QIP II: Implementations, FS 2018 - Questions 2

March, 2018

1) Atom-light interaction:
Consider two levels of the atom (e.g. two eigenstates of Hydrogen or the ground and excited states of a transmon qubit) such that we can write

$$H_{atom} = E_g|g\rangle\langle g| + E_e|e\rangle\langle e|, \tag{1}$$

where $|g\rangle$ denotes the ground state and $|e\rangle$ the excited state. A single atom may interact with light (here we consider a single frequency light source such as a laser or a microwave generator) via the dipole interaction and can be described by the Hamiltonian $H = H_{atom} + H_{int}$ with the interaction given by

$$H_{int} = -\hat{\boldsymbol{d}} \cdot \mathcal{E} = -\hat{d}\,\mathcal{E}_0 \cos(\omega_l t + \phi), \tag{2}$$

where $\hat{d} = e\hat{x}$ is the dipole operator for the atom, $\mathcal{E}_0$ is the electric field strength of the light and ω_l is angular frequency of the light.

Write the energy difference as $\hbar\omega_a = E_e - E_g$ and argue that the Hamiltonian can be written as

$$H = \frac{\hbar\Delta}{2}\sigma_z + \frac{\hbar\Omega}{2}\sigma_x \tag{3}$$

with $\Delta = \omega_a - \omega_z$ and where σ_x and σ_z are the Pauli x and z operators.

Hint 1: In the $|g\rangle, |e\rangle$-basis the Pauli operators can be written as $\sigma_z = |e\rangle\langle e| - |g\rangle\langle g|$ and $\sigma_z = |e\rangle\langle g| + |g\rangle\langle e|$.

Hint 2: Consider the Hamiltonian H_{int} in the interaction picture with the Hamiltonian $H_0 = \frac{\hbar\omega_l}{2}\sigma_z$.

Hint 3: Use the rotation-wave approximation $|\Delta| \ll \omega_a + \omega_l$.

2) Rabi oscillations: Consider the case where $\Delta = 0$. Show that the time evolution operator can be written in the form

$$U(t) = \begin{bmatrix} \cos\frac{1}{2}\Omega t & -i\sin\frac{1}{2}\Omega t \\ -i\sin\frac{1}{2}\Omega t & \cos\frac{1}{2}\Omega t \end{bmatrix}. \tag{4}$$

Determine the qubit state as a function of time t for an initial ground, $|g\rangle$, state and under this time-evolution and illustrate this on the Bloch sphere. What is the role of Ω in the time-evolution? A π-pulse is defined as a π-rotation around the Bloch-sphere. Determine the interaction time t_π needed for a π-pulse.