Dispersive Readout, Rabi- and Ramsey-Measurements for Superconducting Qubits

QIP II (FS 2018) Student presentation by Can Knaut
I. Cavity Quantum Electrodynamics and the Jaynes Cummings Model

II. Circuit Quantum Electrodynamics and Dispersive Qubit Readout

III. Experimental Realizations
   a. Rabi-Measurement
   b. Quantum Jumps
   c. Ramsey-Measurement

IV. Recent Developments

V. Summary
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Jaynes Cumming Model in Cavity Quantum Electrodynamics

- Cavity quantum electrodynamics (CQED) studies light-matter interaction between atoms and a single cavity.

- System can be described by the Jaynes-Cummings Hamiltonian:
  \[ H = \hbar \omega_r a + \hbar \Omega^2 \sigma^z + \hbar g (a \sigma^- + \sigma^+ a) \]

- The rates \( \kappa, \gamma \) denote cavity decay rate and spontaneous emission rate, respectively.

- Strong coupling regime requires \( g \gg \kappa, \gamma \).
For zero detuning, JC interaction lifts the degeneracy of the undressed atom-cavity system.

JC-Hamiltonian yields eigenstates ("dressed states"): 

$$ |+, n\rangle = \frac{1}{\sqrt{2}} |\downarrow, n\rangle + \frac{1}{\sqrt{2}} |\uparrow, n + 1\rangle $$

$$ |-, n\rangle = \frac{1}{\sqrt{2}} |\downarrow, n\rangle - \frac{1}{\sqrt{2}} |\uparrow, n + 1\rangle $$

Energy-splitting between dressed states amounts to:

$$ \Delta E = 2g\sqrt{n + 1} $$
Recall the Rabi interaction-Hamiltonian, describing a two-level system resonantly driven by a coherent drive $\vec{E}_0$ (e.g. laser):

$$H = \frac{\hbar \Omega_R}{2} \sigma_x,$$

with $\Omega_R = \frac{\mu_{\uparrow \downarrow} \vec{E}_0}{\hbar}$

Population of exited state oscillates with Rabi-frequency $\Omega_R \rightarrow$ Rabi oscillations:

$$P_{\downarrow} = \cos \left( \frac{\Omega_R}{2} t \right)^2$$

Bloch sphere depiction: Rotation around x-axis. Controlling interaction time can control state (e.g. chose $t'$ such that $\Omega_R t' = \frac{\pi}{2}$ will drive transition: $|\downarrow\rangle \rightarrow \frac{1}{\sqrt{2}} (|\uparrow\rangle + |\downarrow\rangle)$) $\rightarrow$ Rabi pulses
Jaynes Cumming Spectrum: Dispersive Regime

- Dispersive regime is characterized by large detuning between cavity frequency and atomic transition frequency:

\[ \frac{g}{\Delta} \ll 1 \text{ with } \Delta = \Omega - \omega_r \]

- In the dispersive regime, the JC Hamiltonian can be unitarily transformed and expanded to second order of \( g^2 \) into:

\[ H' = \hbar \left( \omega_r + \frac{g^2}{\Delta} \sigma^z \right) a^\dagger a + \frac{\hbar}{2} \left( \Omega + \frac{g^2}{\Delta} \right) \sigma^z + O(g^3) \]

- In the dispersive limit, the cavity frequency is shifted by \( \pm \frac{g^2}{\Delta} \)
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Circuit Implementation of CQED

- 2-D implementation of CQED consist of a coplanar transmission line resonator («cavity») capacitively coupled to a superconducting qubit («two-level atom») → circuit QED (cQED)

- Input- and output signals are coupled to the resonator via capacitive gaps of the resonator

- Amplitude and phase of output signal can be measured

Superconducting qubit

- Transmon qubit realized using a superconducting quantum interference device (SQUID) → Effective Josephson Energy becomes flux tunable using external flux line

- Presence and absence of Cooper pairs on the charge island in the qubits define states $|\uparrow\rangle$ and $|\downarrow\rangle$

- Hamiltonian of waveguide and qubit can be reduced to JC-Hamiltonian → cQED-Problem can be mapped to CQED-Problem

Blais et al.
In the dispersive regime, for a fixed drive frequency, resonance frequency of waveguide resonator is shifted depending on the qubit state.

If resonator drive frequency $\omega_{\mu w}$ is near resonance, phase shift of transmitted wave strongly depended on qubit state.

Measuring the phase of the transmitted wave can determine state of the qubit without destroying it → Quantum non-demolition (QND) measurement.

\[ H' = \hbar \left( \omega_r + \frac{g^2}{\Delta} \sigma^z \right) a\dagger a + \frac{\hbar}{2} \left( \Omega + \frac{g^2}{\Delta} \right) \sigma^z + O(g^3) \]
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$\omega_s$: Frequency of applied microwave pulse

$\omega_r$: Resonance frequency of transmission line resonator

$\omega_a$: Frequency of qubit: $E_a = \hbar \omega_a$

$\omega_{RF}$: Frequency of measurement signal
Rabi Measurement

- Apply Rabi pulse of arbitrary pulse length $\Delta t$ to qubit in ground state
- The extracted phase shift of the output signal $\phi$ exhibit Rabi oscillations
- Linear relationship between Rabi frequency and drive strength $\epsilon_S$ confirmed
- Decay of Rabi amplitude very small for pulse lengths up to 100 ns
Rabi Measurement: Determination of $T_1$ and Qubit Population

- Qubit in ground state is excited with $\pi$-pulse at frequency $\omega_s \approx \omega_a$

- Qubit-state depended phase shift allows to define $\phi_{\text{max}}, \phi_{\text{min}}$ corresponding to $|\downarrow\rangle, |\uparrow\rangle$

- Extracting decay constant yields qubit lifetime $T_1 \approx 7.3 \mu s$

- Comparing the measurement with the theoretical prediction (red line) yields measurement for $P_{|\uparrow\rangle}$
Observation of quantum jumps

- A continuously monitored qubit prepared in an excited state will decay to its ground state with an abrupt, stochastically occurring jump.
- Using fast, ultralow-noise parametric amplifier allows to resolve the quantum jumps.
- Rabi measurement used to produce single shot time traces.
- Simultaneous excitation and detection yields random signal.

Ramsey Fringes, Pulse Sequence

Ramsey Fringes, Determination of $T_2$

- Precession of Bloch vector around $z$-axis yields oscillation of $P_{\uparrow\uparrow}$

- Decoherence introduces time-dependence of qubit frequency $\omega_a = \omega_a(t)$ → Bloch vectors for given $\Delta t$ are fanning out

- Averaging of fanned out Bloch vectors yields Bloch vector of magnitude smaller than one ($P_{\uparrow\uparrow}$) decays

- Gaussian envelope of experimental results gives decoherence time $T_2 \approx 500$ ns

Wallraff et al.
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Rapid High-Fidelity Single-Shot Dispersive Readout

Rapid High-Fidelity Single-Shot Dispersive Readout

- (Slow) premeasurement and appropriate Rabi pulse prepares qubit in ground state
- Output signal is sampled in $8 \text{ ns}$ time bins
- Goal: Given a fixed maximal error probability, choose shortest $\tau$ realizing this probability
Rapid High-Fidelity Single-Shot Dispersive Readout

- Parametric amplifier allows to minimize noise added during amplification
- Purcell filter reduces spectral overlap between qubit and readout resonator
- 98.25% fidelity in 42ns readout time achievable (minimizing readout time)
- State discrimination now predominantly limited by qubit lifetime
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Summary

- JC model in the dispersive regime predicts split of resonance frequency of cavity accompanied with state depended phase shift of cavity photons.

- Measurement of phase shifts allows QND measurement of qubit state.

- Rabi measurement allow for depiction time-resolved oscillation of qubit population and determination of qubit lifetime $T_1$.

- Ramsey measurement allow determination of qubit decoherence time $T_2$.

- Recent developments of dispersive readouts use parametric amplifiers and Purcell filters to provide single-shot readout, increase fidelity and decrease measurement time.
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