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# Two-Qubit State Tomography Using a Joint Dispersive Readout

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# Overview

- Introduction and main idea
- Important concepts and terms
  - Density Matrix
  - Quantum State Tomography
  - Superconducting Qubits (Cooper pair box & Transmon)
- Historical background
- Measurement
  - Setup
  - Procedure
  - Example results
  - Advantages of new method
- Conclusion

# Introduction

- Important aspect in QIP:
  - Find density matrix of quantum state
  - Find correlations between multiple quantum states ( Entanglement )
- Paper presents a new measurement method:
  - Two qubit systems
  - **Full** density matrix ( including correlations )

# 1. Density Matrix Formalism

**Definition**:

$$\rho(t) \equiv |\psi(t)\rangle\langle\psi(t)|$$

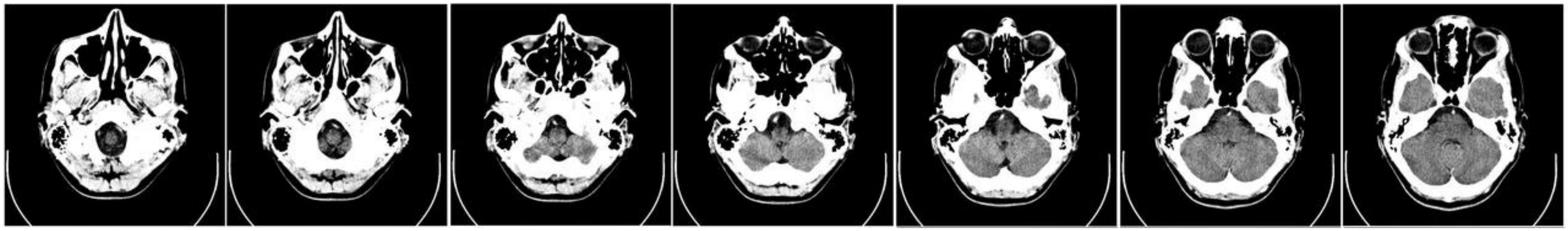
**Properties**:

- $\mathrm{Tr}(\rho) = 1$
- $\rho$ is hermitian: $\rho^\dagger = \rho$
- $\mathrm{Tr}(\rho^2) \leq 1$
  - Pure state: $\mathrm{Tr}(\rho^2) = 1$
  - Mixed state: $\mathrm{Tr}(\rho^2) < 1$

Useful representation of quantum states

# 2. Quantum State Tomography

- **Classical tomography**
  - Reconstruct 3D image
  - Use series of 2D projections
  - Measure **the same** object
  - E.g. CT (Computer tomography)
- **Quantum state tomography**
  - Reconstruct complete quantum state
  - Use measurement in different basis
  - Measure **multiple copies** of the state (Measurement disturbs state)



# 2.1 Single Qubit Quantum State Tomography

- Any single qubit can be described as $|\psi\rangle = \alpha|0\rangle + \beta|1\rangle$
- Tomography requires ensemble of such states
- Problem:
  - Ensemble could contain different pure states
  - Members of ensemble are not pure states

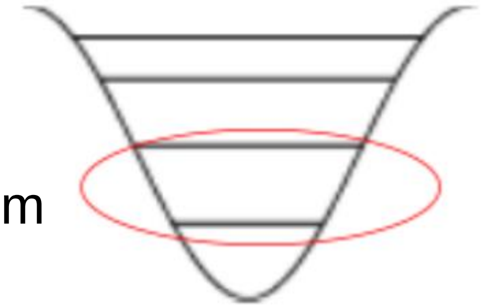
Overall state is mixed ➡ Can be described by probabilistically weighted incoherent sum of pure states

- Representation of any quantum state = ensemble of two orthogonal **pure** states

$$\hat{\rho} = \sum_i P_i |\psi_i\rangle\langle\psi_i|$$

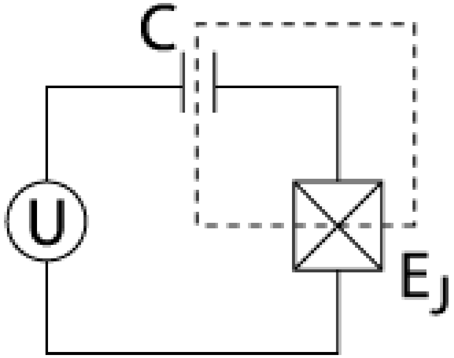
# Concepts and Terms
# 3. Superconducting Qubits

How could we build a Qbit?



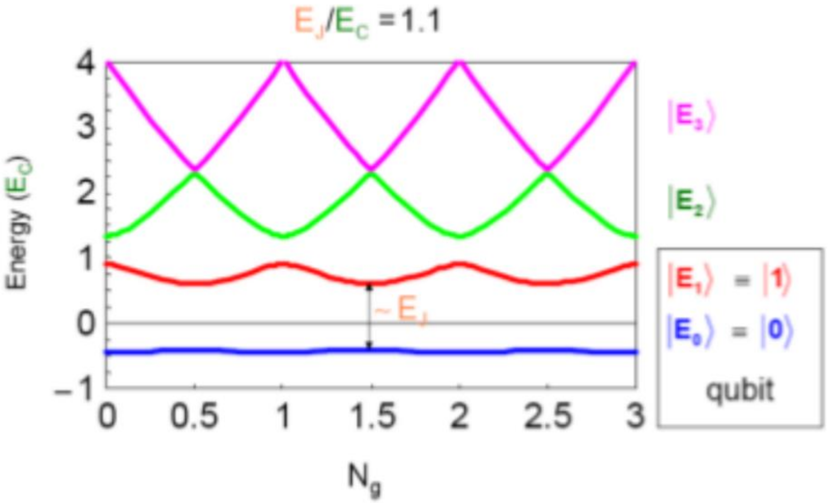
- 2 level system with non uniform energy spectrum
- **Josephson junction**:
  harmonic oscillator with non uniform energy spectrum

- Cooper Pair Box:



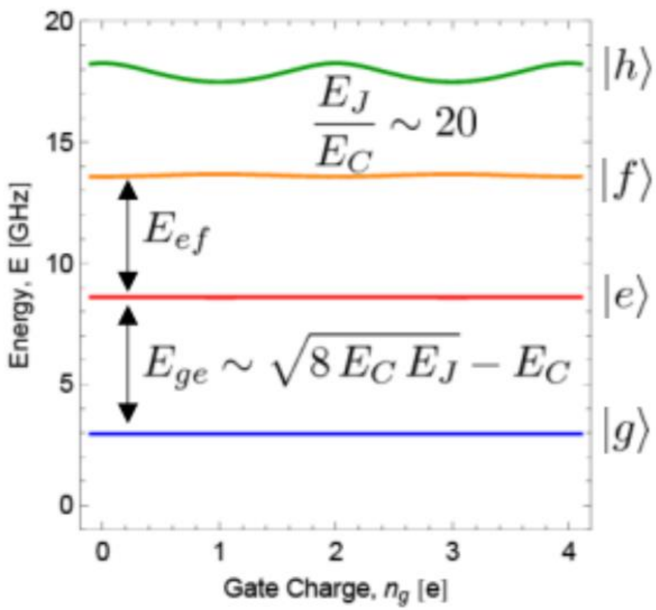
# Concepts and Terms
# 3.1 Superconducting Qubits

- **Qubit state**:
  - absence / presence of extra Cooper-Pairs on the island
  - Modulate Gate Voltage: Tunnel through Jospheson Junction onto island

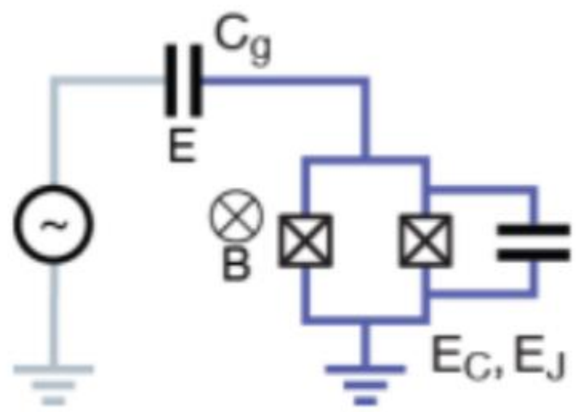


# Concepts and Terms
# 3.2 Superconducting Qubits (Transmons)

- **Goal**: Get more stable two level system
- **How:** Use Transmons => reduces fluctuations




Flat Levels



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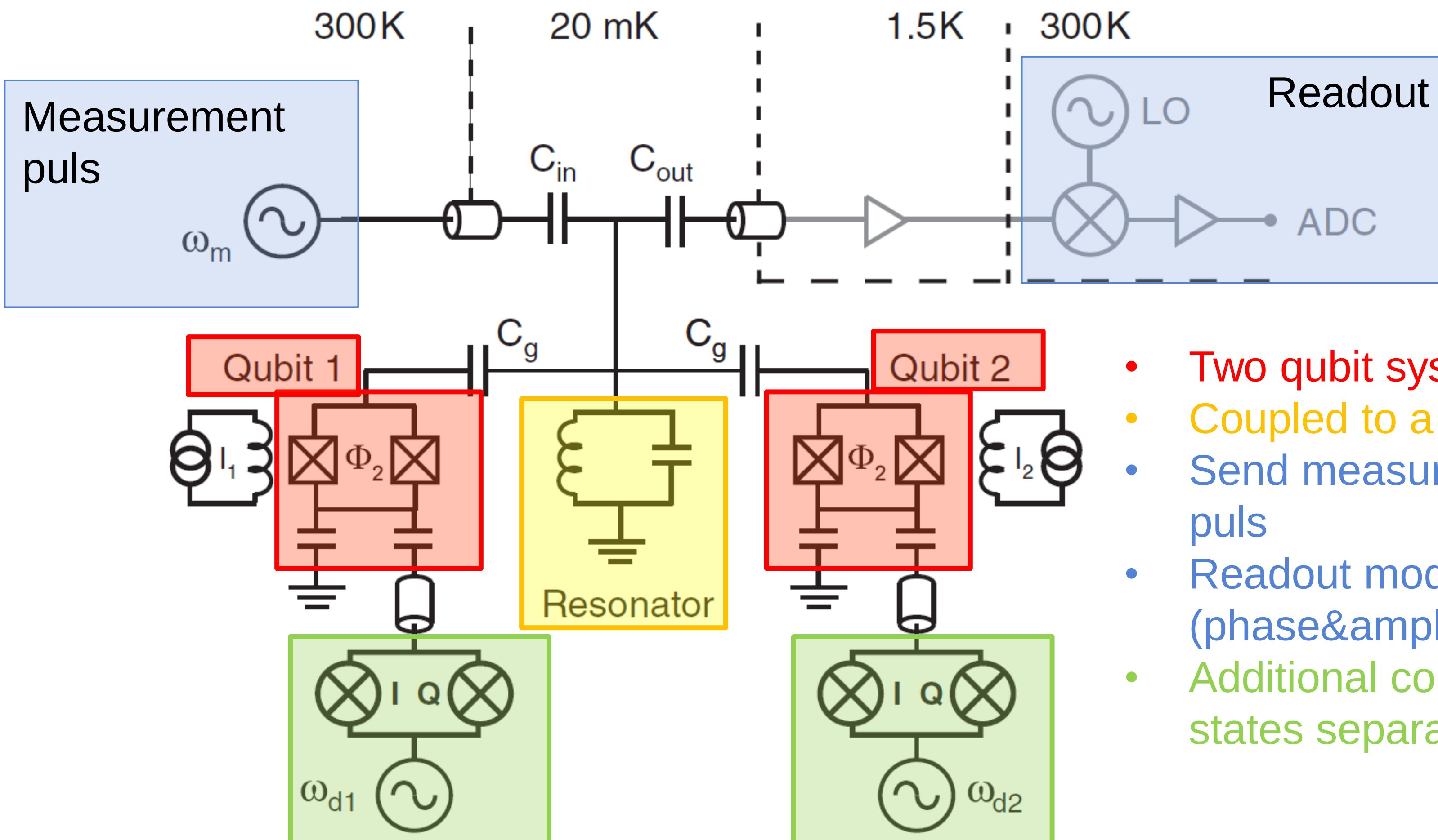
# History of quantum tomography

- Reconstruct Wigner Function of light mode
- Continuous spectra
  - Vibrational states (molecules, ions, atoms)
- Discrete spectra
  - Spin, polarisation of entangles photon pairs
- Individual  two level systems
  - Density matrix of single qubit
  - Correlate individual measurements of one qubit systems to reconstruct two qubit states
- **Now**: Measure two qubit system with only a single measurement

# Measurement Setup




- Two qubit system
- Coupled to a resonator
- Send measurement puls
- Readout modified puls (phase&amplitude)
- Additional control qubit states separately

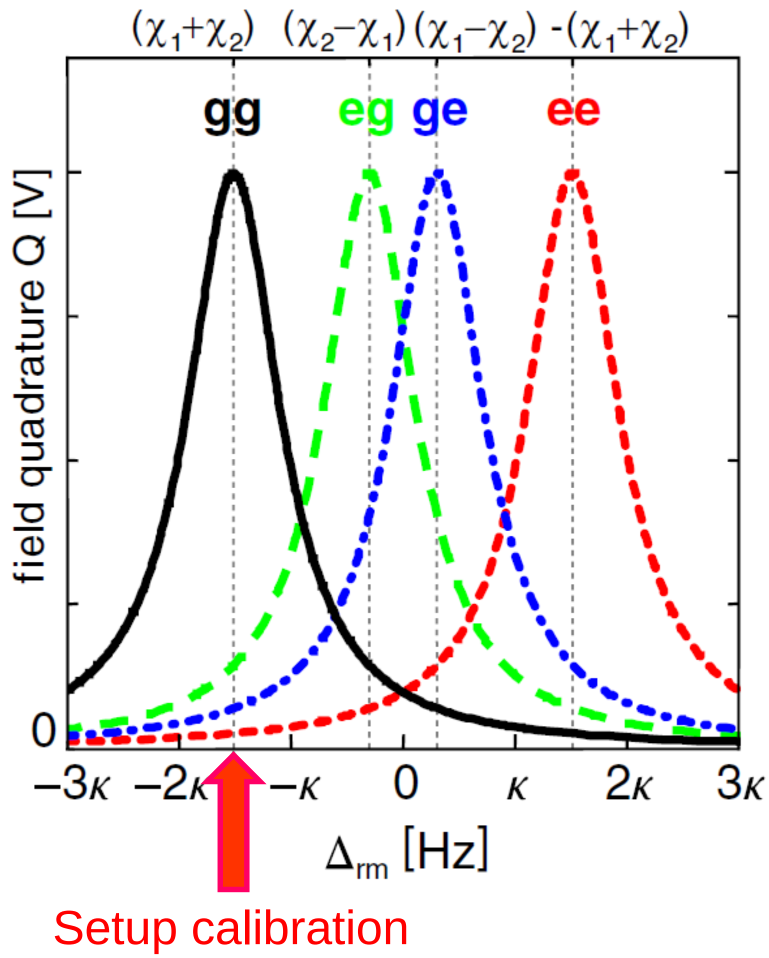
Microwave emitters => control qubit states

# Measurement
# Setup Calibration

$\Delta_{rm}$ = Shift between input and resonator frequency (**Detuning**)

Q = Puls Amplitude

1. Cool system down => $|gg\rangle$
2. Go through all $\Delta_{rm}$ by modifying $\omega_m$ (input)
3. Measure transmission Q($\Delta_{rm}$) and find resonance frequency
4. Set $\omega_m$ such that $\Delta_{rm}$ on $|gg\rangle$ peak



# Measurement
# Procedure 1 (Preparation of States)

1. Cool qubits down => ground state

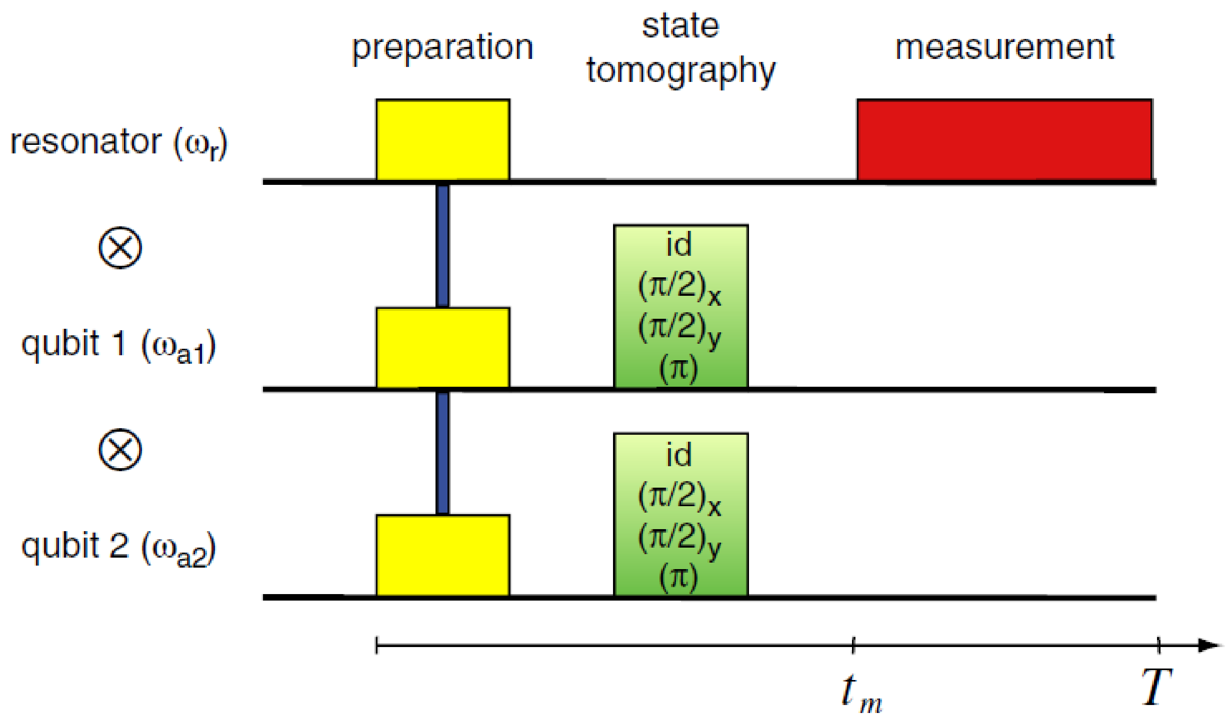
2. Excite them with microwave emitters
   => prepare in desired states $|gg\rangle$, $|eg\rangle$, $|ge\rangle$, $|ee\rangle$

3. States are coupled to a resonator
   => influences effect of the resonator on the input pulse

4. Choose measurement basis with microwave pulses

5. Apply 4x4 rotations to get 16 entries of density matrix

# Measurement Procedure 2




- Rotation ≙ Measurement in certain basis
- Repeat each of the 16 combinations multiple times: $\sim 10^5$
- Average to get matrix entry

# Key Points of New Method & Advantages

- Role of the Resonator:
  - Couple the two qubits
  - Measurement Operator has a new component (correlation term):

$$\widehat{M} = \ldots + \beta_{11}\, \hat{\sigma}_{z1}\hat{\sigma}_{z2}$$

$|gg\rangle = |ee\rangle$ => 1*1 = (-1)*(-1) = 1

$|eg\rangle = |ge\rangle$ => (-1)*1 = 1*(-1) = -1

$\hat{\sigma}_z|g\rangle$ = 1$|g\rangle$

$\hat{\sigma}_z|e\rangle$ = -1$|e\rangle$

Differentiate between superposition and entangled state:

$(|g\rangle + |e\rangle)\otimes(|g\rangle + |e\rangle)$ = $|ee\rangle + |gg\rangle + |eg\rangle + |ge\rangle = 0$

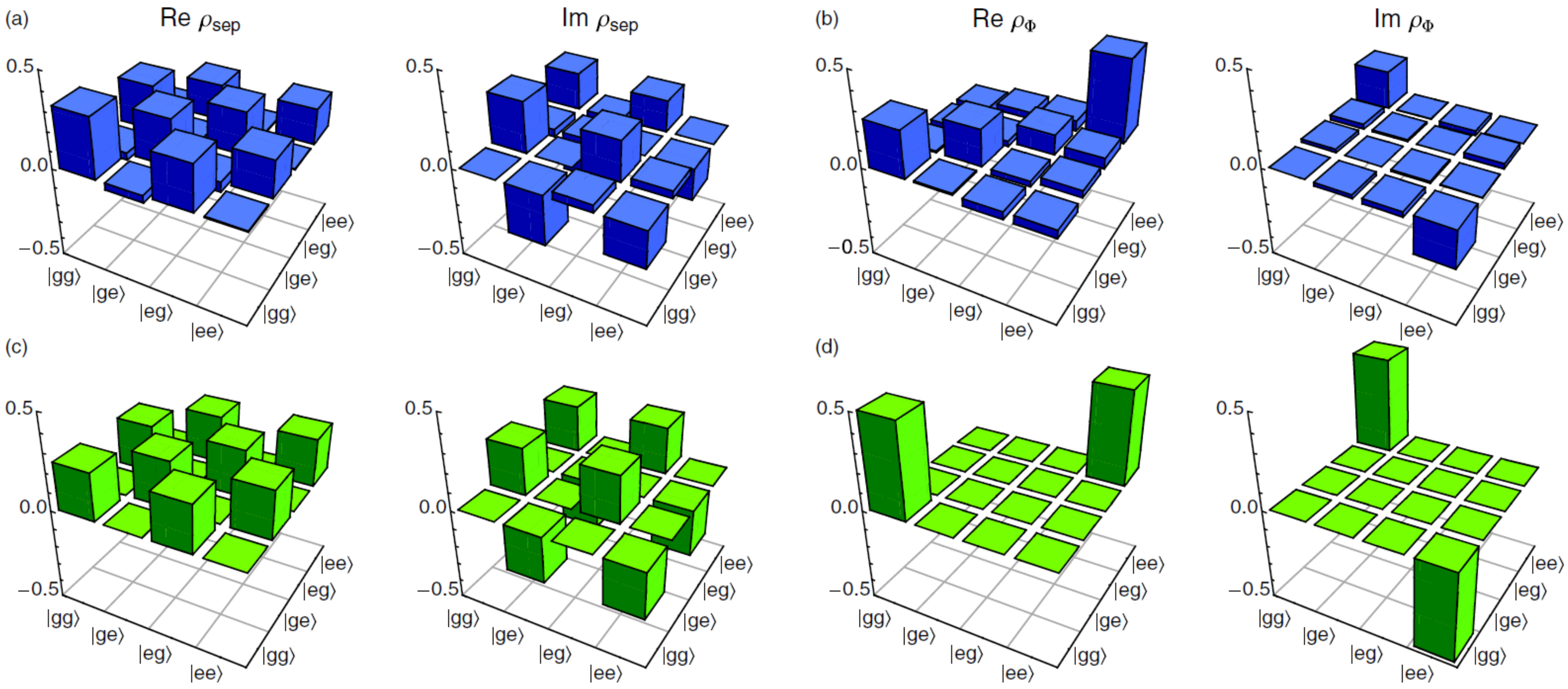
$|gg\rangle + |ee\rangle = 2$

- **New Advantage:** Measure entangled states with averaging method

# Result Examples

$$|\Psi_{SEP}\rangle = \frac{1}{\sqrt{2}}(|g\rangle + |e\rangle) \otimes \frac{1}{\sqrt{2}}(|g\rangle + i|e\rangle)$$

$$|\Psi_{Bell}\rangle = \frac{1}{\sqrt{2}}[|g\rangle \otimes |g\rangle - i|e\rangle \otimes |e\rangle]$$



# Conclusion

- New method for quantum state tomography
- Simultaneously and jointly read out of quantum state of two qubits
- Extract the correlation between the qubits