Experimental Repetitive Quantum Error Correction

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Introduction

• Goal: protect quantum information from errors due to decoherence and other quantum noise.
• Essential to achieve fault-tolerant quantum computation.
• Deal with noise on stored quantum information, faulty quantum gates, faulty quantum preparation, and faulty measurements.

Key idea:

• **encode** the message - add redundant information to the message
• **redundancy** protects the message against the noise
• **decode** the message so that original message is recovered
Classical example

- Binary symmetric channel:

  ![Binary symmetric channel diagram]

  Figure: Binary symmetric channel.

- Encoding: repetition code \( \Rightarrow 0 \rightarrow 000 \) and \( 1 \rightarrow 111 \).
- Decoding: majority voting.
- New error probability: \( p_e = 3p^2 - 2p^3 \Rightarrow \) more reliable when \( p < 1/2 \).
Quantum error-correcting codes

Difficulties:

- **No cloning**: duplicating the quantum states is forbidden.
- **Errors are continuous**: appears that we need infinite precision.
- **Measurements destroy quantum information**: observations would make recovery impossible.

Example: Bit flip channel, $a |0\rangle + b |1\rangle \rightarrow a |1\rangle + b |0\rangle$

Probability $p$: $|\psi\rangle \rightarrow X |\psi\rangle$.

Encoding:

- $|\psi\rangle = a |0\rangle + b |1\rangle$
- $|0\rangle \rightarrow |0_L\rangle \equiv |000\rangle$
- $|1\rangle \rightarrow |1_L\rangle \equiv |111\rangle$
- $a |000\rangle + b |111\rangle$

Figure: Bit flip code encoding.
Error correction procedure

- **Error-detection**: error syndromes correspond to four projection operators:

  \[ P_0 \equiv |000\rangle \langle 000| + |111\rangle \langle 111| \text{ no error} \]
  \[ P_1 \equiv |100\rangle \langle 100| + |011\rangle \langle 011| \text{ bit flip on qubit 1} \]
  \[ P_2 \equiv |010\rangle \langle 010| + |101\rangle \langle 101| \text{ bit flip on qubit 2} \]
  \[ P_1 \equiv |001\rangle \langle 001| + |110\rangle \langle 110| \text{ bit flip on qubit 3} \]

  \[ \langle \psi | P_2 | \psi \rangle = 1 \Rightarrow \text{ bit flip on qubit 2} \]

  Syndrome measurement does not change the state \( a|010\rangle + b|101\rangle \)
  
  We cannot infer anything about the values of \( a \) and \( b \).

- **Recovery**: error syndrome tells us what procedure to use to recover the initial state.

- Probability for an error: \( 3p^2 - 2p^3 \), improvement for \( p < 1/2 \).
Phase flip code

Probability $p$: $a\,|0\rangle + b\,|1\rangle \longrightarrow a\,|0\rangle - b\,|1\rangle$.

Idea: switch to basis $|+\rangle = (a\,|0\rangle + b\,|1\rangle) / \sqrt{2}$, $|-\rangle = (a\,|0\rangle - b\,|1\rangle) / \sqrt{2}$.

Operator Z acts like a bit flip, it takes $|+\rangle$ to $|-\rangle$.

Encoding:

$|\psi\rangle = a\,|0\rangle + b\,|1\rangle$

$|0\rangle \longrightarrow |0_L\rangle \equiv |+++\rangle$

$|1\rangle \longrightarrow |1_L\rangle \equiv |---\rangle$

$a\,|+++\rangle + b\,|---\rangle$

Figure: Phase flip code encoding.
The first experimental implementation with multiple quantum error corrections (QEC) cycles.

- Measurement-free QEC algorithm.
- $^{40}Ca^+$ ions as qubits.
Each QEC cycle consists of:

- encoding the system qubit \( \{|0\rangle, |1\rangle\} \) and two auxiliary qubits (ancillas) into an entangled state,
- error incidence,
- detecting and correcting the error,
- resetting the ancillas.

**Figure:** Schematic view of QEC cycles.
• Initially: $|\psi\rangle = \alpha |+\rangle + \beta |-\rangle$, ancillas are both in $|1\rangle$.
• Encoding: mapping to $\alpha |++\rangle + \beta |--\rangle$.
• Phase flip changes $|\pm\rangle$ to $|\mp\rangle$.
• Decoding and correction: majority vote.
• Resetting the ancillas, while preserving the information of system qubit.

**Figure:** Quantum circuit for QEC - textbook implementation.

4 CNOT gates and 1 Toffoli gates $\rightarrow$ fidelities CNOT(92%) Toffoli (80%).
Major improvement with global Mølmer–Sørensen entangling operations (99%).

Optimization toolbox:

1. high-fidelity quantum operations,
2. optimized pulse sequence,
3. qubit-reset technique with negligible effect on the system of qubits.

**Figure:** Optimized pulse sequence for a single error-correction cycle.

- D commutes with any phase errors:
  
  \[
  D (\text{Error}) D^{-1} = (\text{Error}) DD^{-1} = (\text{Error}).
  \]

- U increases the space of optimized solutions.
• Method to construct states: \[ |\psi\rangle = \frac{1}{\sqrt{2}} \left[ e^{i\phi_g} |gg \cdots g\rangle + e^{i\phi_e} |ee \cdots e\rangle \right]. \]

• Address each ion through single laser, quantum logic gates through off-resonant laser pulses.

• Use it together with ancillas to perform encoding procedure.

• Independent of quantum vibrations and temperature.
Interaction Hamiltonian: \( H_{\text{int}} = \frac{\hbar \Omega}{2} \mathbf{E} \cdot \mathbf{S} = \sum_i \frac{\hbar \Omega_i}{2} \left( \sigma_+ e^{i \left[ \eta_i (a + a^\dagger) \right]} e^{-i \omega_i t} \right) + \text{h.c.} \).

Lamb-Dicke parameter: \( \eta = 2\pi z_0 / \lambda, \quad z_0 = \sqrt{\hbar / 2NM\nu} \).

Rabi frequency between \( |ggn\rangle \) and \( |een\rangle \) in the second order perturbation theory:

\[
\left( \frac{\Omega}{2} \right)^2 = \frac{1}{\hbar^2} \sum_m \left| \langle e e n | H_{\text{int}} | m \rangle \langle m | H_{\text{int}} | g g n \rangle \right|^2 \Rightarrow \Omega = \frac{(\Omega \eta)^2}{2(\nu - \delta)}. 
\]

\( \eta_1 = \eta_2 = \eta \) and \( \Omega_1 = \Omega_2 = \Omega, \quad \delta = \omega_1 - \omega_{eg} \).
When summed over all ions, the couplings lead to the spin Hamiltonian:

\[ H = 4 \frac{\chi}{\hbar} J_x^2, \quad \chi = \frac{\eta^2 \Omega^2 \nu}{2 (\nu^2 - \delta^2)}. \]

Time evolution operator: \( U(t) = \exp \left( -i \frac{4 \chi}{\hbar} J_x t \right) \) → population becomes distributed on all states.

**Figure:** Time evolution of the population of the joint ground state, and the joint excited state. Different curves correspond to different number of ions.

At time \( t = \frac{\pi}{8\chi} \) populations equal one half → maximally entangled states of high fidelity.
In our experiment: three \(^{40}\text{Ca}^+\) ions, each qubit in the \(|1\rangle = 4S_{1/2}(m_J = -1/2)\) and \(|0\rangle = 3D_{5/2}(m_J = -1/2)\) states.

Universal set of gates:

- Collective local operations: \(X(\theta) = \exp\left(-iS_x\theta/2\right)\) and \(Y(\theta) = \exp\left(-iS_y\theta/2\right)\).
- Single-qubit operations: \(Z_k(\theta) = \exp\left(-i\sigma_z^{(k)}\theta/2\right)\).
- Collective entangling Molmer-Sorensen operations: \(Y^2(\theta) = \exp\left(-iS^2_y\theta/2\right)\), with \(S_{x,y} = \sum_{k=1}^{3} \sigma_x^{(k)}\).

Experimental cycle:
1. cooling of ions to the motional ground state,
2. applying the manipulating laser pulses,
3. measuring the population of the qubit states.

Procedure repeated 1000 times.
Reset of ancillas

1. Ancilla qubits in $|0\rangle$ transferred into $|S'\rangle = 4S_{1/2}(m_J = 1/2)$.

2. Population in $|S'\rangle$ excited to the $4P_{1/2}(m_J = -1/2)$ by a laser beam at 397nm.

3. Population from the $4P_{1/2}$ spontaneously decays to the $4S_{1/2}$.

* population loss into $3D_{3/2}$ avoided by repump laser resonant with $3D_{3/2} - 4P_{1/2}$ transition.

Figure: Schematic of the reset procedure.
Operational quality of the QEC protocol

Expose it to correctable errors → single-qubit phase-flips.
Ideally, encoded qubit experiences identity operation.
Experimentally, process characterized by quantum process tomography →
process matrix $\chi$.
Process fidelity: $F_{\text{proc}} = Tr(\chi \cdot \chi_{\text{id}})$.

<table>
<thead>
<tr>
<th>Number of QEC cycles</th>
<th>No error $F_{\text{none}}$</th>
<th>Optimized no error $F_{\text{opt}}$</th>
<th>Single-qubit errors $F_{\text{single}}$</th>
<th>Optimized single-qubit errors $F_{\text{sopt}}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>97(2)</td>
<td>97(2)</td>
<td>–</td>
<td>–</td>
</tr>
<tr>
<td>1</td>
<td>87.5(2)</td>
<td>90.1(2)</td>
<td>89.1(2)</td>
<td>90.1(2)</td>
</tr>
<tr>
<td>2</td>
<td>77.7(4)</td>
<td>79.8(4)</td>
<td>76.3(2)</td>
<td>80.1(2)</td>
</tr>
<tr>
<td>3</td>
<td>68.3(5)</td>
<td>72.9(5)</td>
<td>68.3(3)</td>
<td>70.2(3)</td>
</tr>
</tbody>
</table>

Process fidelity for a single uncorrected qubit as well as for one, two, and three error-correction cycles. $F_{\text{none}}$ is the process fidelity without inducing any errors. $F_{\text{single}}$ is obtained by averaging over all single-qubit errors. $F_{\text{opt}}$ and $F_{\text{sopt}}$ are the respective process fidelities where constant operations are neglected.
Figure: Mean single-qubit process matrices $\bar{\chi}_n$ (absolute value) for $n$ QEC cycles with single-qubit errors. Transparent bars show the identity process matrix, and the red bar denotes a phase-flip error. Process matrices were reconstructed from a data set averaged over all measured expectation values.
Behavior of the QEC algorithm in the presence of two most prominent noise types:

- Uncorrelated: short laser pulse on the detection transition after encoding.
- Correlated: fluctuations in the magnetic field strength and the laser frequency.

Presence of noise type verified by the probability of simultaneous n-qubit flips.

**Figure:** Probability of simultaneous two-qubit phase flips as a function of the single-qubit phase flip probabilities for uncorrelated (square) and correlated (circle) noise measured by a Ramsey-type experiment.
**Figure:** Process fidelity of the QEC algorithm in the presence of correlated (circle) and uncorrelated (square) phase noise as a function of the single-qubit phase flip probability. The theory is shown for an unencoded qubit (solid line), a corrected qubit in presence of correlated (dashed line), and uncorrelated noise (dash-dot line).
Summary and outlook

- Implementation of a repeatable QEC algorithm with 3 trapped ions.
- Used global-entangling and local-qubit operations in an optimized pulse sequence.
- For uncorrelated errors, a corrected qubit performs better than an uncorrected qubit for a range of probabilities.
- Algorithm extended to 5 qubits.\(^1\)
- Work in other models:

\(^1\)Optimal control of entangling operations for trapped-ion quantum computing, V. Nebendahl, H. Häffner, and C. F. Roos, Phys. Rev. A 79, 012312