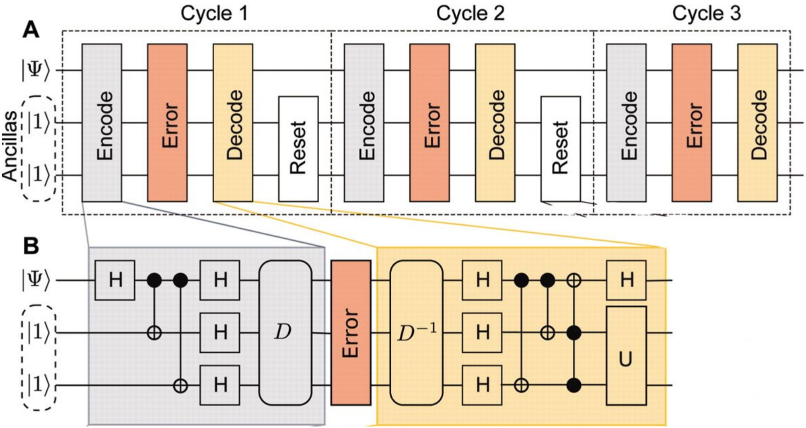
- $|\psi\rangle\,|1\rangle\,|1\rangle$
$= (\alpha\,|+\rangle + \beta\,|-\rangle)\,|1\rangle\,|1\rangle$
- $(\alpha\,|0\rangle + \beta\,|1\rangle)\,|1\rangle\,|1\rangle$
- $\alpha\,|0,0,0\rangle + \beta\,|1,1,1\rangle$
- $\alpha\,|+,+,+\rangle + \beta\,|-,-,-\rangle$
- $\alpha\,|+,-,+\rangle + \beta\,|-,+,-\rangle$
- $\alpha\,|0,1,0\rangle + \beta\,|1,0,1\rangle$
- $\alpha\,|0,0,1\rangle + \beta\,|1,0,1\rangle$
- $(\alpha\,|0\rangle + \beta\,|1\rangle)\,|0\rangle\,|1\rangle$
- $(\alpha\,|+\rangle + \beta\,|-\rangle)\,|0\rangle\,|1\rangle$

Error correction cycle - two errors

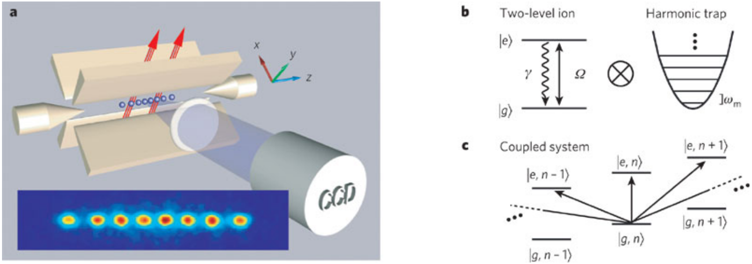


- $|\psi\rangle|1\rangle|1\rangle$
$= (\alpha|+\rangle + \beta|-\rangle)|1\rangle|1\rangle$
- $(\alpha|0\rangle + \beta|1\rangle)|1\rangle|1\rangle$
- $\alpha|0,0,0\rangle + \beta|1,1,1\rangle$
- $\alpha|+,+,+\rangle + \beta|-,-,-\rangle$
- $\alpha|-,-,+\rangle + \beta|+,+,-\rangle$
- $\alpha|1,1,0\rangle + \beta|0,0,1\rangle$
- $\alpha|1,1,0\rangle + \beta|0,1,0\rangle$
- $(\alpha|1\rangle + \beta|0\rangle)|1\rangle|0\rangle$
- $(\alpha|-\rangle + \beta|+\rangle)|0\rangle|1\rangle$

Repetitive error correction

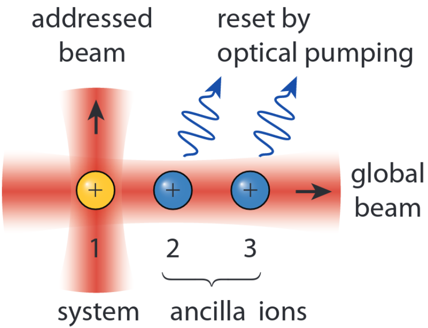


Paul Trap

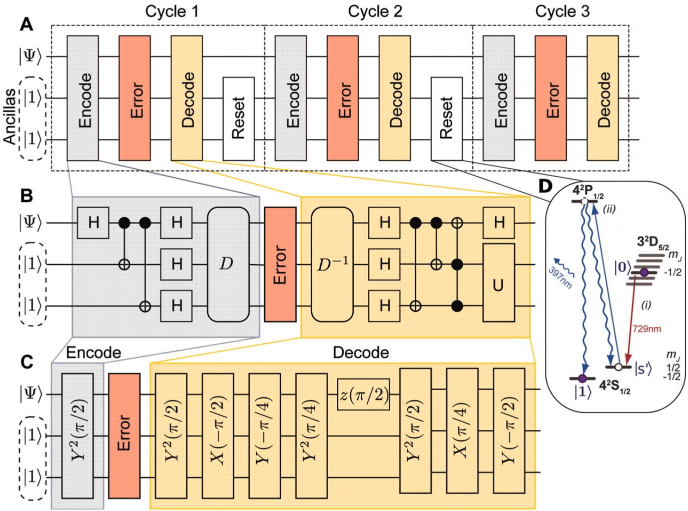


a
x
y
z
CCD
b
Two-level ion
Harmonic trap
|e⟩
|g⟩
γ
Ω
ω_m
c
Coupled system
|e, n − 1⟩
|e, n⟩
|e, n + 1⟩
|g, n − 1⟩
|g, n⟩
|g, n + 1⟩

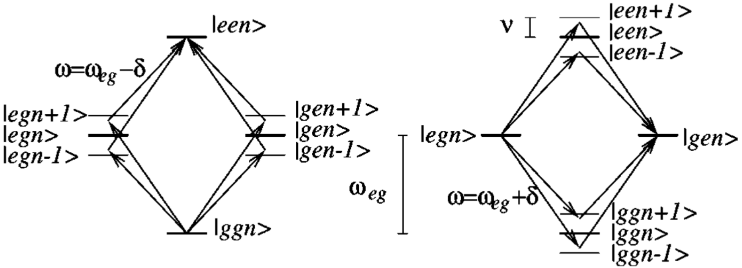
Adressing the Ions



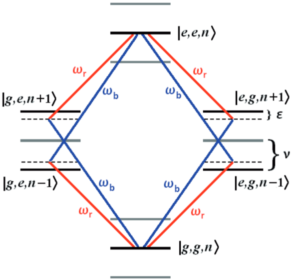
Experimental realization



Mølmer - Sørensen Gate



Mølmer - Sørensen Gate



$$\tilde{\Omega} = \frac{1}{\hbar} \sum_m \frac{\langle een|H_{int}|m\rangle\langle m|H_{int}|ggn\rangle}{E_m - (E_{ggn} + \hbar\omega_1)}$$

$|m\rangle = |e, g, n\rangle$ or $|g, e, n\rangle$

Red first: $\hbar(\nu - \varepsilon)$

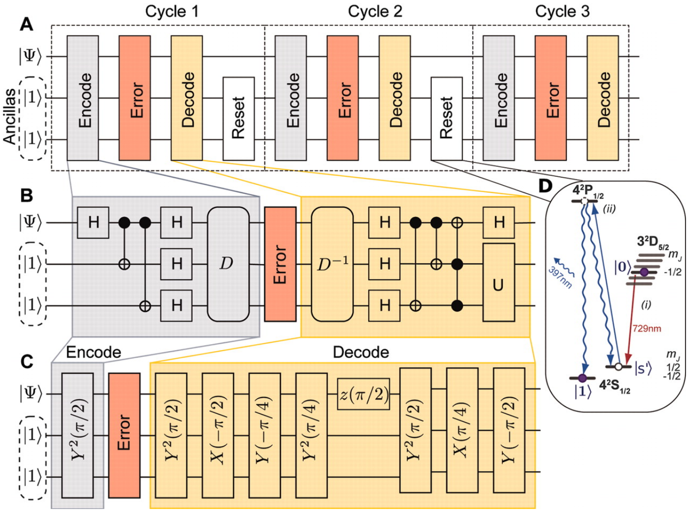
Blue first: $-\hbar(\nu - \varepsilon)$

$|m\rangle = |e, g, n-1\rangle$ or $|e, g, n+1\rangle$

Red first: $\hbar\varepsilon$

Blue first: $-\hbar\varepsilon$

Overview



A
Cycle 1
Cycle 2
Cycle 3
|Ψ⟩
Ancillas
|1⟩
|1⟩
Encode
Error
Decode
Reset
B
H
D
Error
D^{-1}
U
C
Encode
Decode
$Y^2(\pi/2)$
Error
$X(-\pi/2)$
$Y(-\pi/4)$
$Y^2(\pi/4)$
$z(\pi/2)$
$X(\pi/4)$
$Y(-\pi/2)$
D
$4^2P_{1/2}$
(ii)
$3^2D_{5/2}$
m_J
-1/2
|0⟩
397nm
(i)
729nm
m_J
1/2
-1/2
|s'⟩
|1⟩
$4^2S_{1/2}$

Implementation of the Toffoli Gate

- "And" operation on c_2 and c_3, writing the result in the vibrational quantum number
- C-Not from the vibrational mode onto the first qubit
- decoding the state

Implementation of the Toffoli Gate

After the pulses for the "AND" operation: we get (just looking at the last two qubits + oscillation):

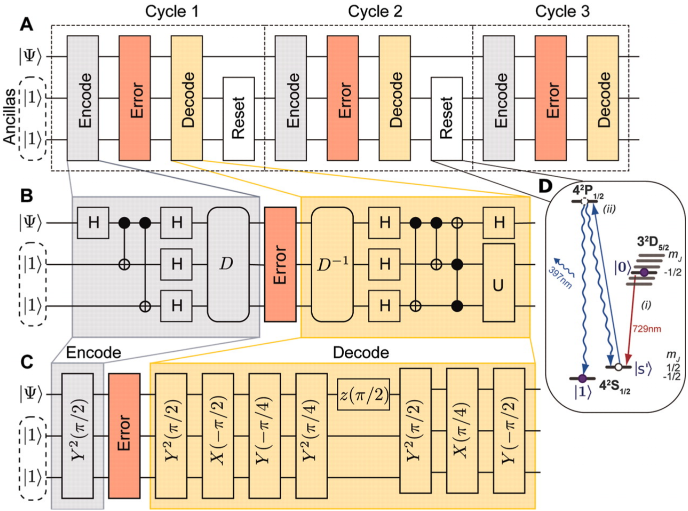
$$|c_2c_3, 0\rangle = |SS, 0\rangle \rightarrow |SD, 1\rangle$$

$$|c_2c_3, 0\rangle = |DS, 0\rangle \rightarrow \sin\frac{\pi}{2\sqrt{2}}|SD, 0\rangle + \cos\frac{\pi}{2\sqrt{2}}|DS, 0\rangle$$

$$|c_2c_3, 0\rangle = |SD, 0\rangle \rightarrow \cos\frac{\pi}{2\sqrt{2}}|SD, 0\rangle - \sin\frac{\pi}{2\sqrt{2}}|DS, 0\rangle$$

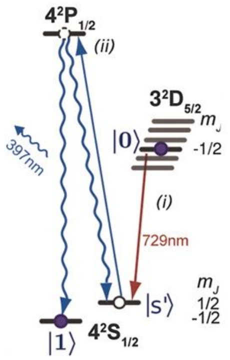
$$|c_2c_3, 0\rangle = |DD, 0\rangle \rightarrow |DD, 0\rangle$$

Overview



A
Cycle 1
Cycle 2
Cycle 3
|Ψ⟩
Ancillas
|1⟩
|1⟩
Encode
Error
Decode
Reset
B
H
D
D^{-1}
U
C
Encode
Decode
Y^2(π/2)
Y^2(π/2)
X(−π/2)
Y(−π/4)
Y^2(π/4)
z(π/2)
X(π/4)
Y(−π/2)
D
4^2P_{1/2}
(ii)
3^2D_{5/2}
m_J
-1/2
|0⟩
397nm
(i)
729nm
|1⟩
|s'⟩
4^2S_{1/2}
1/2
-1/2

Resetting the ancillas



- $3^2D_{5/2} \rightarrow |S'\rangle = 4^2S_{1/2}$ with $m_j = +\frac{1}{2}$
- $|S'\rangle \rightarrow 4^2P_{1/2}$ with $m_j = -\frac{1}{2}$
- $4^2P_{1/2} \rightarrow |1\rangle = 4^2S_{1/2}$ with $m_j = -\frac{1}{2}$

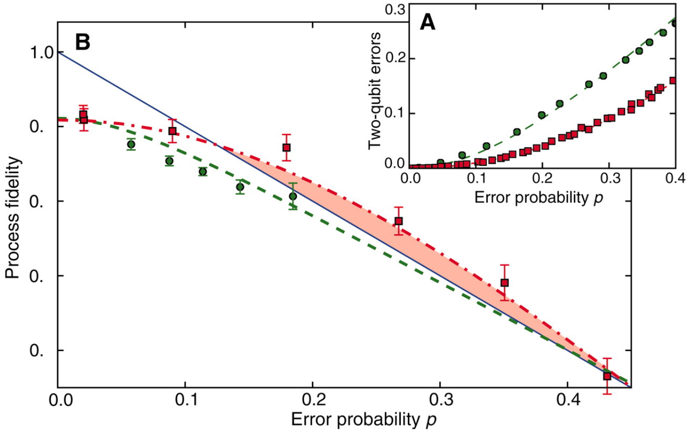
Process fidelity

Process: $\rho \mapsto \varepsilon(\rho) = \sum_{m,n=1}^{4} \chi_{mn} A_m \rho A_n^\dagger$

where $\{A_m\} = \{\mathbb{1}, \sigma_x, \sigma_y, \sigma_z\} \Rightarrow \mathbb{1}_{\text{Process}} \hat{=} \chi_{\mathbb{1}} = \delta_{m0}\delta_{n0}$

Process fidelity: $F_{proc} = \text{Tr}(\chi \chi_{\mathbb{1}})$

Results



B
1.0
0.
0.
0.
0.
Process fidelity
0.0
0.1
0.2
0.3
0.4
Error probability p
A
0.3
0.2
0.1
0.0
Two-qubit errors
0.0
0.1
0.2
0.3
0.4
Error probability p