QIP II: Implementations, FS 2018 - Questions 1

26. Februar 2018

1. **The Cooper-pair box Hamiltonian**
   As will be shown in one of the lectures, many realizations of superconducting qubits are described by the following Hamiltonian
   $$\hat{H} = 4E_C(\hat{n} - n_g)^2 - E_J \cos\hat{\phi}.$$
   Here, the charging energy E_C, the Josephson energy E_J, and the offset charge n_g are system parameters and the charge number $\hat{n}$ and the phase $\hat{\phi}$ are quantum-mechanical operators. The goal of this exercise is to get familiar with important properties and the relevant parameter regimes of this Hamiltonian. We refer to Koch *et al.* PRA 76, 042319 (2007) for further details.

   (a) Derive the stationary Schroedinger equation with respect to the wave-function $\psi(\phi) = \langle\phi|\psi\rangle$, where $\hat{\phi}|\phi\rangle = \phi|\phi\rangle$. In this basis the operator $\hat{n}$ acts on the wave function according to $\langle\phi|\hat{n}|\psi\rangle = -i\frac{\partial}{\partial\phi}\psi(\phi)$. The eigenstates and eigenenergies for this Schroedinger equation are found to be given by Mathieu functions. Plot the first three eigenenergies for different ratios of E_J/E_C and vs. the offset charge number n_g. Interpret your result.

   (b) Consider the limit $E_J/E_C \gg 1$ and $n_g = 0$. Argue why in this limit the zero-point fluctuations in the phase variable are small $\langle\hat{\phi}^2\rangle \ll 1$ and why in this case a low order Taylor expansion of the cosine potential $\cos\hat{\phi} \approx 1 - \hat{\phi}^2/2 + \hat{\phi}^4/24 + ...$ is justified. The resulting approximate Hamiltonian describes an oscillator with a finite anharmonicity. Express this Hamiltonian in terms of the standard annihilation a and creation operator $a^\dagger$ and determine the degree of anharmonicity. Discuss and interpret your observations.