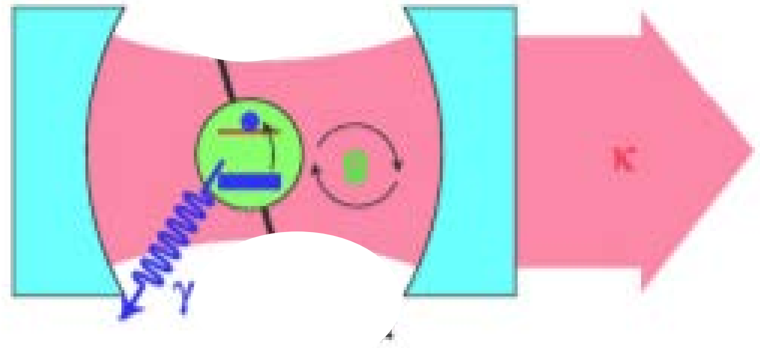
Readout of SC qubits

Cavity Quantum Electordynamics

- Interaction between atoms and the quantized electromagnetic modes inside a cavity



- Decay rate κ
- Coupling strength g
- Two level system with spontaneous decay rate γ

Cavity Quantum Electordynamics

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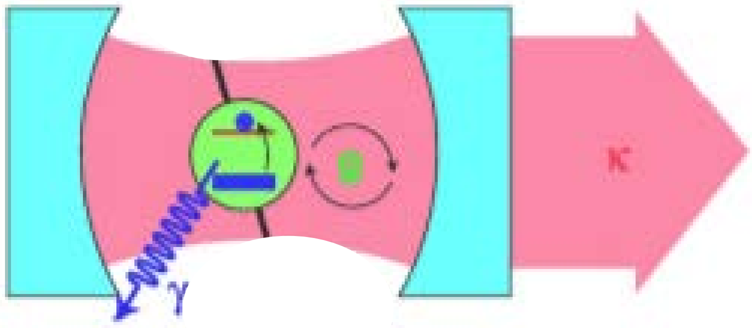
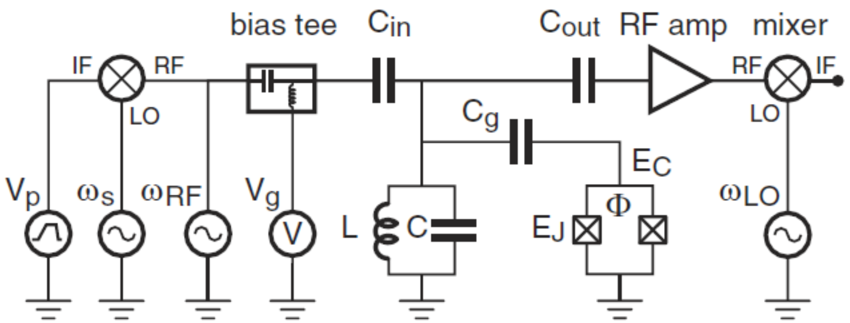
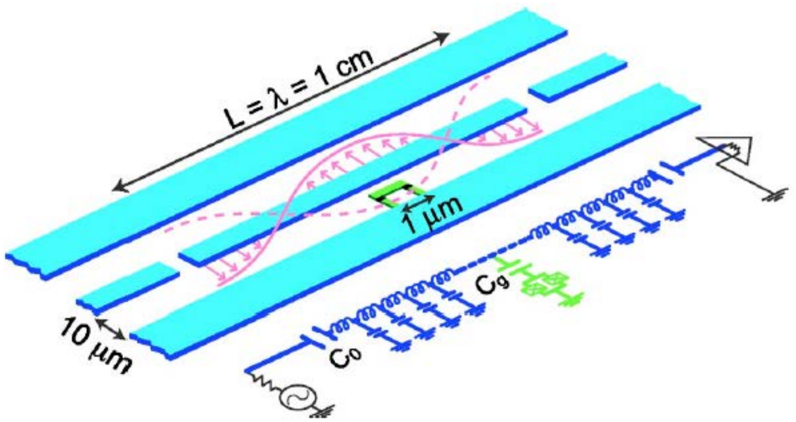


Applications

- High fidelity dispersive read-out
- Coupling qubits and realizing 2-qubit gates

Tuning Frequency





Cooper pair box

- Acting as a two level system
- Level separation $E_a = \hbar\omega_a = \sqrt{E_{el}^2 + E_J^2}$

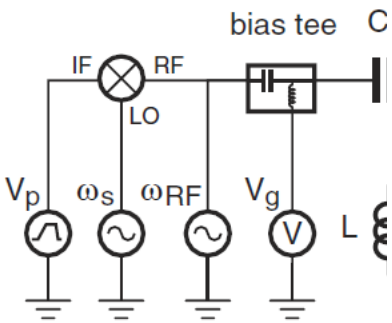
coupled capacitively to a single mode of the electromagnetic field of a transmission line resonator with resonance frequency ω_r

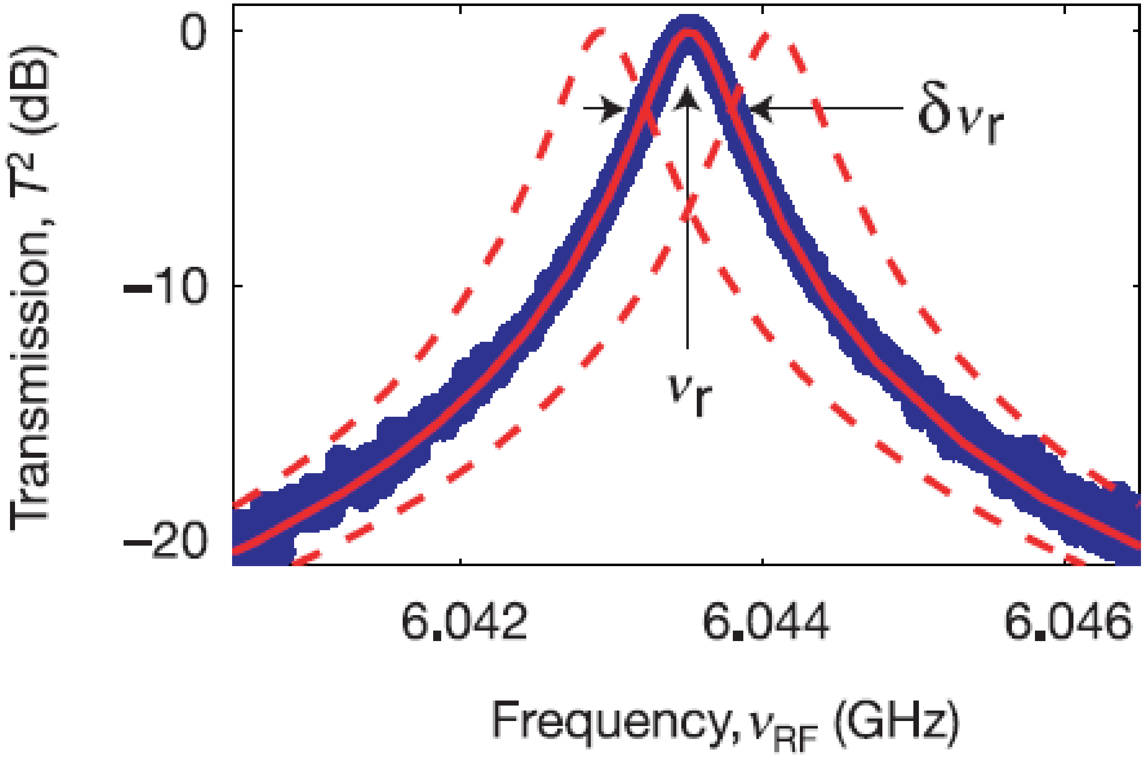
$$\omega_{1,2} = \omega_{1,2}^{max}\sqrt{|\cos(\pi\Phi/\Phi_0)|}$$

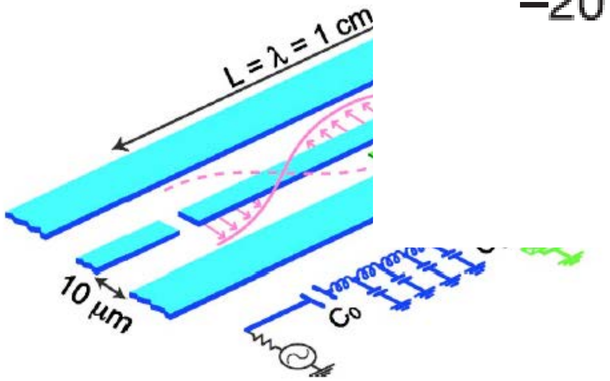
in situ tuning of the transition frequency of the qubits

Tuning Frequency

Cooper pair box







system

$E_a = \hbar\omega_a = \sqrt{E_{el}^2 + E_J^2}$

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$\mathrm{os}\,(\pi\Phi/\Phi_0)|$

in situ tuning of the transition frequency of the qubits

Readout

For $\omega_r = \omega_a$:

Jaynes-Cumming Hamiltonian:

$$H = \hbar\omega_r\left(a^\dagger a + \frac{1}{2}\right) + \frac{\hbar\omega_a}{2}\sigma^z + \hbar g(a^\dagger\sigma^- + a\sigma^+)$$

Quantized field

qubit

coupling

What happens in the limit of large detuning?

$|\Delta| = |\omega_a - \omega_r| >> g$

Readout

Jaynes-Cumming Hamiltonian:

$$H = \hbar\omega_r\left(a^\dagger a + \frac{1}{2}\right) + \frac{\hbar\omega_a}{2}\sigma^z + \hbar g(a^\dagger\sigma^- + a\sigma^+)$$

Unitary transformation

$$\tilde{H} = UHU^\dagger$$

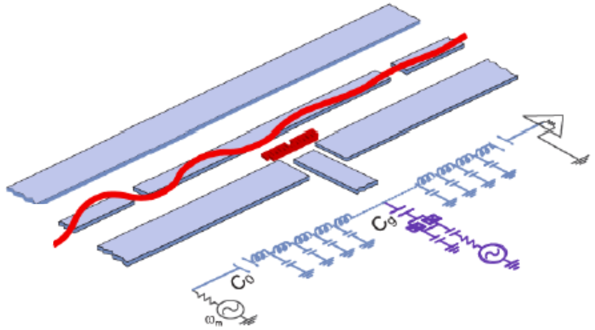
with $U = \exp\frac{g}{\Delta}(a\sigma^+ - a^\dagger\sigma^-)$

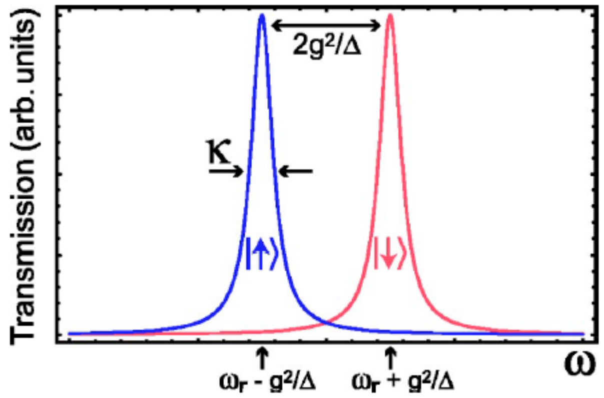
and $\Delta = \omega_a - \omega_r$

Results in dispersive approximation up to 2nd order in g

$$\tilde{H} \approx \hbar\left(\omega_r + \frac{g^2}{\Delta}\sigma_z\right)a^\dagger a + \frac{1}{2}\hbar\left(\omega_a + \frac{g^2}{\Delta}\right)\sigma_z$$

Readout





Jaynes-Cumming Hamiltonian

$$H \approx \hbar\left(\omega_r + \frac{g^2}{\Delta}\sigma_z\right)a^\dagger a + \frac{1}{2}\hbar\left(\omega_a + \frac{g^2}{\Delta}\right)\sigma_z$$

=

Cavity frequency shift

- **Dispersive Shift** of $\pm\, g^2/\Delta$

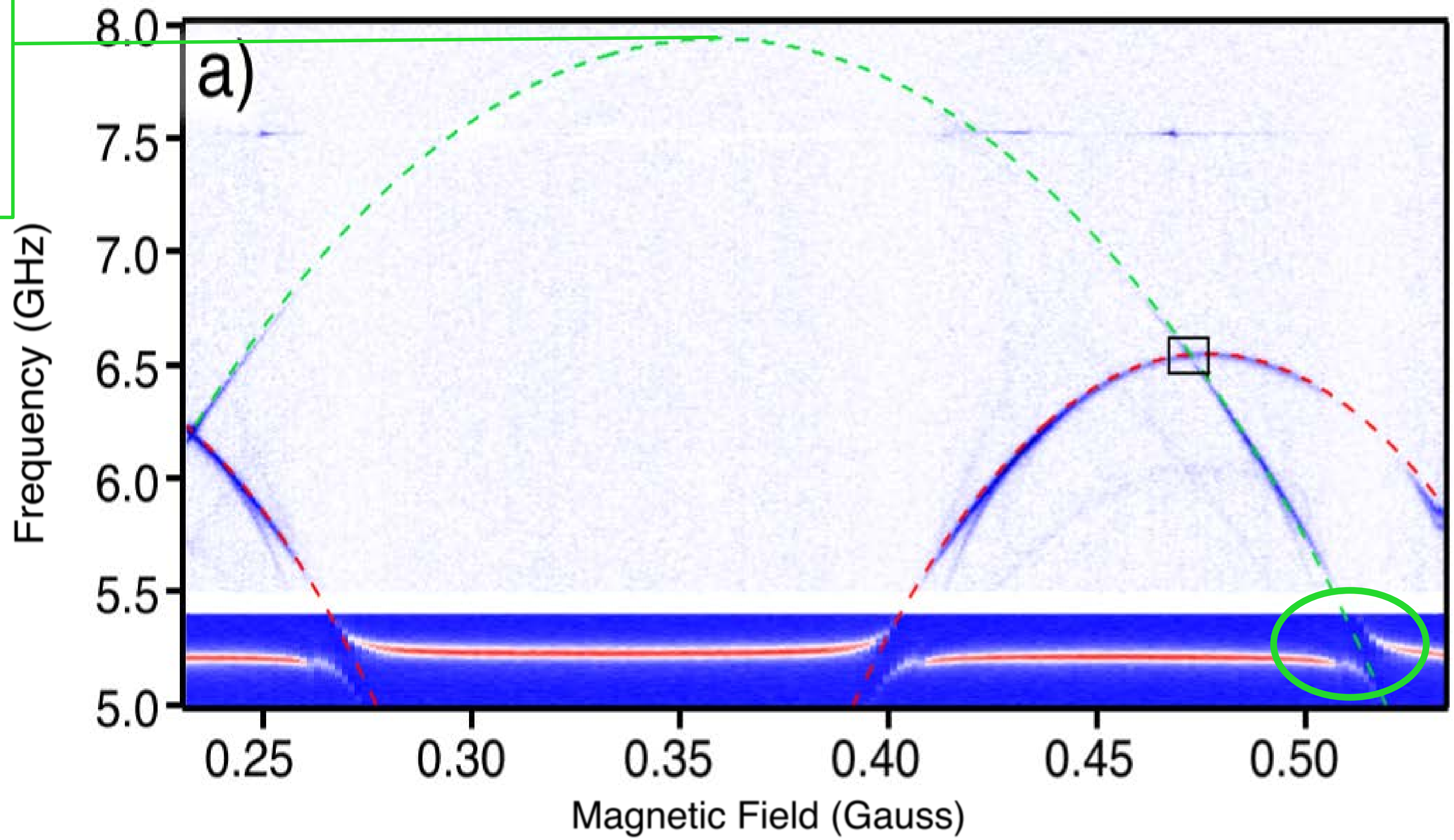
→ Cooper pair box pulls the cavity frequency depending on the state of the qubit

- Rabi splitting

Resonant Regime

$$\omega_{1,2} = \omega_{1,2}^{max} \sqrt{|\cos(\pi\Phi/\Phi_0)|}$$

Nonresonat Regime
$\Delta = \omega_a - \omega_r > g$



Resonant regime
$\omega_a = \omega_r$

Experiment

- Nonresonant (dispersive) regime → Qubit and resonator "uncoupled"

  → **Quantum nondemolition measurement**

- No transition induced between ground and excited state

Rabi oscillations:

- Two level system $\{|1\rangle, |2\rangle\}$ in an oscillating field with a frequency ω near the transition frequency ω_{21}
- For $\omega = \omega_{21}$ the population density oscillates with the **Rabi frequency** Ω

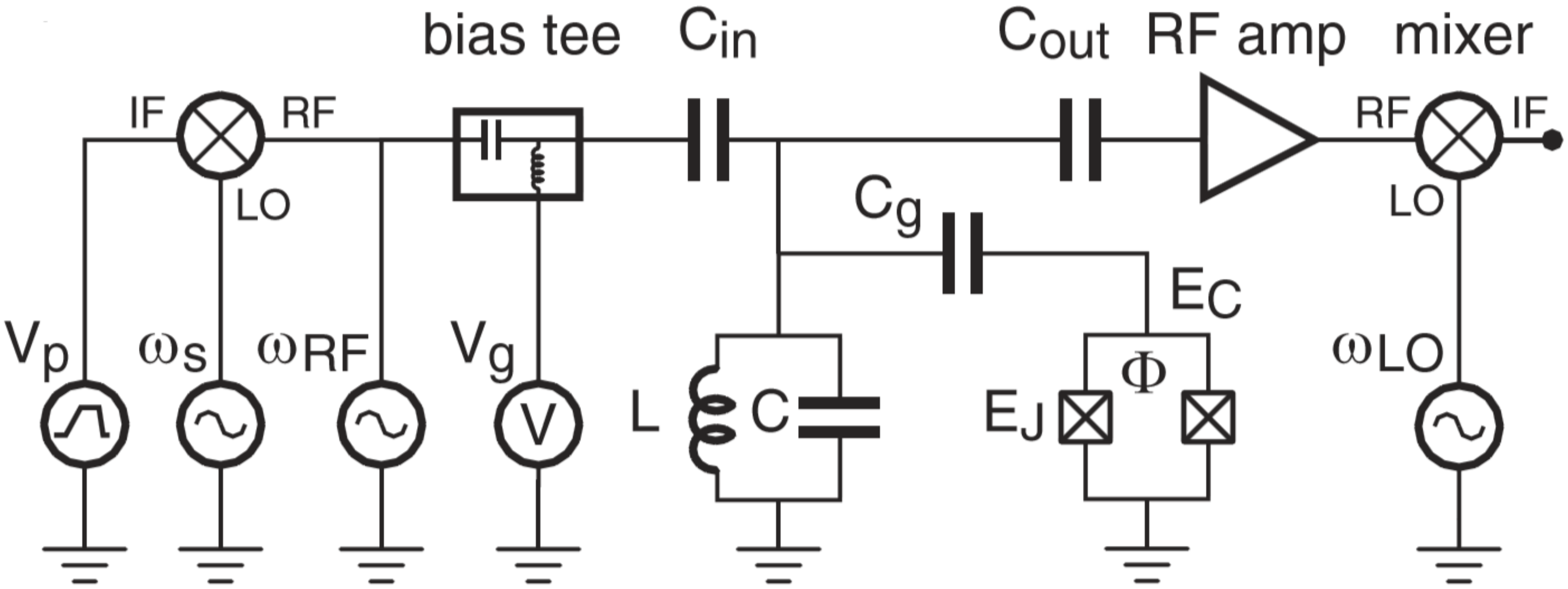
$$P_1(t) = \cos^2(\Omega t/2), \qquad P_2(t) = \sin^2(\Omega t/2),$$

- For $\omega = \omega_{21} - \Delta$ the Rabi frequency changes as $\Omega' = (\Omega^2 + \Delta^2)^{1/2}$

$$P_2(t) = (\Omega^2/\Omega'^2) \sin^2(\Omega' t/2),$$

Here: $\Omega = 2\pi\nu_{Rabi} = 2\sqrt{n_s}g$

n_s number of photons, g coupling



Controlling the qubit:

Microwave pulses with frequency $\omega_s \sim \omega_a$

Induce Rabi-oscillations

Shape of pulse given by V_p

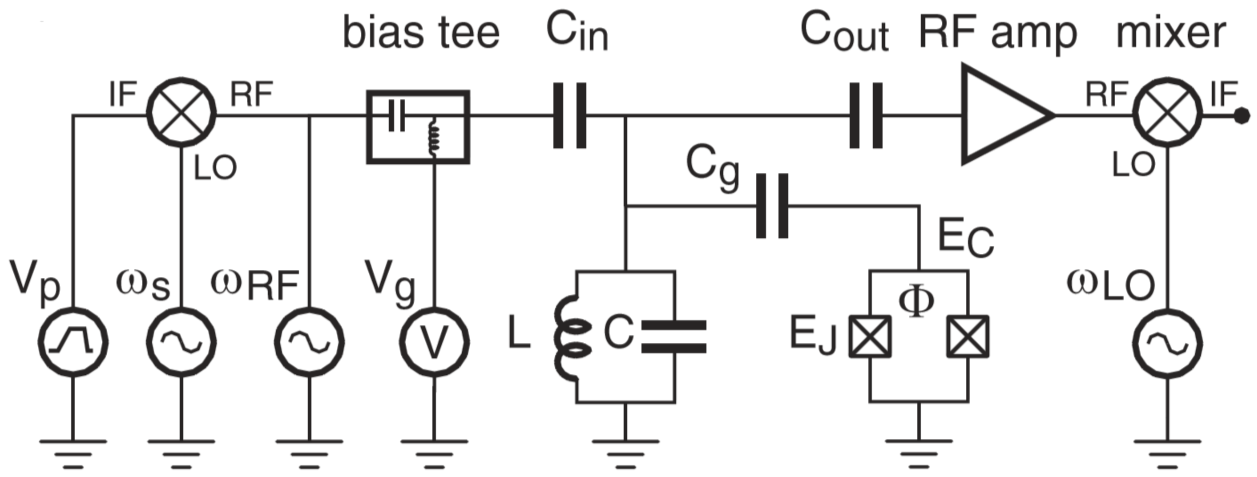
Measuring:

Microwave with frequency $\omega_{RF} \sim \omega_r$

Mixed and amplified with frequency ω_{LO}

Calculate population from phase shift

For $\omega_{RF} = \omega_r$ the population density is encoded in the phase shift φ in the measured signal

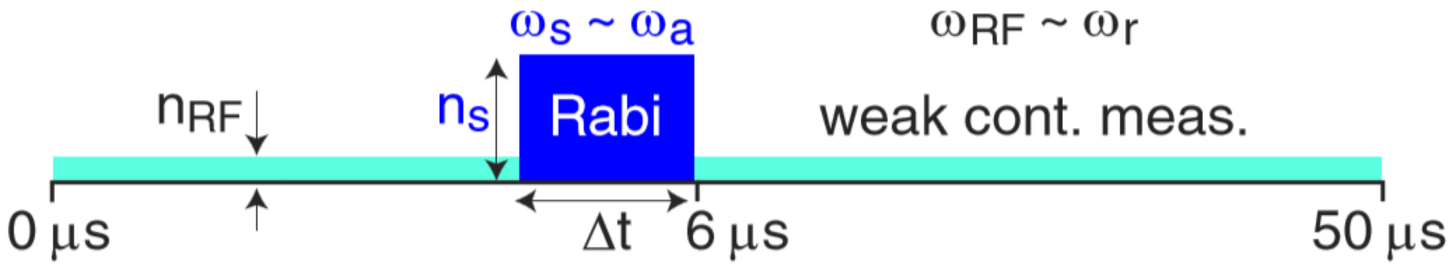


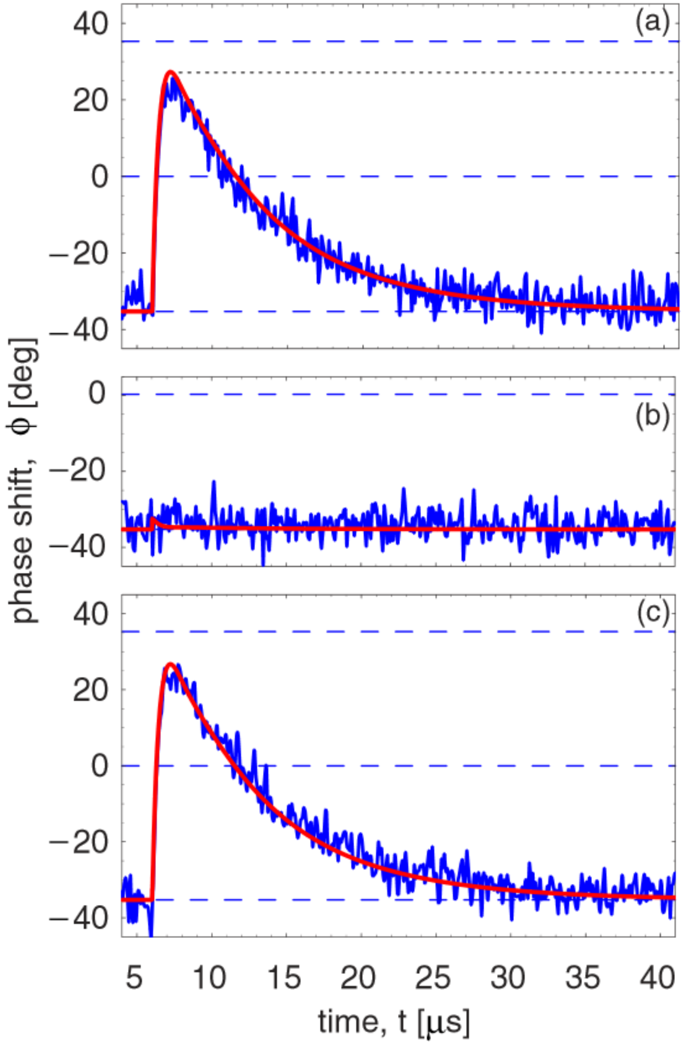
Calibrate meter:

1. Maximize detuning Δ → interaction $\frac{g^2}{\Delta} \rightarrow 0$
   define as φ = 0
2. System in ground state $|\downarrow\rangle$
3. Detuning Δ/2π ≈ -1.1 GHz corresponding to maximum Josephson energy E_J
   $\rightarrow \varphi_{|\downarrow\rangle}$
4. Long microwave pulse $\rightarrow P_{|\downarrow\rangle} = P_{|\uparrow\rangle} = 1/2$
   measure, expect φ = 0
   $\rightarrow \varphi_{|\uparrow\rangle} = -\varphi_{|\downarrow\rangle}$

Single shot measurements:

One short pulse





π-pulse (16ns)

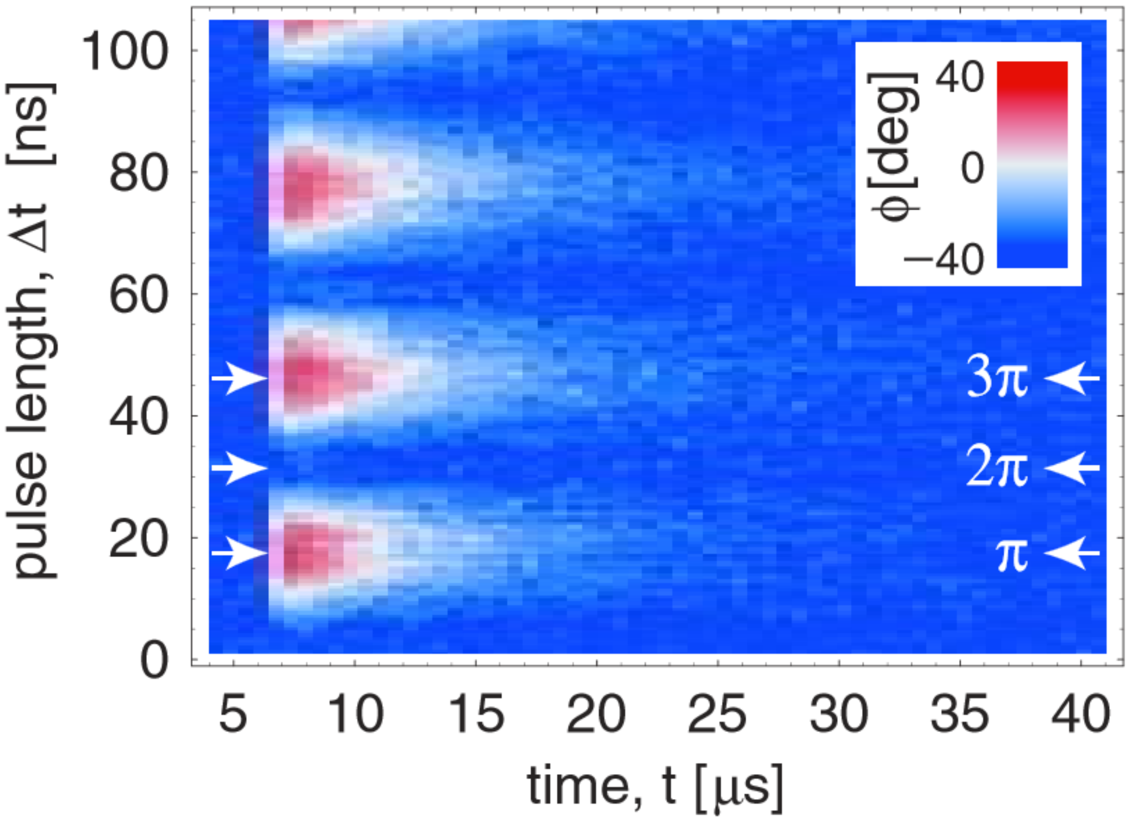
2π-pulse (32ns)

3π-pulse

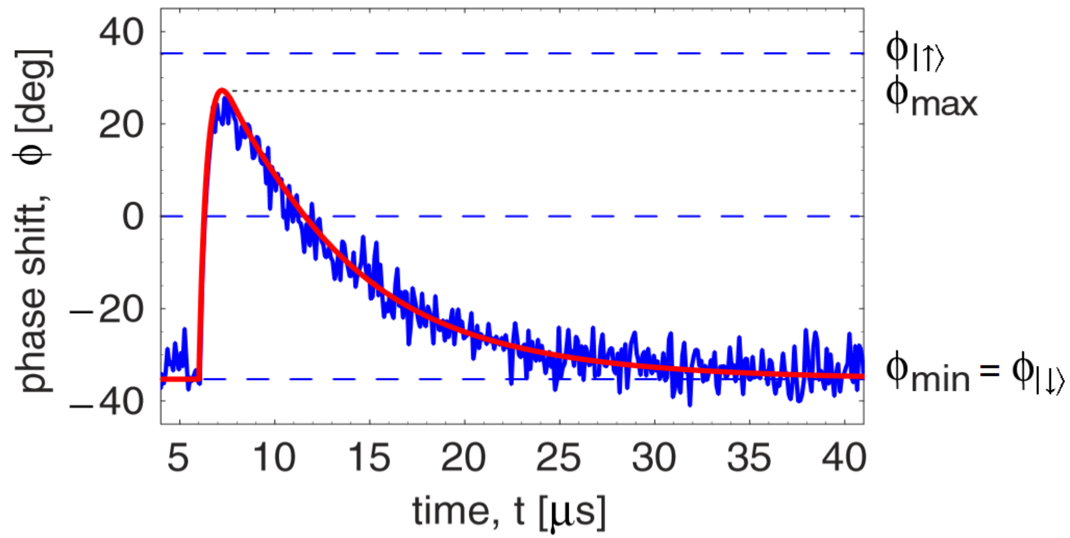
blue: measured data

red: prediction

Color density plot



blue: measured data, red: prediction

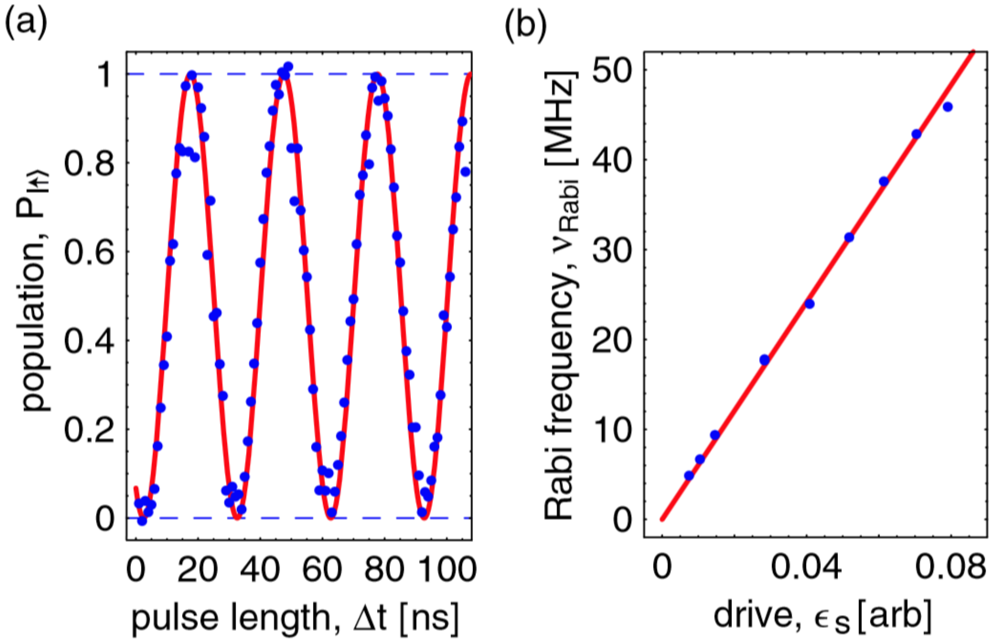


Contrast $C = \frac{\varphi_{max} - \varphi_{min}}{\varphi_{|\uparrow\rangle} - \varphi_{|\downarrow\rangle}} \sim 85\%$

Pulse duration much smaller than energy relaxation time

Calculate population density from normalized dot product between φ and prediction

Visibility 95 ± 6% (max. achieved population density)



(a) Decay over pulse length of 100ns very small → visible Rabi oscillations

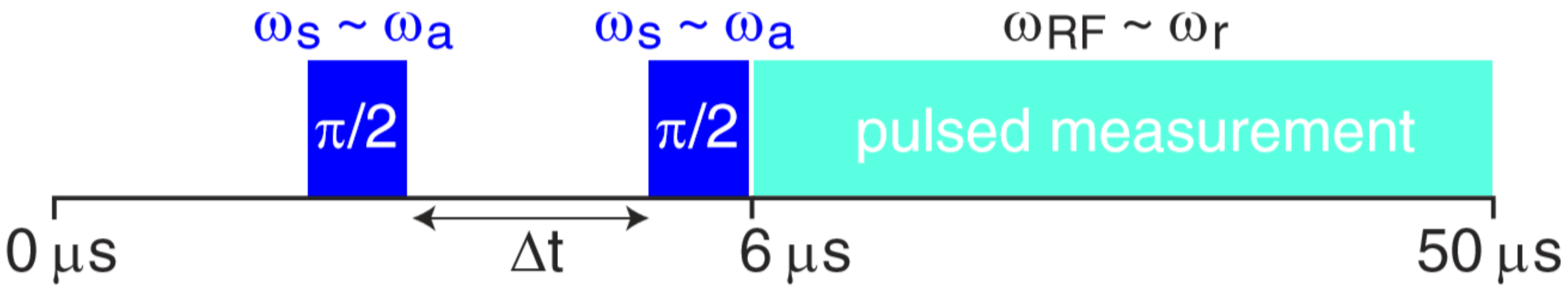
(b) Measured Rabi frequency ν_{Rabi} vs. pulse amplitude ε_s (blue dots) and linear fit

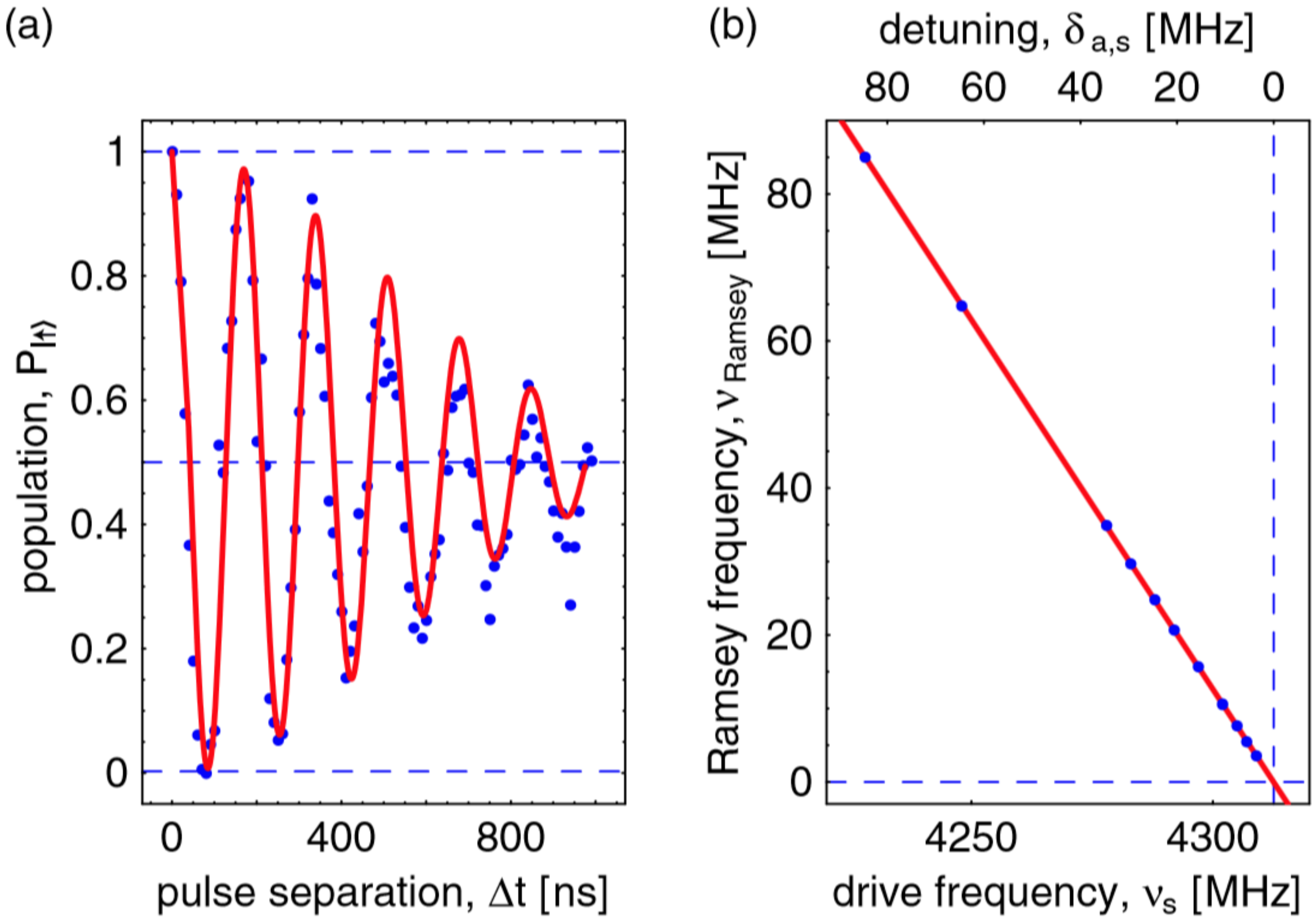
$$\varepsilon_s \propto \sqrt{n_s}$$

$$(\nu_{\text{Rabi}} = \sqrt{n_s}\, g/\pi)$$

Ramsey experiment:
Two π/2-pulses seperated by a longer time T
For a hard pulse and detuning Δ: $P(T,\Delta) = \cos^2 (T\,\Delta/2)$
The oscillating pattern is called **Ramsey fringes**

Here: Used to measure the coherence time of the Cooper pair box
Measurement beam only after second pulse to avoid dephasing





a) Ramsey fringes oscillate with detuning frequency $\delta_{a,s} = \omega_a - \omega_s$
Long coherence time $T_2 \sim 500$ns

qubit phase quality factor
$Q_\varphi = T_2\omega_a/2$

b) Linear dependence of ν_{Ramsey} on detuning $\delta_{a,s}$

Can be used to measure qubit transition frequency
$\omega_a = \omega_s + 2\pi\, \nu_{Ramsey}$