Lecture 3, March 8, 2018

Last week:

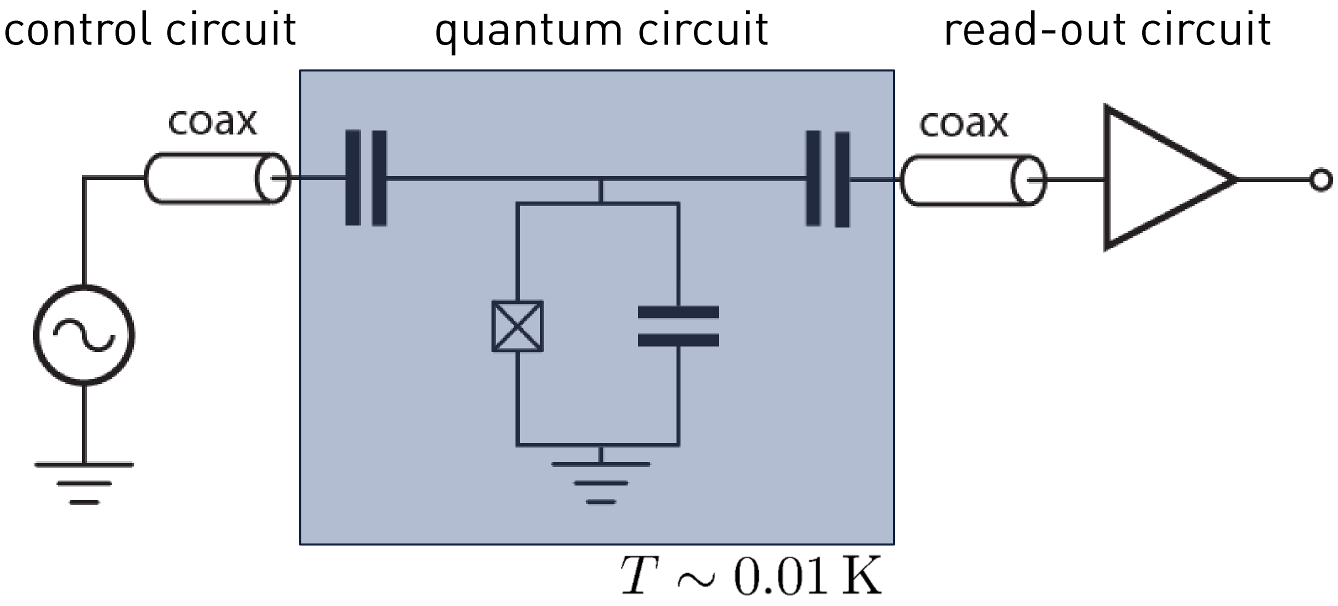
- Building a quantum computer from electronic circuits
  - Quantum electronic harmonic oscillators
    - The role of loss
  - Quantum bits: non-linear oscillators
    - The Josephson effect
    - The Cooper pair box qubit

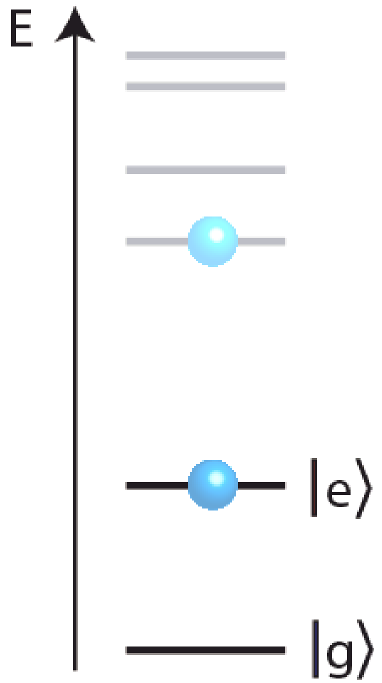
This week:

- Building a quantum computer from electronic circuits
  - Cavity quantum electrodynamics (QED)
    - General aspects
    - Circuit QED
  - Qubit Readout
    - Dispersive regime
    - Qubit Spectroscopy
    - Quantum jumps

Please take a seat
in the front center part of the lecture hall
if you do not mind.

How to Operate Electronic Circuits Quantum Mechanically?



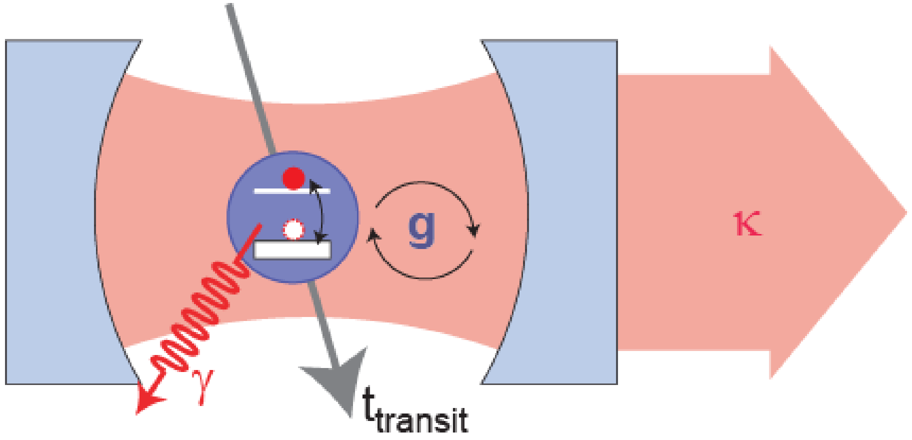


Recipe for preserving coherence:

- avoid dissipation
- work at low temperatures
- isolate quantum circuit from environment

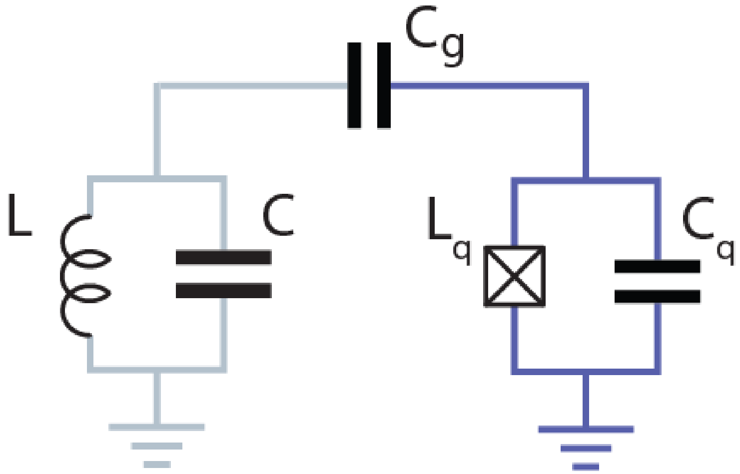
Review: M. H. Devoret, A. Wallraff and J. M. Martinis, *arXiv:condmat/0411172* (2004)

Cavity Quantum Electrodynamics (QED) with Superconducting Circuits



coherent quantum mechanics
with individual photons and qubits ...

... basic approach:

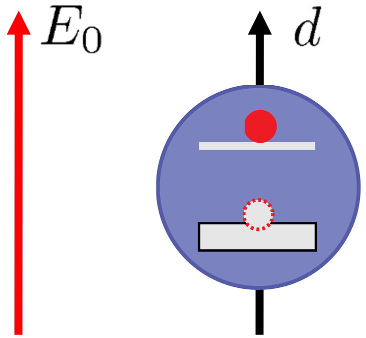


What is this good for?

- Isolating qubits from their electromagnetic environment
- Maintain addressability of qubits
- Reading out the state of qubits
- Coupling qubits to each other
- Converting stationary qubits to flying qubits

Controlling Light-Matter Interactions

challenging on the level of single (artificial) atoms and single photons



- mode-matching (controlling the absorption probability)
- single photon fields E_o (small in 3D)
- dipole moment d (usually small ~ ea_o)
- photon/dipole interaction $\hbar g \sim dE_0$ (usually small)

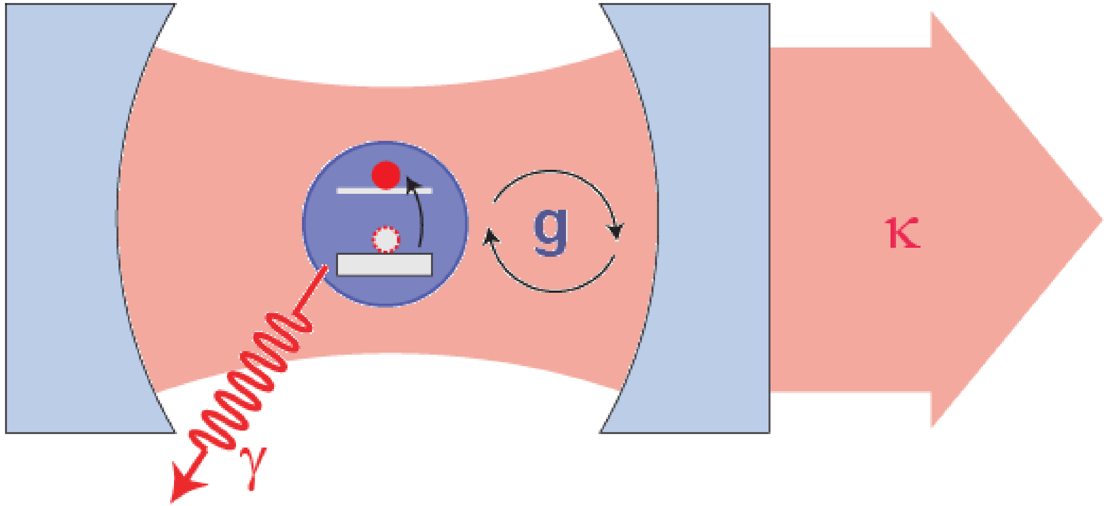
What to do?

- confine atom and photon in a cavity (cavity QED)
- engineer matter/light interactions, e.g. in solid state circuits

D. Walls, G. Milburn, Quantum Optics, *Spinger-Verlag, Berlin*, (1994)

Cavity Quantum Electrodynamics

interaction of atom and photon in a cavity



Jaynes-Cummings Hamiltonian

$$H = \hbar\omega_r \left(a^\dagger a + \frac{1}{2} \right) + \frac{\hbar\omega_a}{2}\sigma^z + \hbar g (a^\dagger \sigma^- + a\sigma^+) + H_\kappa + H_\gamma$$

strong coupling limit:

$$g = dE_0/\hbar > \gamma,\, \kappa\,,\, 1/t_{\text{transit}}$$

D. Walls, G. Milburn, Quantum Optics, Spinger-Verlag, Berlin (1994)

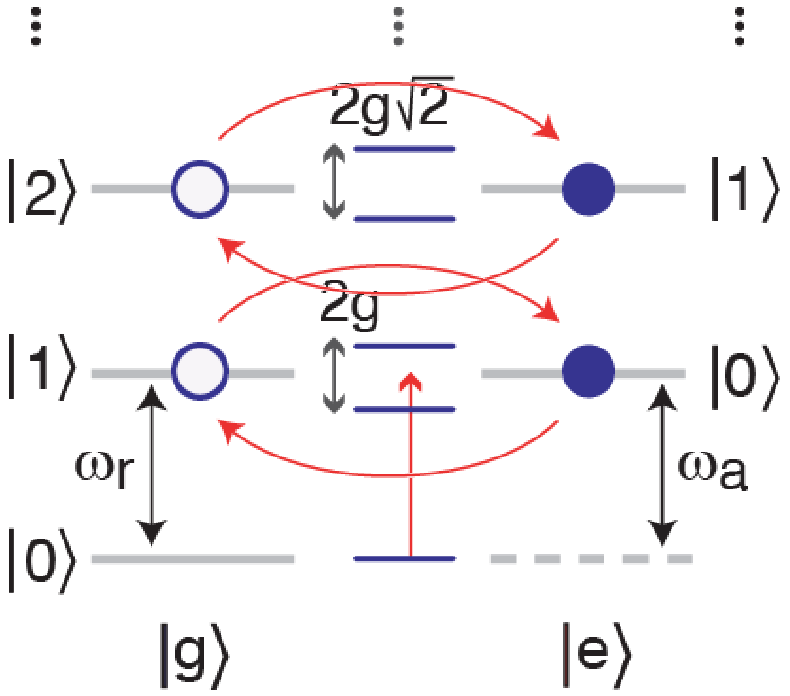
S. Haroche & J. Raimond, Exploring the Quantum, OUP Oxford (2006)

Dressed States Energy Level Diagram

$$H = \hbar\omega_r\left(a^\dagger a + \frac{1}{2}\right) + \frac{\hbar\omega_a}{2}\sigma^z + \hbar g(a^\dagger\sigma^- + a\sigma^+)$$

on resonance: $\omega_a - \omega_r = \Delta = 0$

strong coupling limit: $g = \frac{dE_0}{\hbar} > \gamma,\, \kappa$



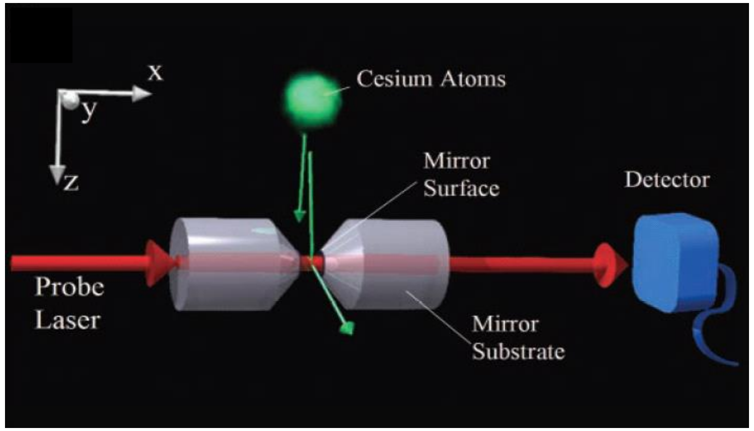
Jaynes-Cummings Ladder

atomic cavity QED reviews:

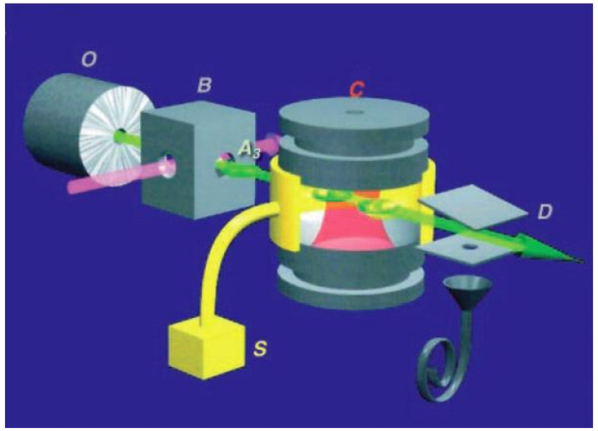
J. Ye., H. J. Kimble, H. Katori, *Science* **320**, 1734-1738 (2008)

S. Haroche & J. Raimond, Exploring the Quantum, *OUP Oxford* (2006)

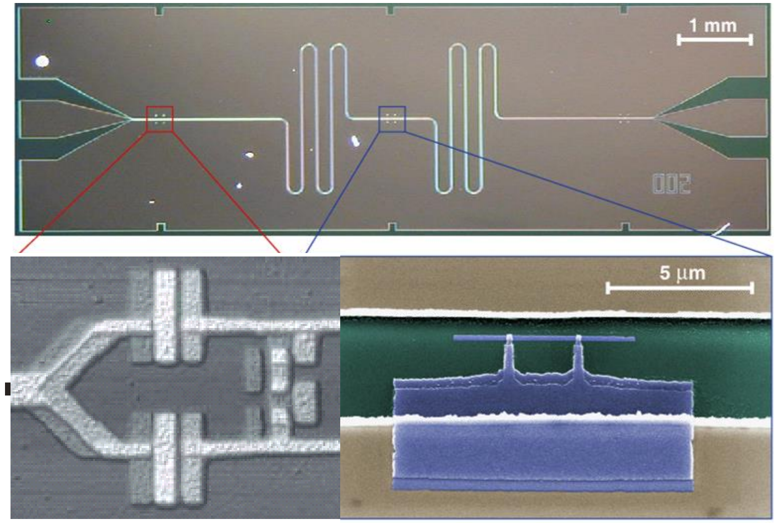
Strong Coupling Cavity Quantum Electrodynamics (QED)



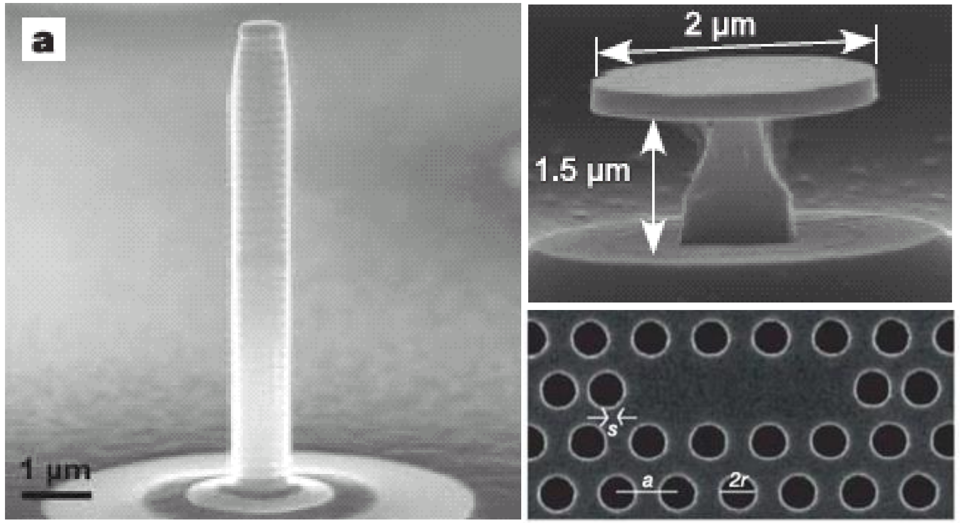
alkali atoms
MPQ, Caltech, ...



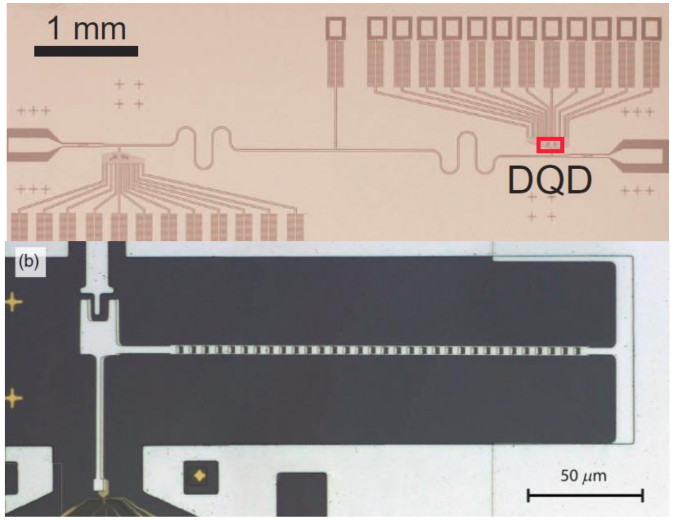
Rydberg atoms
ENS, MPQ, ...



superconductor circuits
Yale, Delft, NTT, ETHZ, NIST, ...



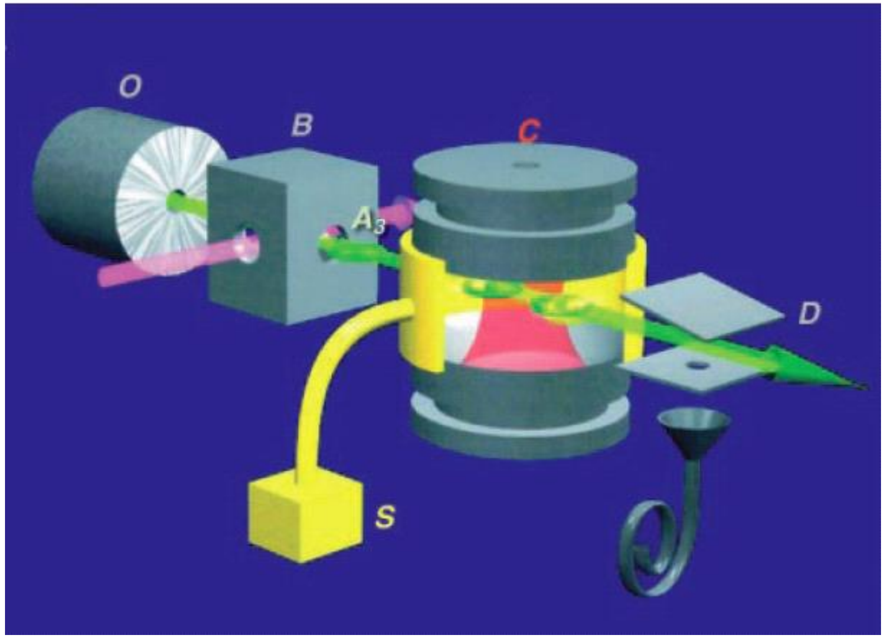
semiconductor optical quantum dots
Wurzburg, ETHZ, Stanford ...



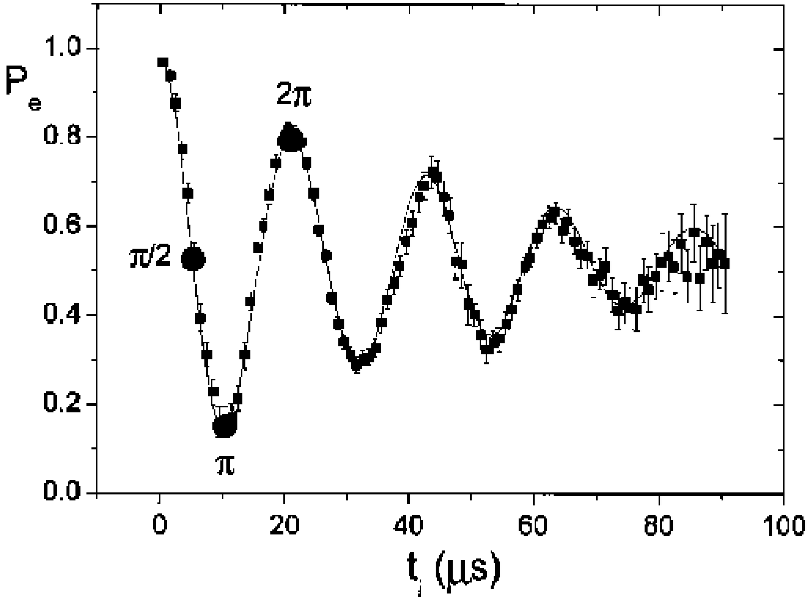
microwave freq. quantum dots
Princeton, ENS, ETHZ, ...

First Vacuum Rabi Oscillations with Rydberg Atoms

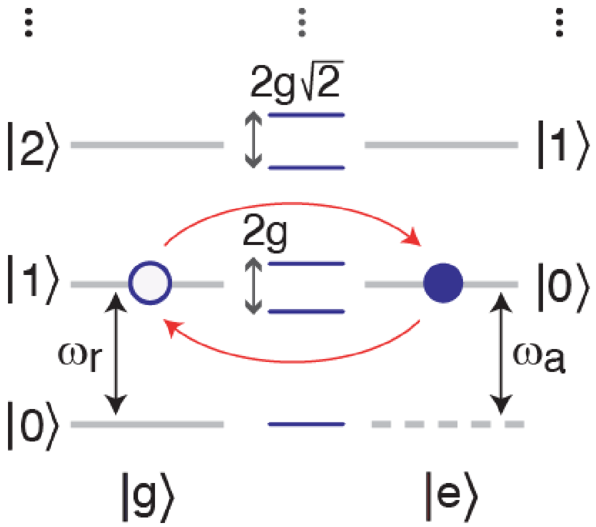
Setup:



Data:



Concept:

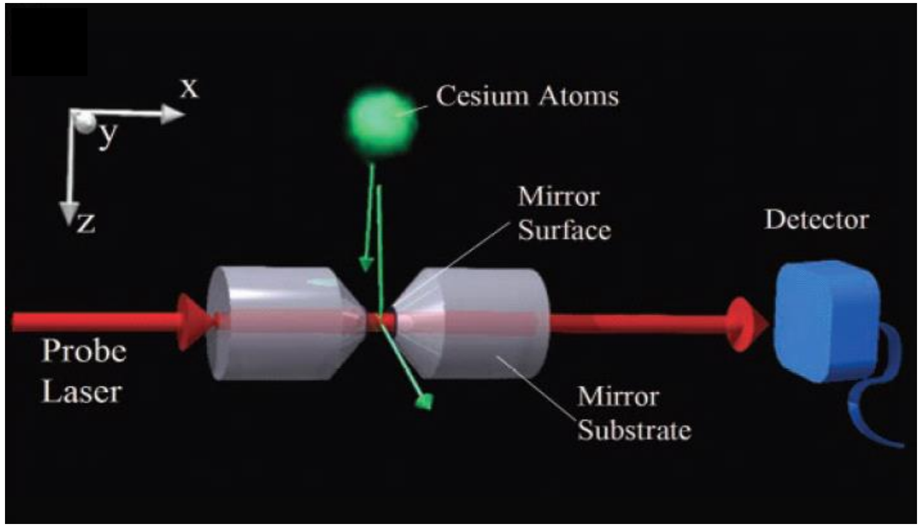


M. Brune, ..., J. M. Raimond, and S. Haroche, *Phys. Rev. Lett.* **76**, 1800 (1996)

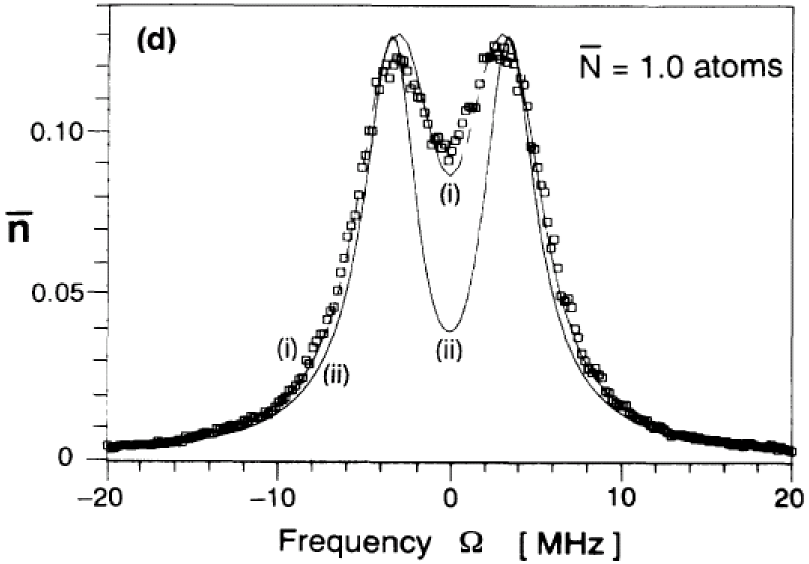
Review: J. M. Raimond, M. Brune, and S. Haroche, *Rev. Mod. Phys.* **73**, 565 (2001)

First Vacuum Rabi Mode Splitting with Alkali Atoms

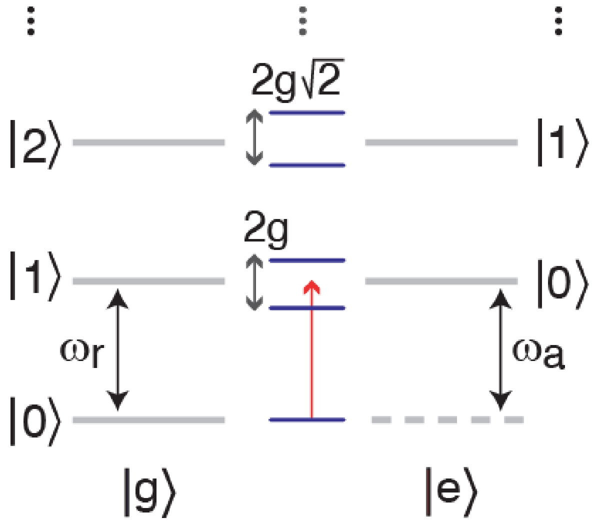
Setup:



Data:



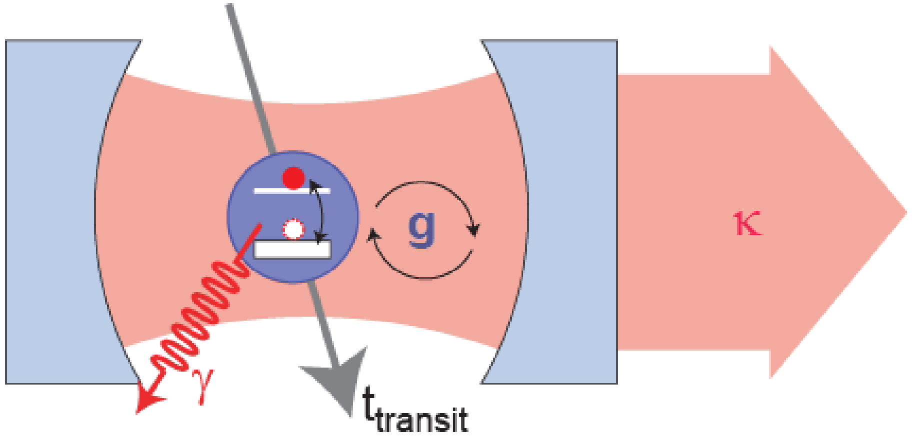
Concept:



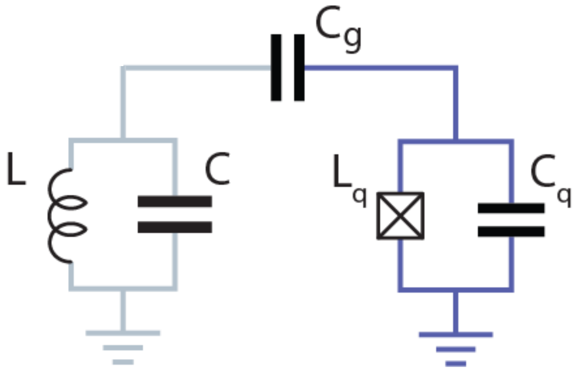
R. J. Thompson, G. Rempe, & H. J. Kimble, *Phys. Rev. Lett.* 68 1132 (1992)

A. Boca, ... , J. McKeever, & H. J. Kimble, *Phys. Rev. Lett.* 93, 233603 (2004)

Proposals for Cavity QED with Superconducting Circuits



coherent quantum mechanics
with individual photons
and qubits ...



a number of approaches suggested at the time:

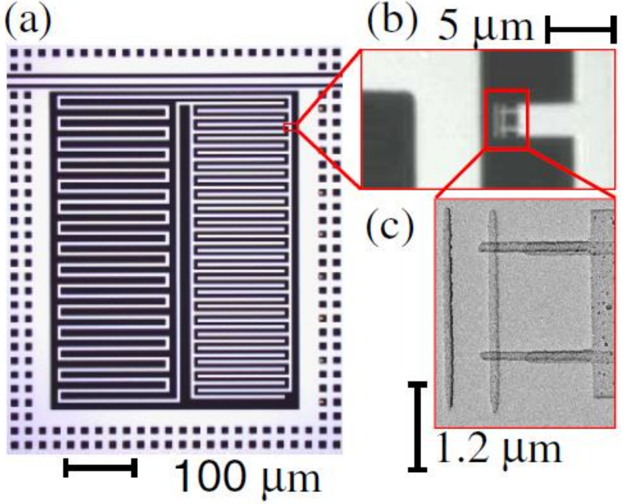
discrete LC circuits:
- Y. Makhlin, G. Schön, and A. Shnirman, Rev. Mod. Phys. 73, 357 (2001).
- O. Buisson and F. Hekking, in *Macroscopic Quantum Coherence and Quantum Computing*, edited by D. V. Averin, B. Ruggiero, and P. Silvestrini (Kluwer, New York, 2001).

large Josephson junctions:
- F. Marquardt and C. Bruder, Phys. Rev. B 63, 054514 (2001).
- F. Plastina and G. Falci, Phys. Rev. B 67, 224514 (2003).
- A. Blais, A. Maassen van den Brink, and A. Zagoskin, Phys. Rev. Lett. 90, 127901 (2003).

3D cavities:
- W. Al-Saidi and D. Stroud, Phys. Rev. B 65, 014512 (2001).
- C.-P. Yang, S.-I. Chu, and S. Han, Phys. Rev. A 67, 042311 (2003).
- J. Q. You and F. Nori, Phys. Rev. B 68, 064509 (2003).

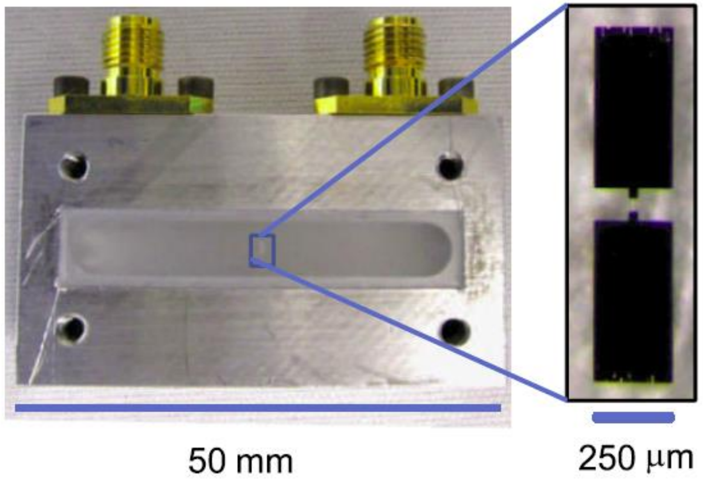
Circuit QED and its Different Realizations

lumped element resonator:



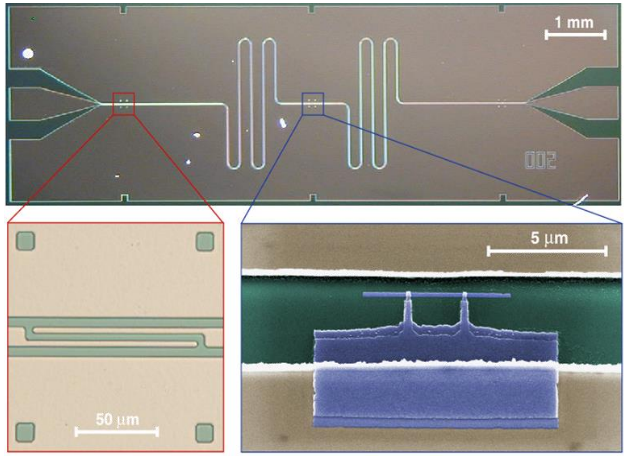
Kim *et al.*, *PRL* **106**, 120501 (2011)

3D cavity:



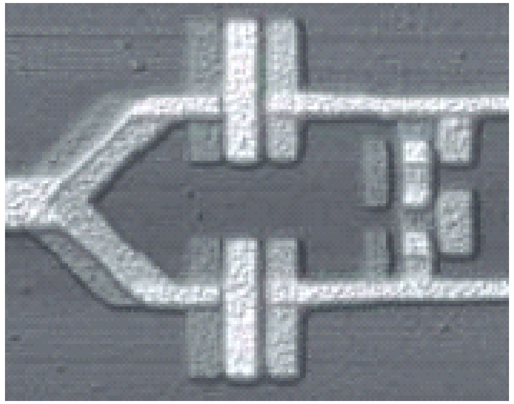
Paik *et al.*, *PRL* **107**, 240501 (2011)

planar transmission line:



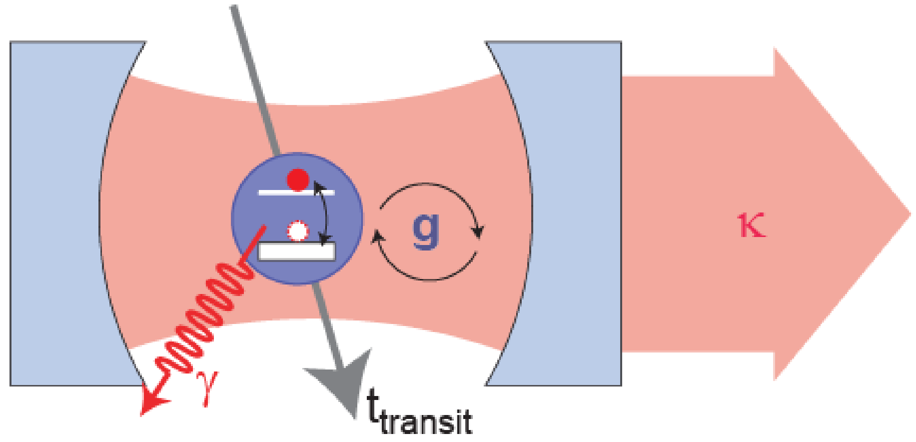
Wallraff *et al.*, *Nature* **431**, 162 (2004)

weakly nonlinear junction:



Chiorescu *et al.*, *Nature* **431**, 159 (2004)

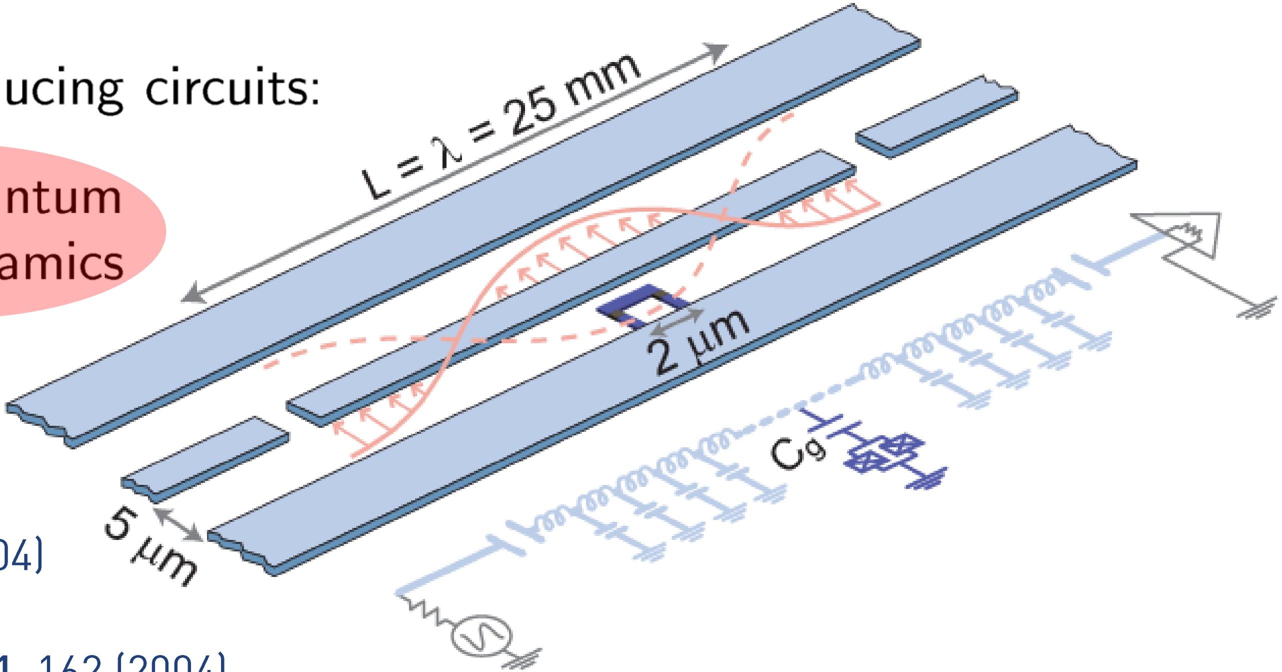
First Cavity QED Experiments with Superconducting Circuits



coherent quantum mechanics
with individual photons and qubits ...

... in superconducing circuits:

circuit quantum
electrodynamics



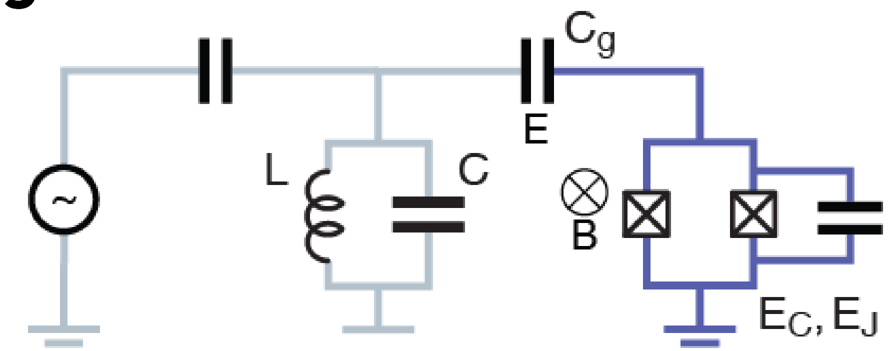
Elements:

- the cavity: a superconducting 1D transmission line resonator with large vacuum field E_0 and long photon life time $1/\kappa$
- the artificial atom: a Cooper pair box with large E_J/E_C with large dipole moment d and long coherence time $1/\gamma$

A. Blais, et al. ,
PRA **69**, 062320 (2004)

A. Wallraff et al.,
Nature (London) **431**, 162 (2004)

Qubit/Photon Coupling



Hamilton operator of qubit coupled to resonator:

$$\hat{H} = \hbar\omega_r(a^\dagger a + \frac{1}{2}) + 4E_C(\hat{N} - \hat{N}_g))^2 - E_J \cos\hat{\phi} + 2\beta e V_{rms}(a + a^\dagger)\hat{N}$$

with $V_{rms} = \sqrt{\hbar\omega_r/2C}$

$\beta \approx C_g/\sqrt{(C_g + C)(C_g + C_q)}$ and C_q the capacitance of the qubit.

A. Blais, et al. , *PRA* **69**, 062320 (2004)

Jaynes-Cummings Hamiltonian

Consider the two lowest lying (qubit) states of the Cooper pair box and use

$$\langle i|\,\hat N\,|i+1\rangle \approx \sqrt{i+1}\left(\frac{E_J}{32E_C}\right)^{1/4}$$

for the matrix elements of N with respect to the qubit eigenstate, to find

$$\hat H \quad = \quad \hbar\omega_r(a^\dagger a+\frac{1}{2})+\hbar\frac{\omega_{ge}}{2}\hat{\sigma_z}+\hbar g(a+a^\dagger)\hat{\sigma_x} \qquad \text{with} \qquad g\approx\frac{1}{2}\beta\sqrt{\omega_r\omega_{ge}}$$

Quantum Rabi Hamiltonian

Coupling term in the rotating wave approximation (RWA) qubit raising and lowering operators $\hat\sigma_x=\hat\sigma^+ +\hat\sigma^-$

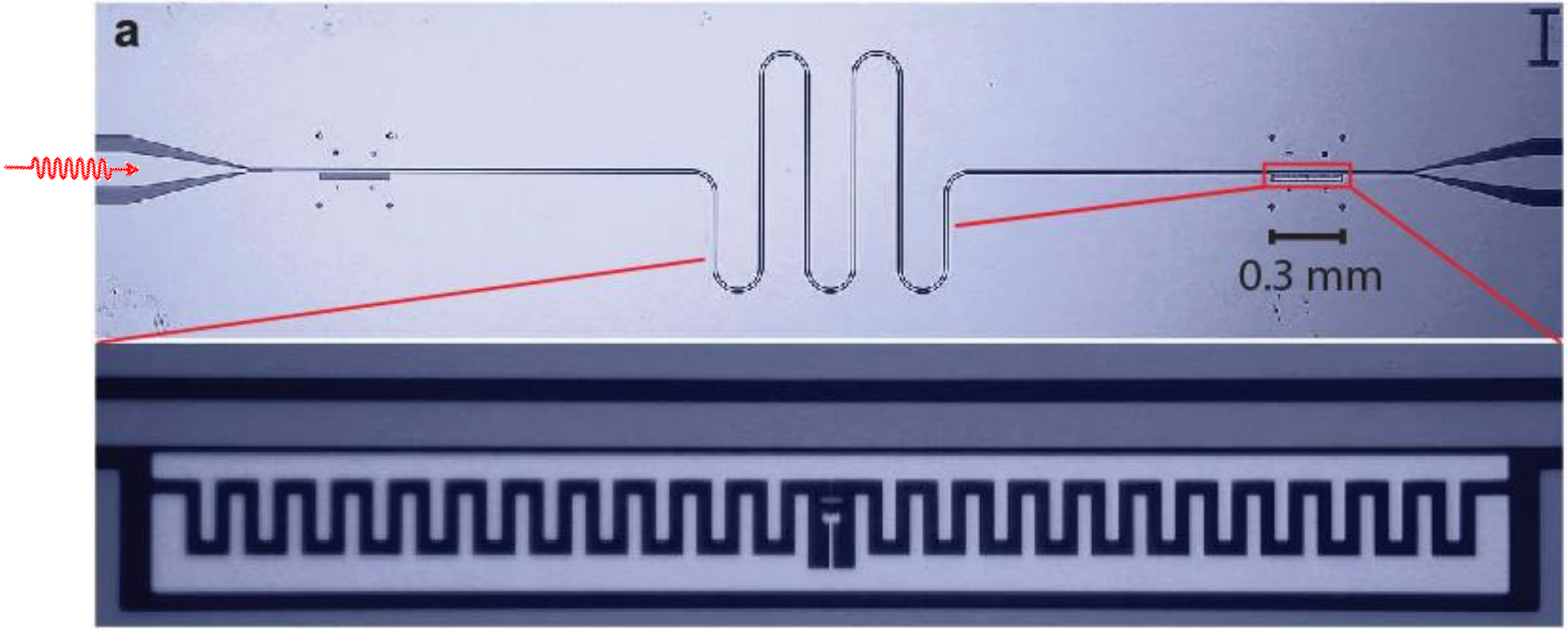
$$\hat H_g \quad = \quad \hbar g(\hat a^\dagger\hat\sigma^- + \cancel{\hat a}\hat\sigma^- + \hat a^\dagger\cancel{\hat\sigma}^+ + \hat a\hat\sigma^+) \qquad \approx \hbar g(\hat a^\dagger\hat\sigma^- + \hat a\hat\sigma^+)$$

Jaynes Cummings Hamiltonian

Coupling strength
Vacuum-Rabi frequency $\nu_R=\dfrac{2g}{2\pi}\approx 1\dots300\,\mathrm{MHz}$ $\qquad g\gg[\kappa,\gamma]$ possible!

A. Blais, et al. , *PRA* **69**, 062320 (2004)

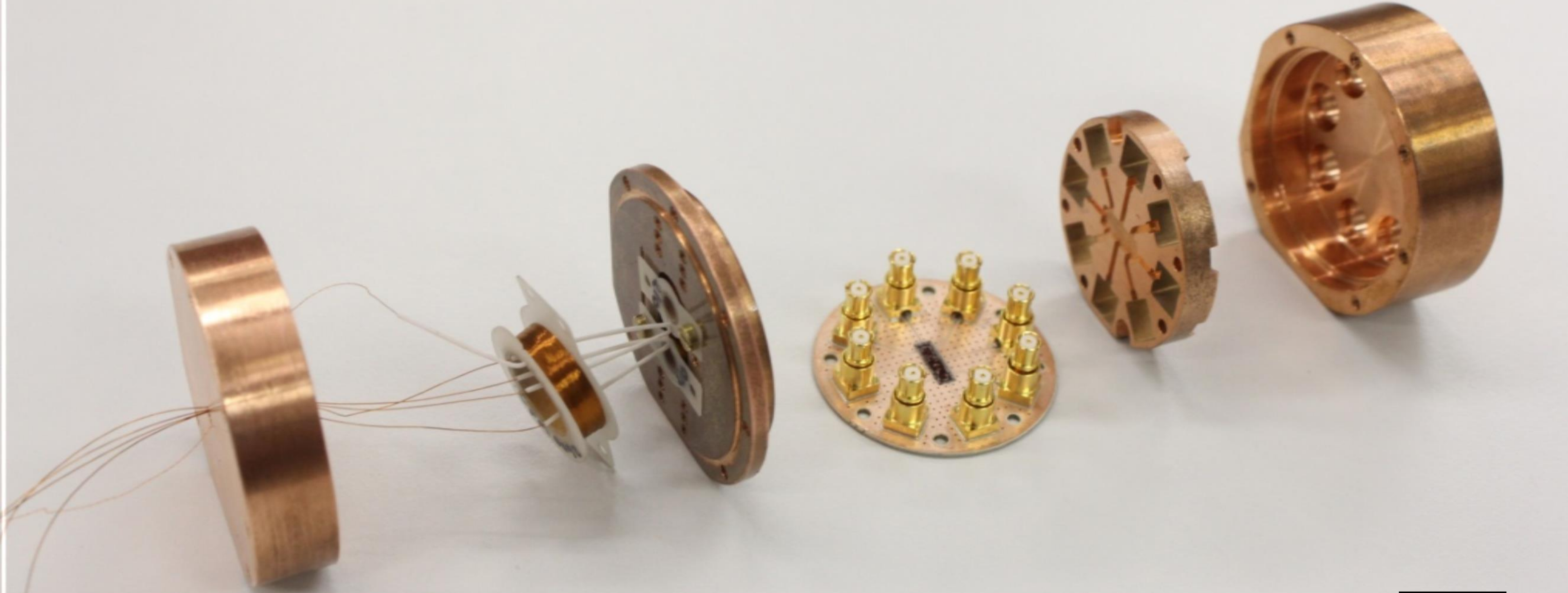
Basic Circuit QED Experiments



First experiment: A. Wallraff *et al., Nature (London)* **431**, 162 (2004)
device fabrication: L. Frunzio et al., *IEEE Trans. on Appl. Supercon.* **15**, 860 (2005)

Electron Micrograph by J. Mlynek et al., Quantum Device Lab, ETH Zurich (2012)

Sample Mount

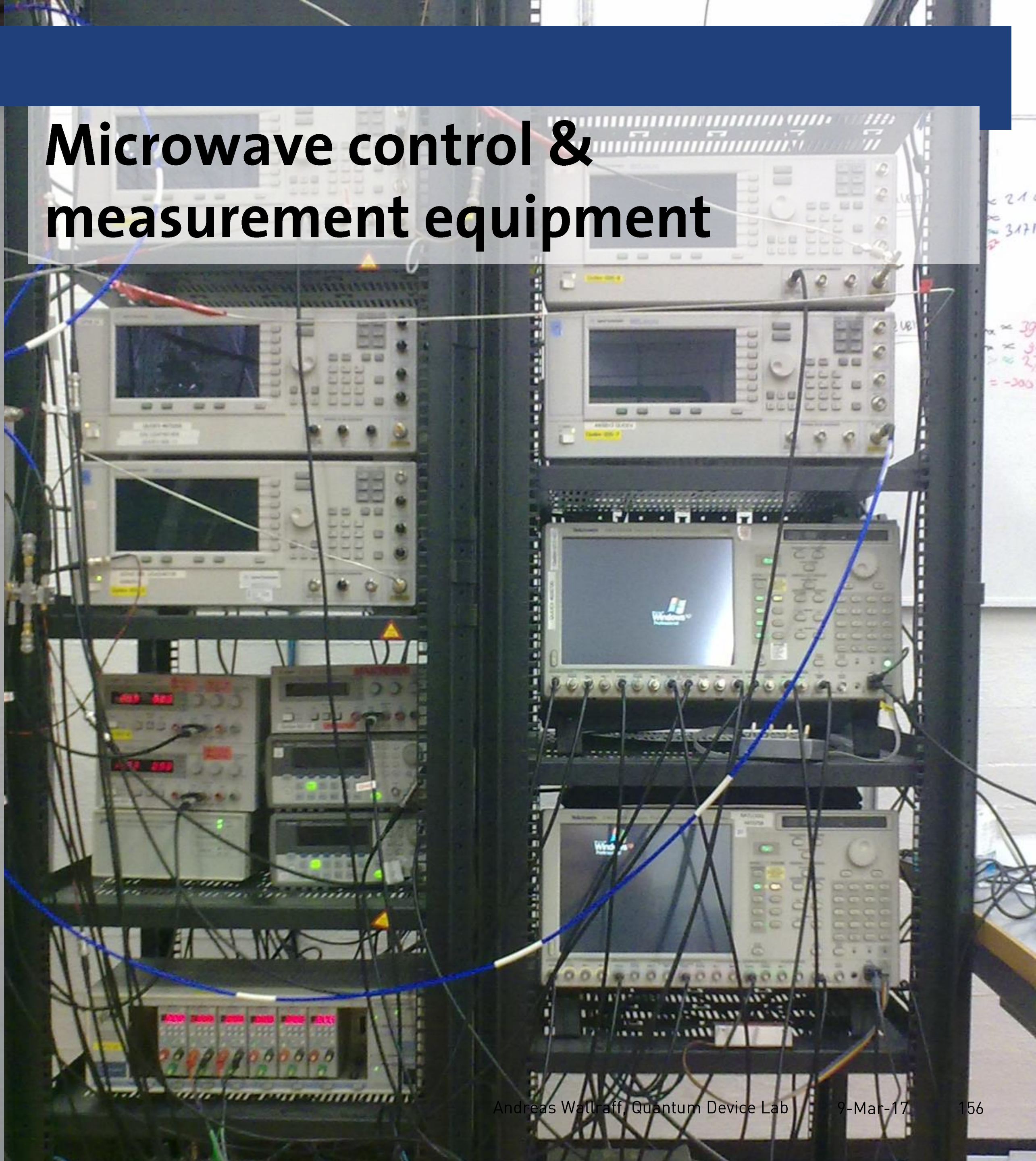


~ 2 cm

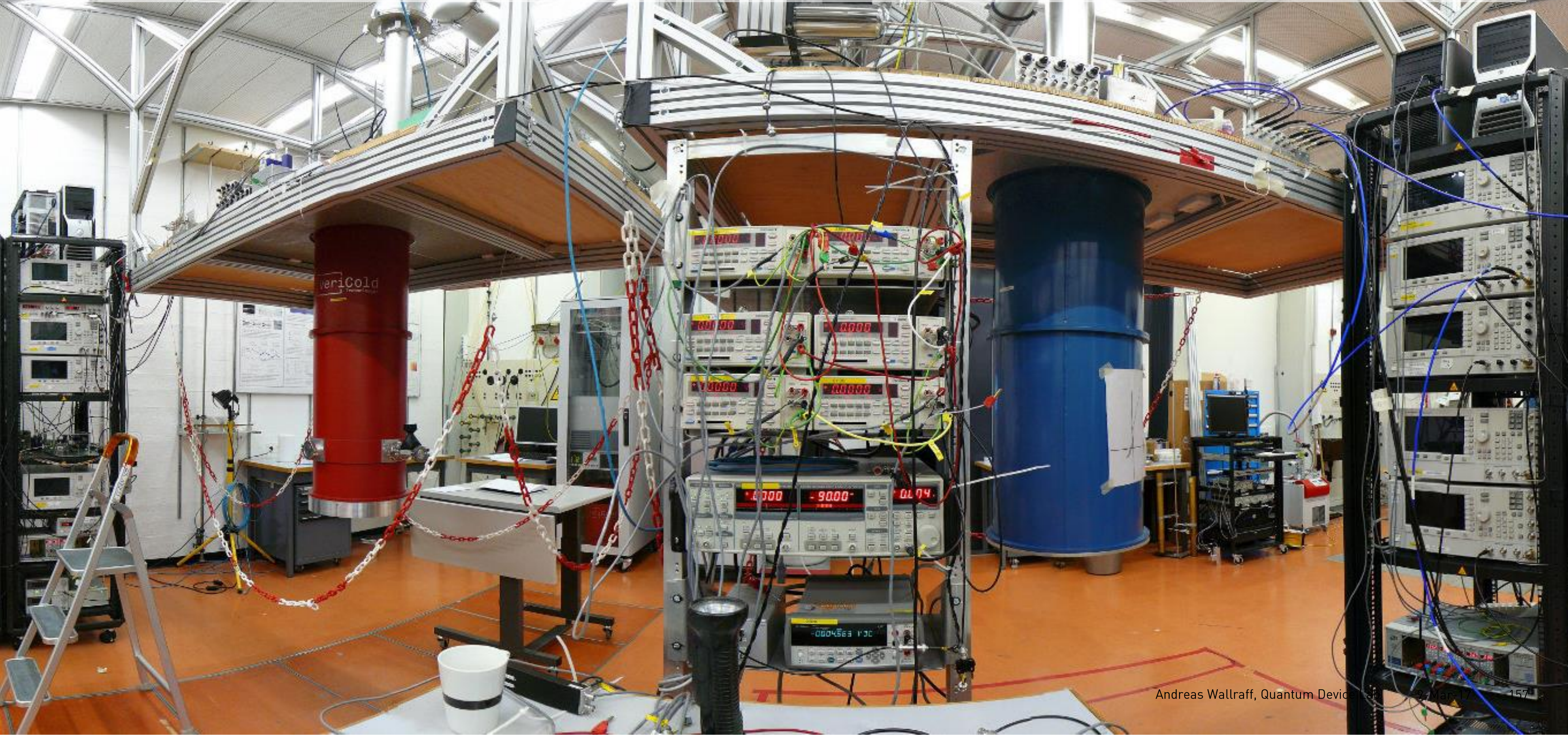
M. Peterer *et al.*, Quantum Device Lab, ETH Zurich (2012)

Cryostat for temperatures down to 0.02 K

Microwave control & measurement equipment



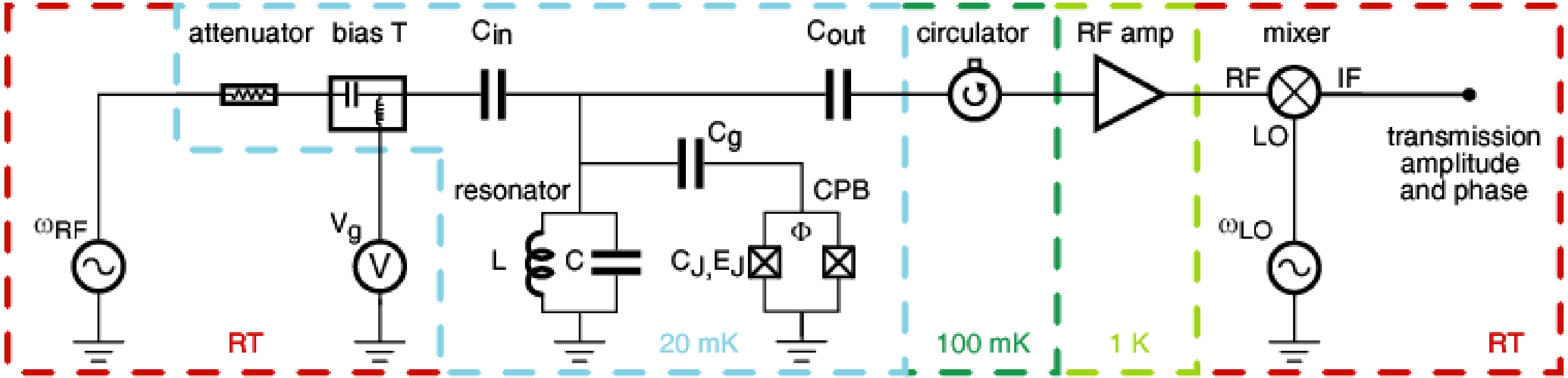
A Cavity QED with Circuits Lab: Quantum Device Lab, ETH Zurich

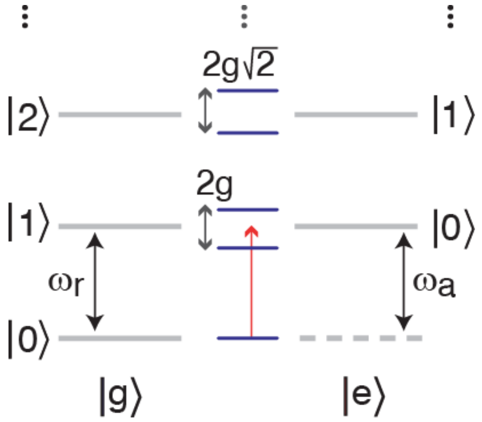


Why and How to Detect Microwave Fields at the Single Photon Level?

- average power to be detected

$$\langle n = 1\rangle \hbar\omega_r\kappa \approx P_{RF} = -140\,\mathrm{dBm} = 10^{-17}\,\mathrm{W}$$

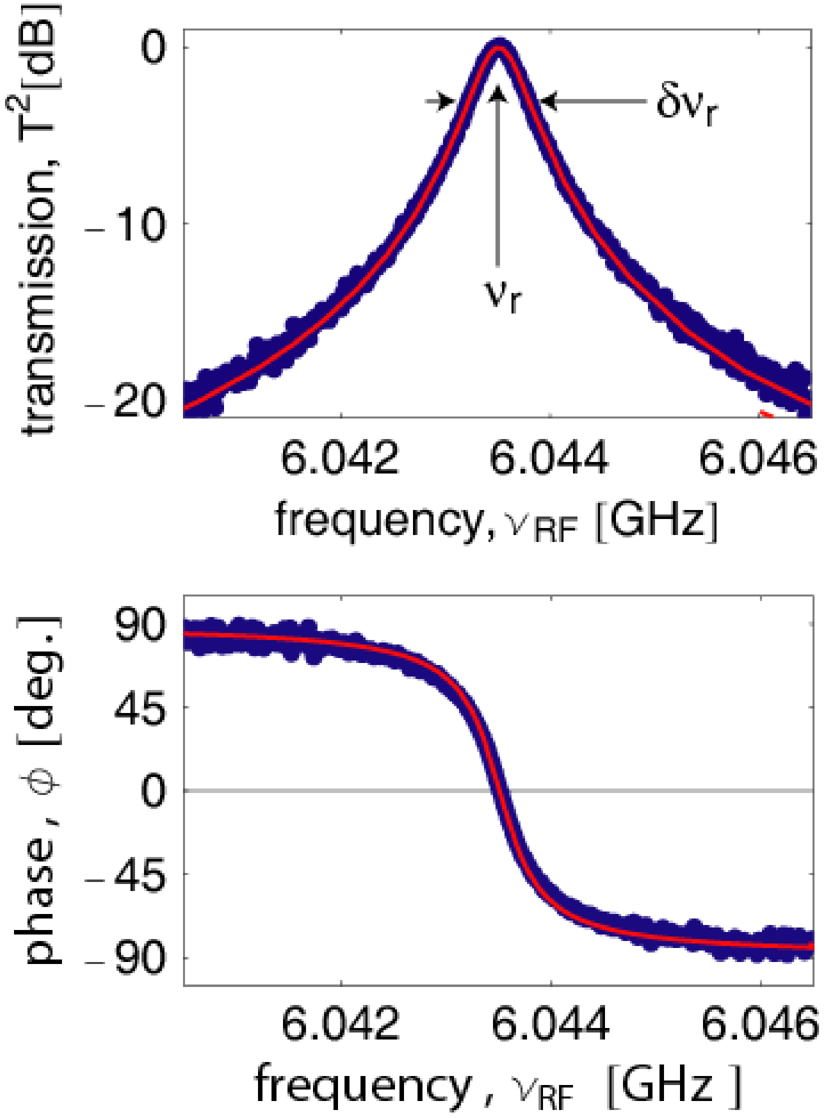




- controllable intra-cavity photon number (n ~ 10^3 to n << 1)
- efficient with cryogenic low noise HEMT amplifier (T_N = 6 K) and parametric amplifiers (quantum noise limited)
- prevent leakage of thermal photons (cold attenuators and circulators)

A. Wallraff *et al.*, *Nature (London)* **431**, 162 (2004)

Resonator Spectroscopy in Transmission



resonance frequency:

$$\nu_r = 6.04\,\mathrm{GHz}$$

quality factor:

$$Q = \frac{\nu_r}{\delta\nu_r} \approx 10^4$$

photon decay rate:

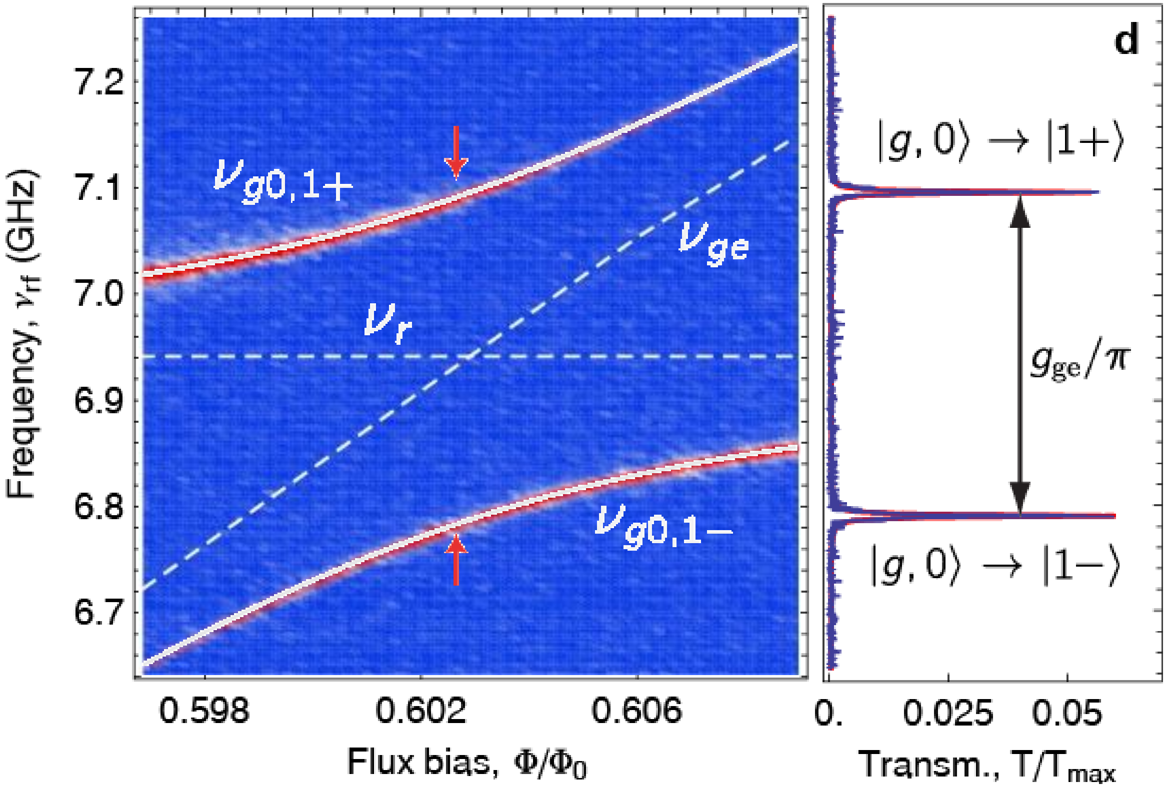
$$\frac{\kappa}{2\pi} = \frac{\nu_r}{Q} \approx 0.8\,\mathrm{MHz}$$

photon lifetime:

$$T_\kappa = 1/\kappa \approx 200\,\mathrm{ns}$$

Resonant Vacuum Rabi Mode Splitting

Measured with one photon (*n=1*):

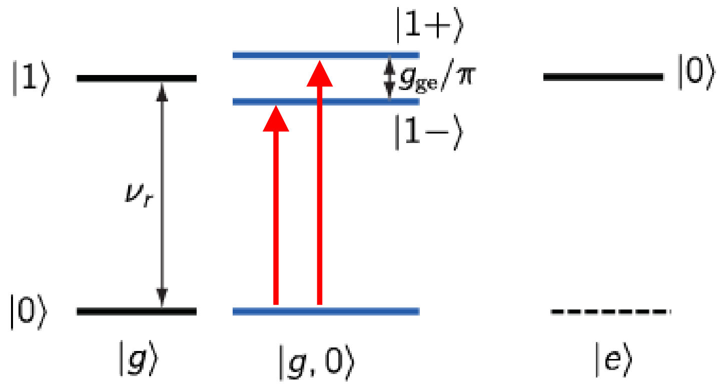


very strong coupling:

$$g_{ge}/\pi = 308\,\mathrm{MHz}$$

$$\kappa, \gamma < 1\ \mathrm{MHz}$$

$$g_{\mathrm{ge}} \gg \kappa, \gamma$$



forming a
'molecule' of a
qubit and a photon

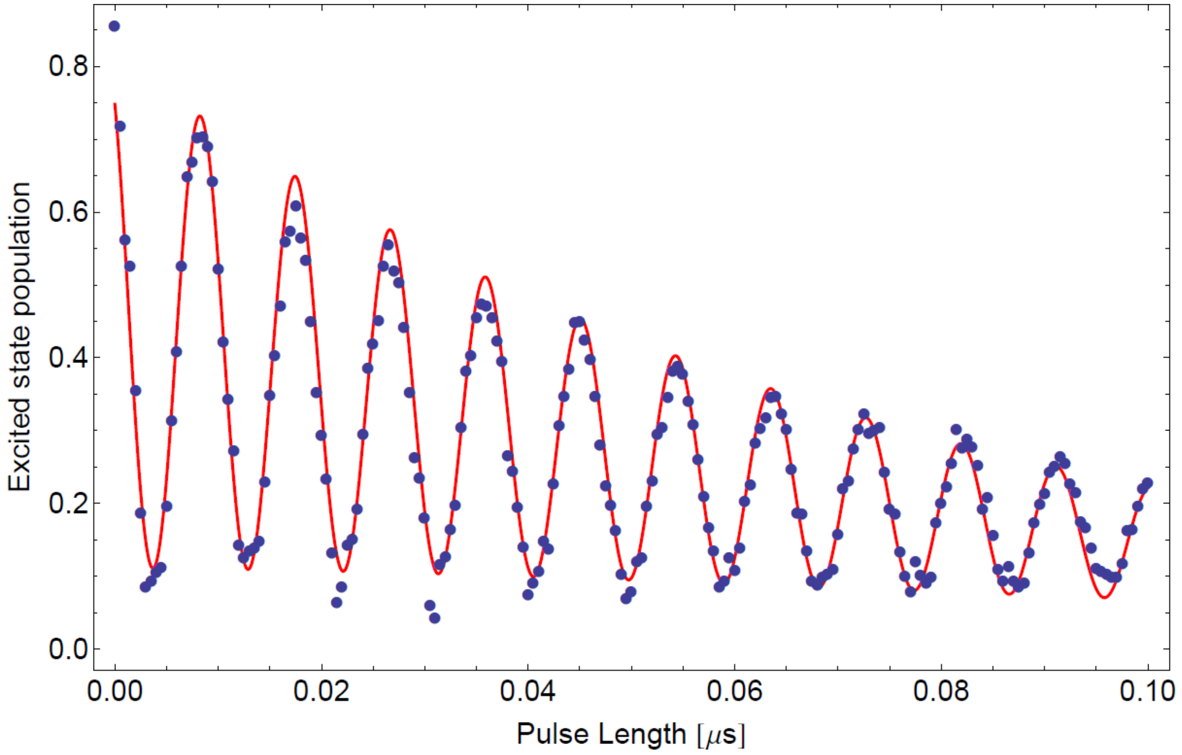
first demonstration in a solid: A. Wallraff et *al.*, *Nature (London)* **431**, 162 (2004)

this data: J. Fink et al., *Nature (London)* **454**, 315 (2008)

R. J. Schoelkopf, S. M. Girvin, *Nature (London)* **451**, 664 (2008)

Vacuum Rabi Oscillations

Measured with one photon (*n=1*):



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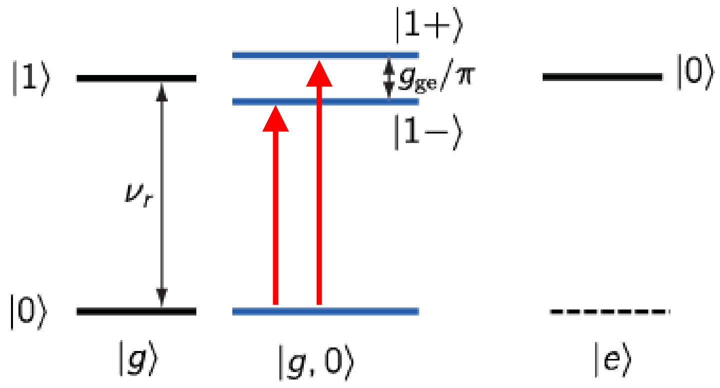


forming a 'molecule' of a qubit and a photon

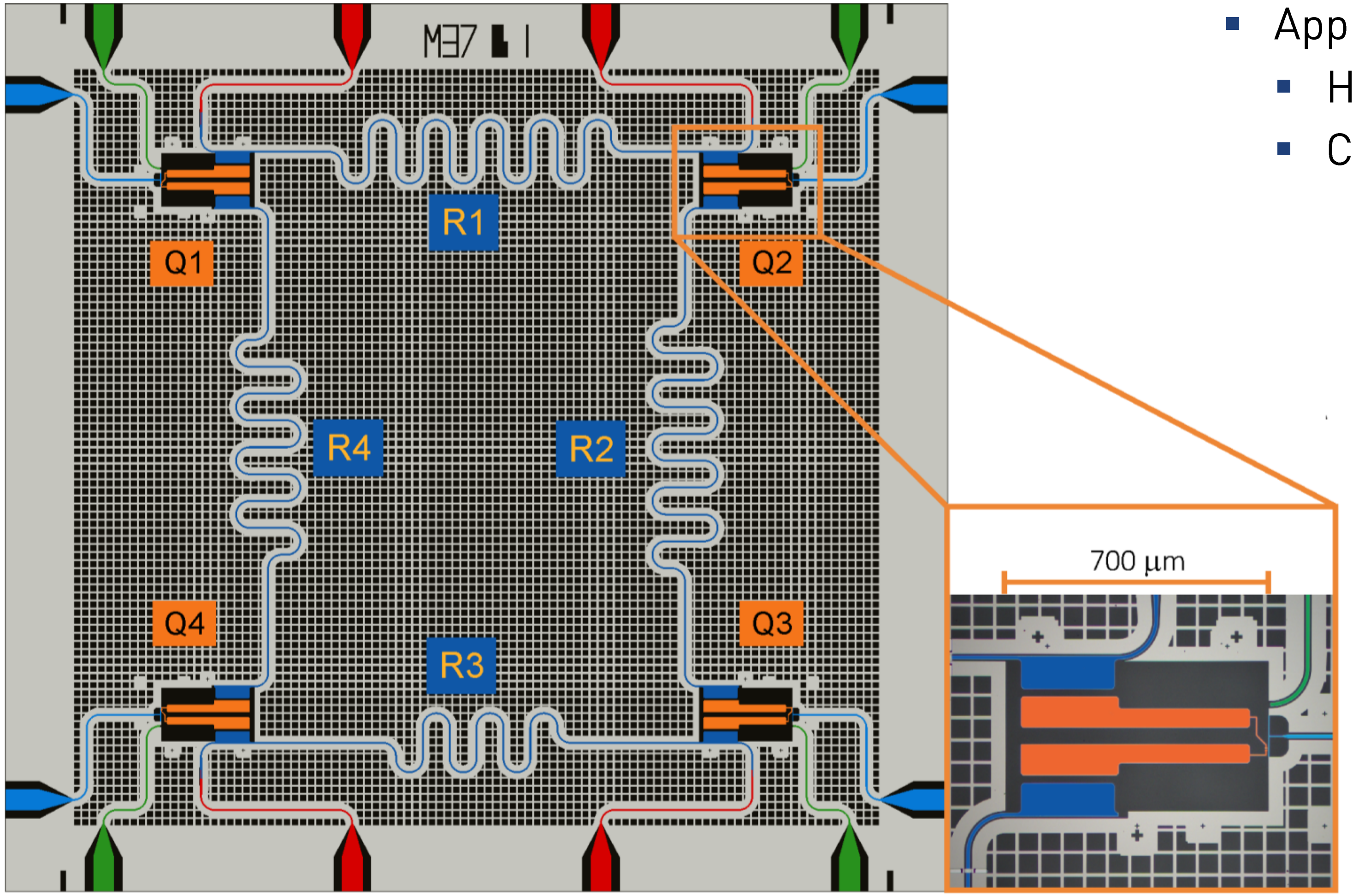
first demonstration with s.c. circuits: M. Hofheinz et *al.*, *Nature (London)* **454**, 310 (2008)

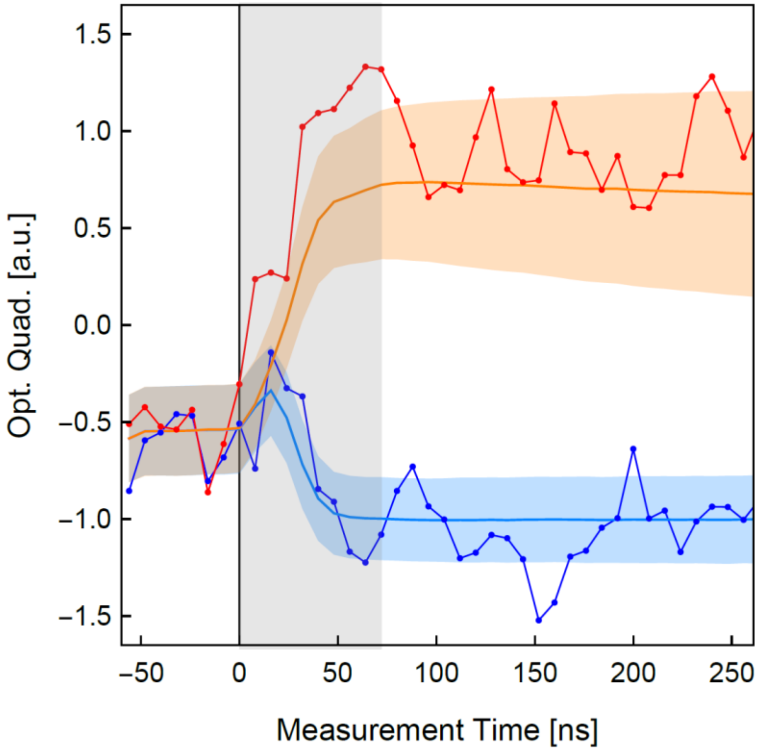
this data: *Quantum Device Lab* (2008)

Applications of Cavity QED

Building a quantum computer from electronic circuits

- Applications of cavity QED
  - High fidelity dispersive read-out
  - Coupling qubits and realizing 2-qubit gates

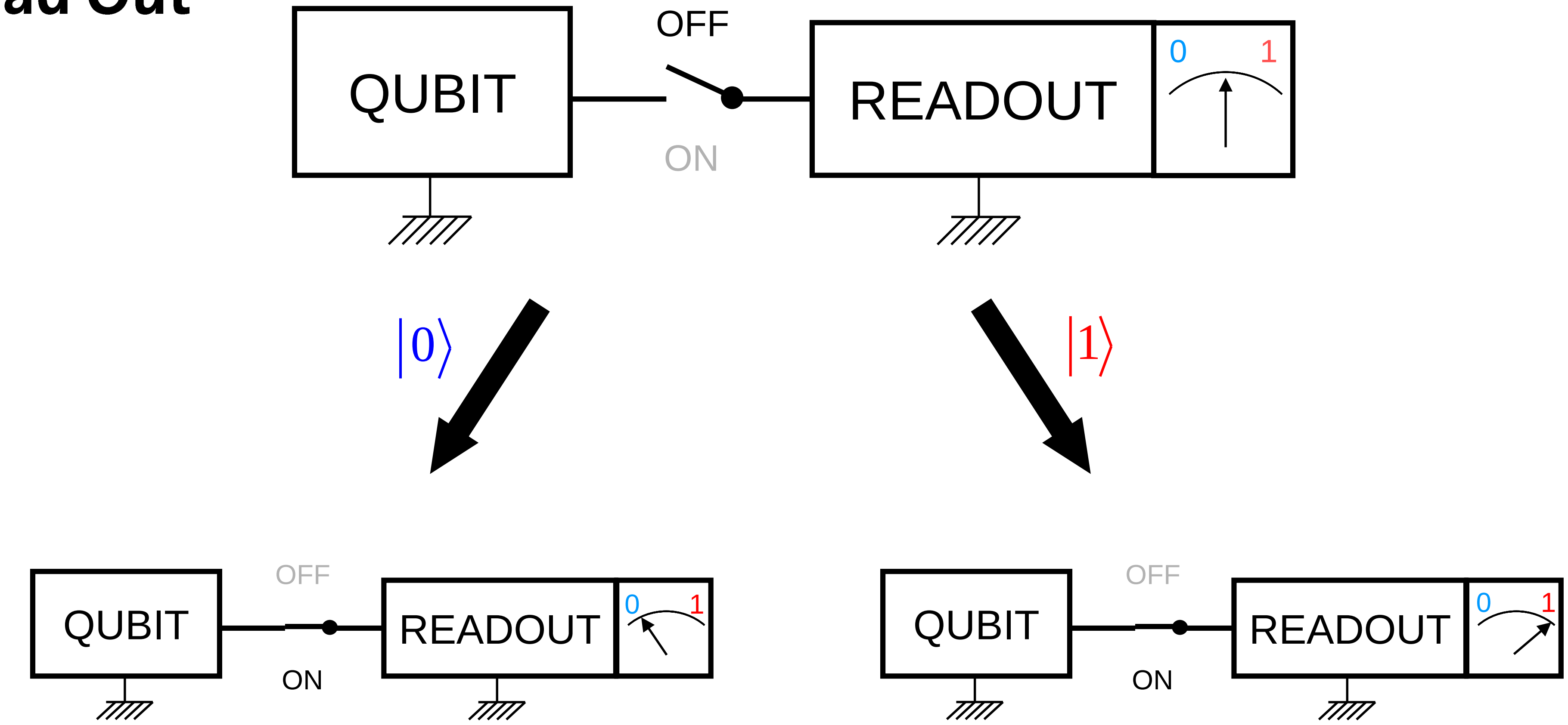




Read-Out …

… of superconducting qubits

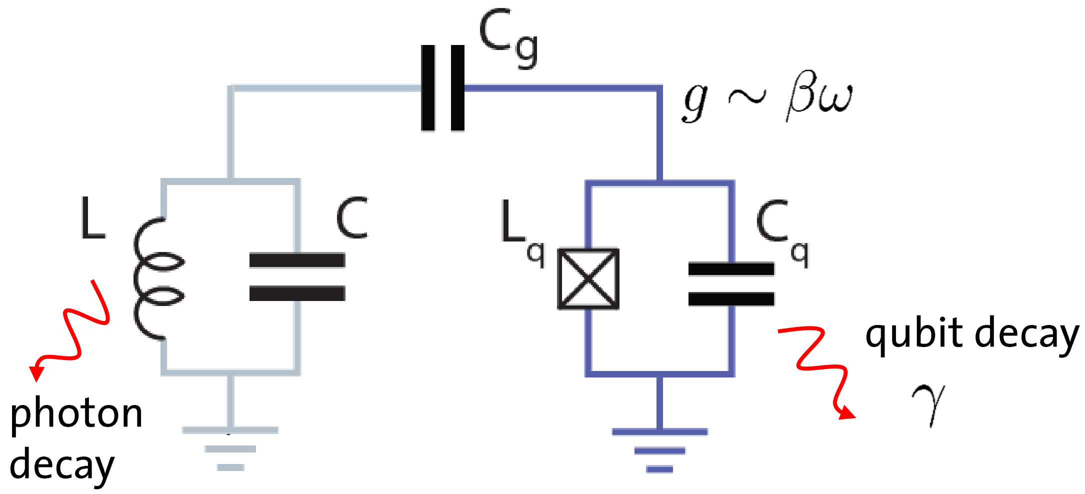
Qubit Read Out

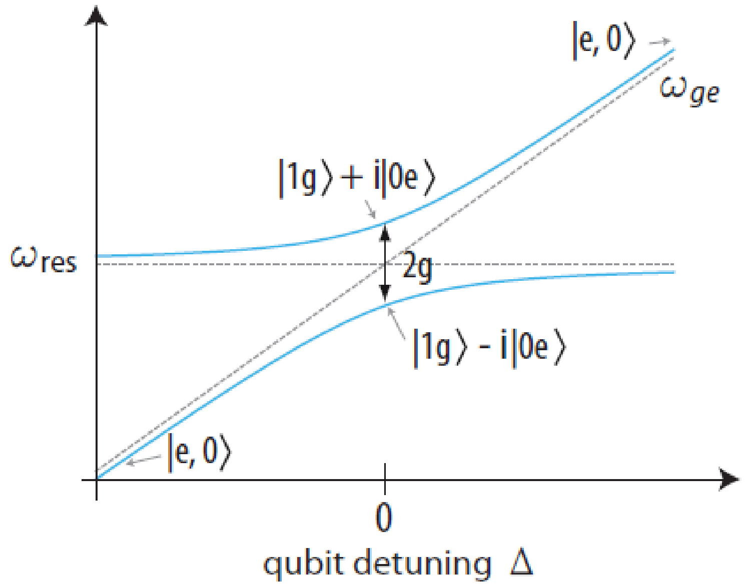


Desirable properties:

- Good ON/OFF ratio
- No spontaneous decay/excitation due to measurement apparatus
- Projective and QND
- Fast and high fidelity

Coupled Transmon - Cavity system





Jaynes-Cummings Hamiltonian:

$$H/\hbar = \underset{\text{quantized field}}{\boxed{\omega_r a^\dagger a}} + \underset{\text{qubit}}{\boxed{\frac{\omega_{ge}}{2}\sigma^z}} + \underset{\text{coupling}}{\boxed{g(a^\dagger \sigma^- + a\sigma^+)}}$$

$$\downarrow$$

$$\boxed{\chi \sigma_z a^\dagger a}$$

Dispersive coupling

Strong coupling limit:

$$g > \gamma, \kappa$$

What happens
in the limit of large detuning?

$$|\Delta| = |\omega_r - \omega_{ge}| \gg g$$

A. Wallraff *et al.*, *Nature* **431**, 162 (2004)

Derivation of Dispersive Hamiltonian

Discussion on the black board.

Dispersive Approximation of the J-C Hamiltonian

Jaynes-Cummings Hamiltonian

$$H = \hbar\omega_r\left(a^\dagger a + \frac{1}{2}\right) + \frac{\hbar\omega_a}{2}\sigma^z + \hbar g(a^\dagger\sigma^- + a\sigma^+)$$

Unitary transformation (Schrieffer-Wolf transformation)

$$\tilde{H} = UHU^\dagger \quad \text{with} \quad U = \exp\frac{g}{\Delta}(a\sigma^+ - a^\dagger\sigma^-)$$

$$\text{and} \quad \Delta = \omega_a - \omega_r$$

Results in dispersive approximation up to 2nd order in g/Δ

$$\tilde{H} \approx \hbar\left(\omega_r + \frac{g^2}{\Delta}\sigma_z\right)a^\dagger a + \frac{1}{2}\hbar\left(\omega_a + \frac{g^2}{\Delta}\right)\sigma_z$$

A. Blais, *et al.*, *PRA* 69, 062320 (2004)

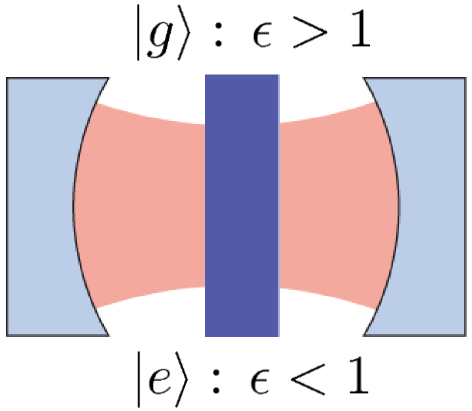
Dispersive Read-Out

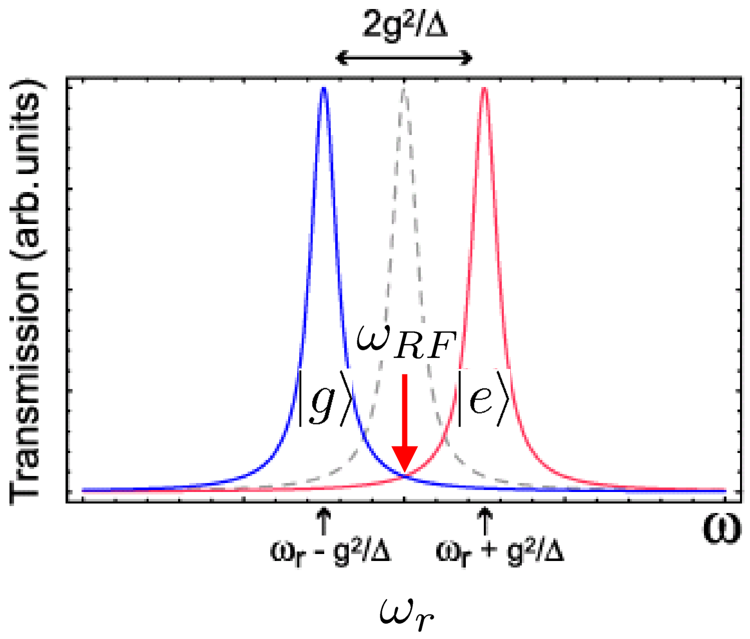
approximate diagonalization in the dispersive limit $|\Delta| = |\omega_a - \omega_r| \gg g$

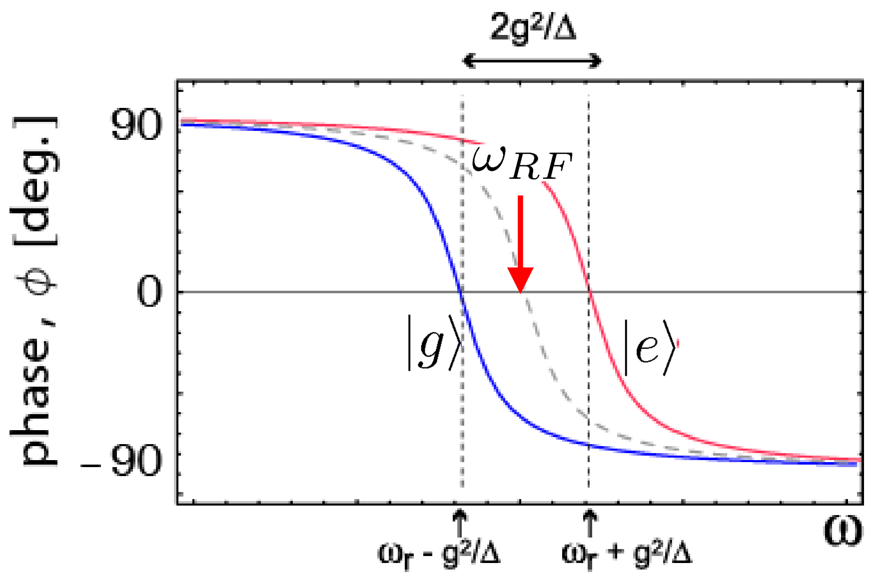
$$H \approx \hbar\left(\omega_r + \frac{g^2}{\Delta}\sigma_z\right) a^\dagger a + \frac{1}{2}\hbar\left(\omega_a + \frac{g^2}{\Delta}\right)\sigma_z$$

//

cavity frequency shift

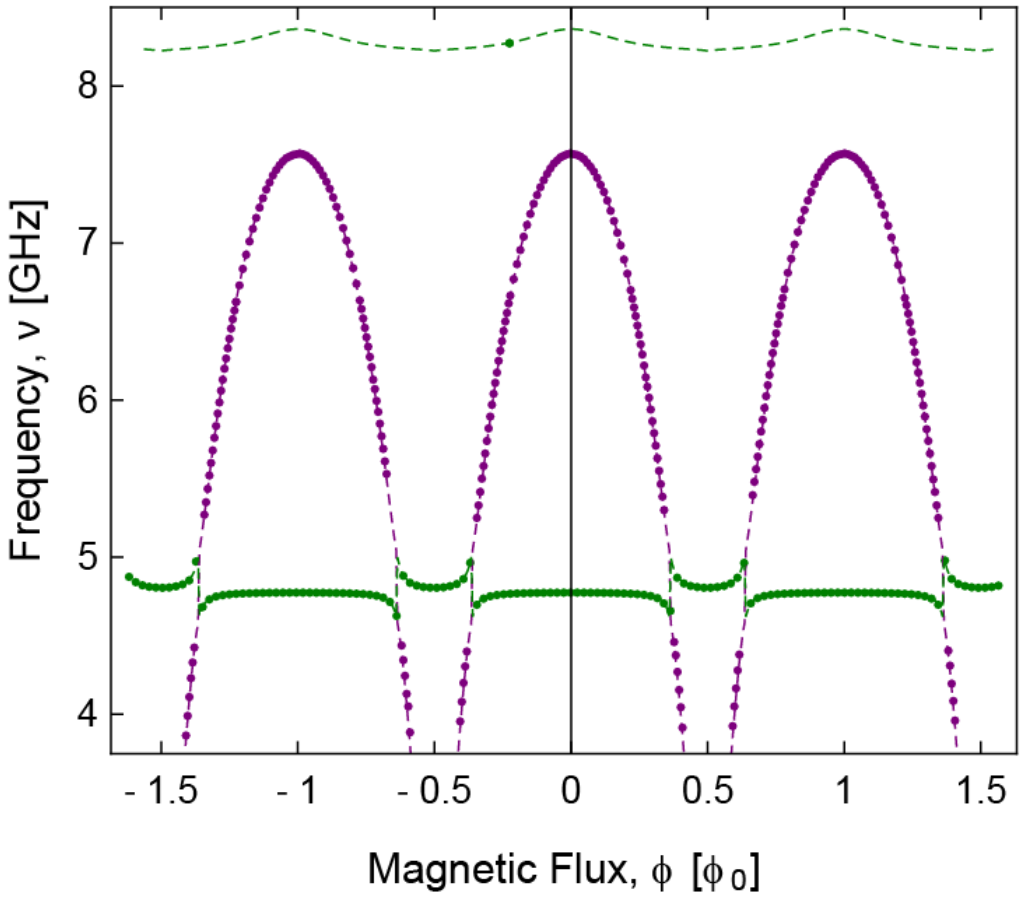






A. Blais *et al.*, *PRA* 69, 062320 (2004)

Characterizing Qubit and Resonator in Spectroscopy



Measured qubit/resonator freq. (*,*)

Fit to full Hamiltonian (-,-)

Qubit Spectroscopy:

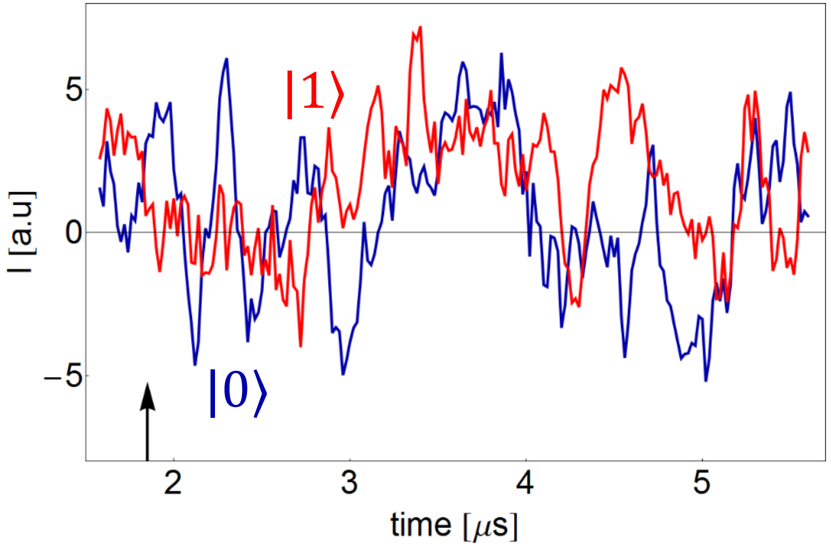
- Probe the resonator at constant frequency
- Drive the qubit at variable frequency
- Observe change in the probe field response, when drive frequency matches the qubit transition frequency.

Discussion on the black board.

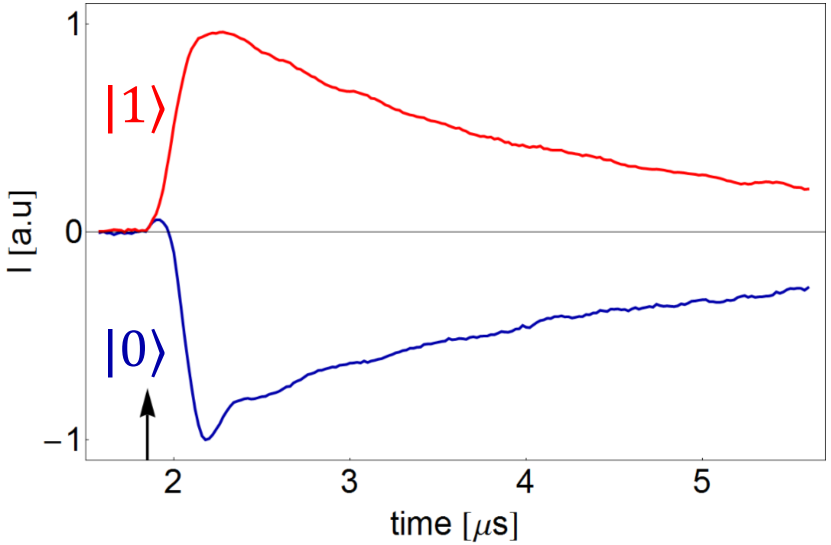
Qubit-Readout (Averaged)

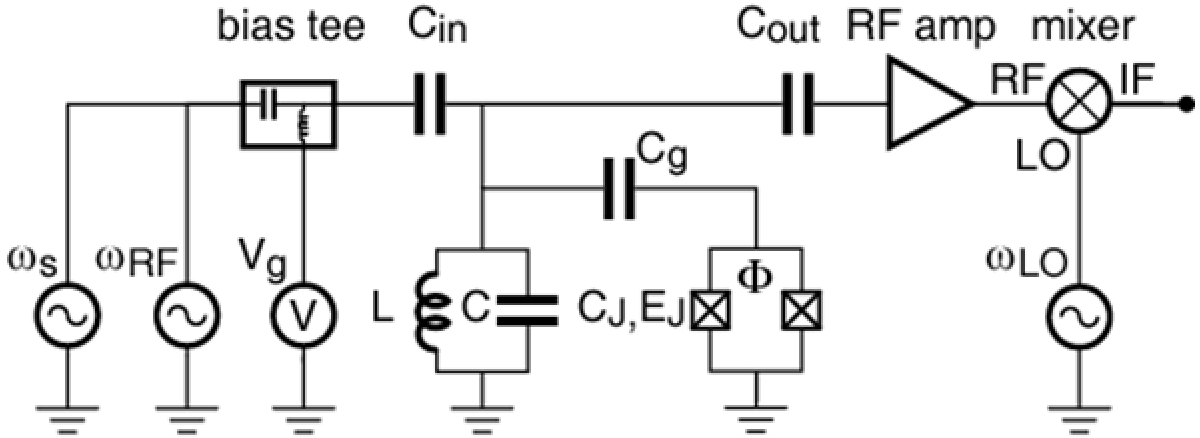
Conventional HEMT

single-shot measurements:



averaged measurements (8 10^4):

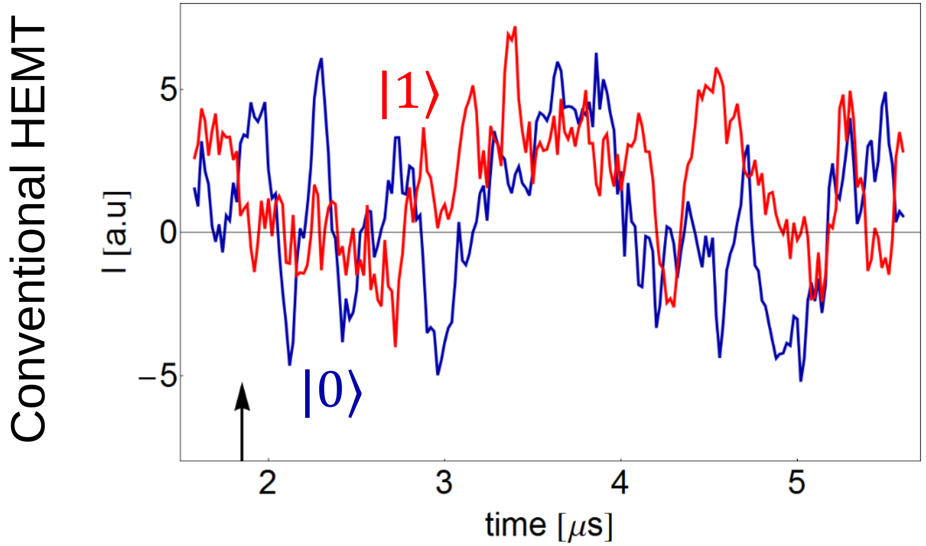




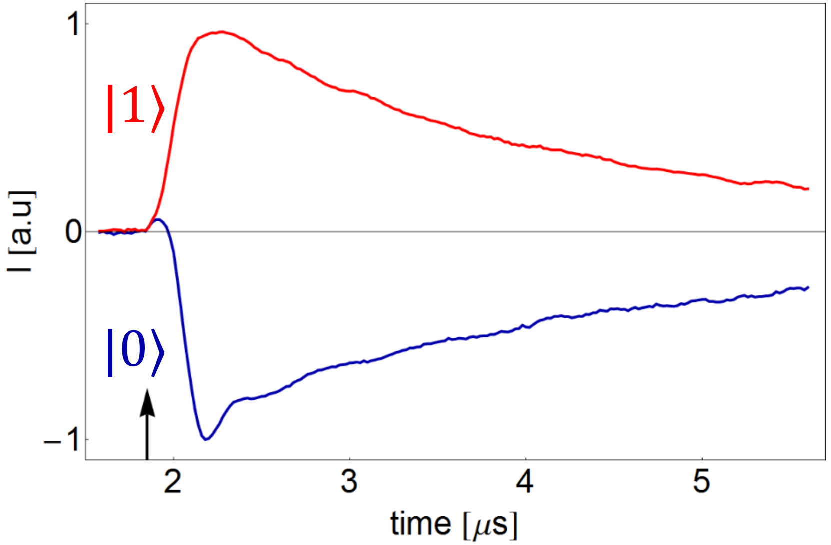
P. Kurpiers, Y. Salathe *et al*, *ETH Zurich* (2013)
R. Vijay *et al.*, *PRL* 106, 110502 (2011)

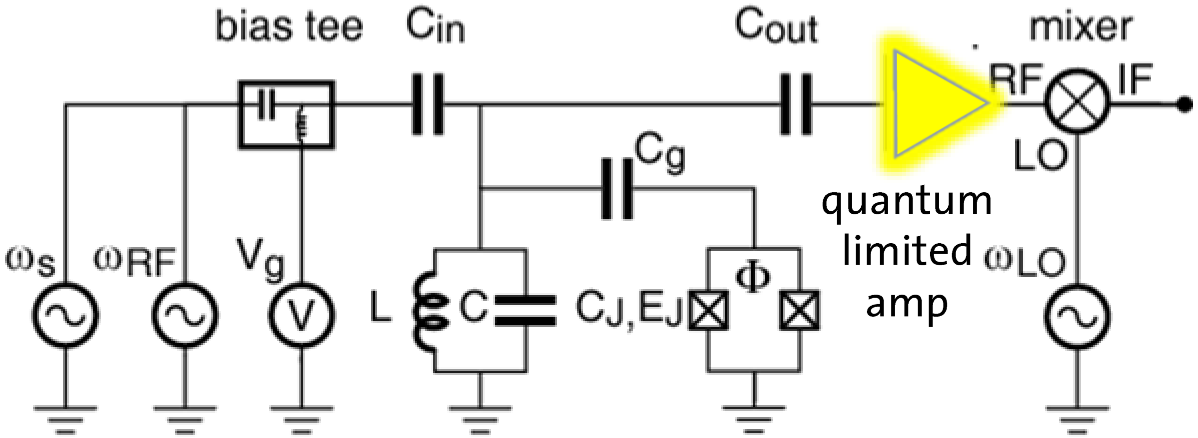
Qubit-Readout (Averaged)

single-shot measurements:



averaged measurements (8 10^4):

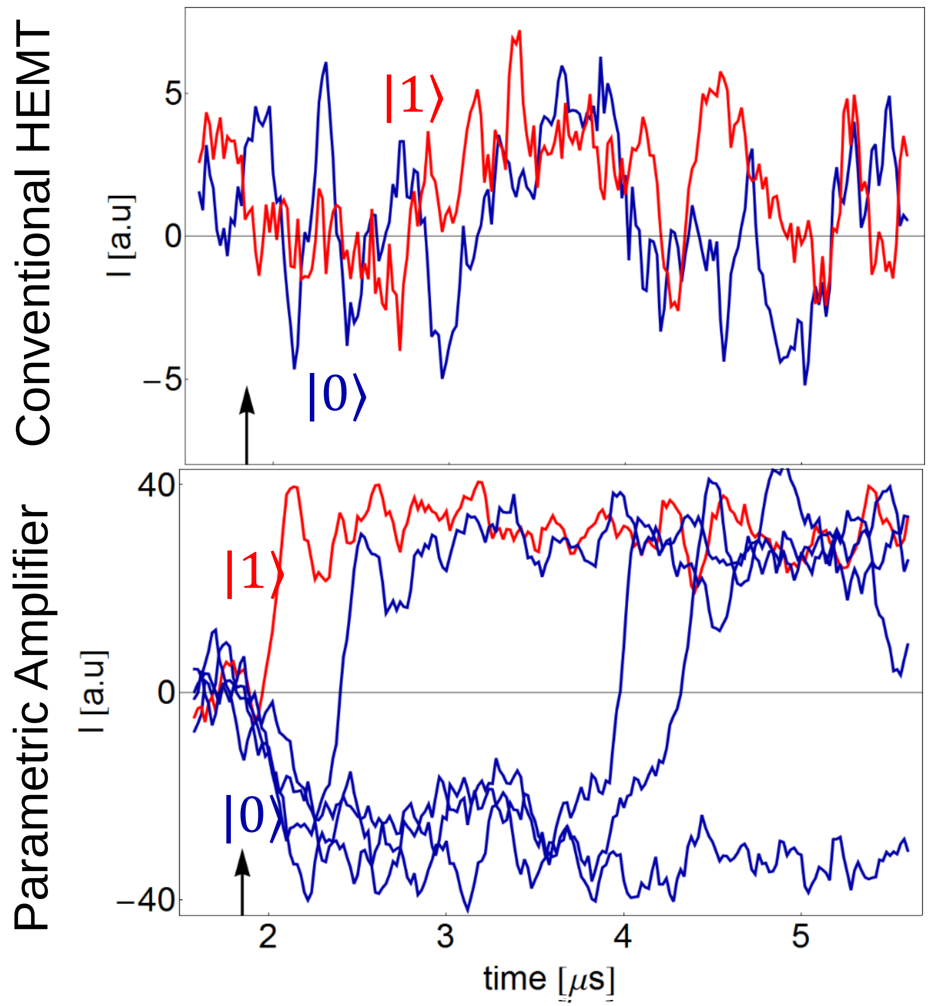




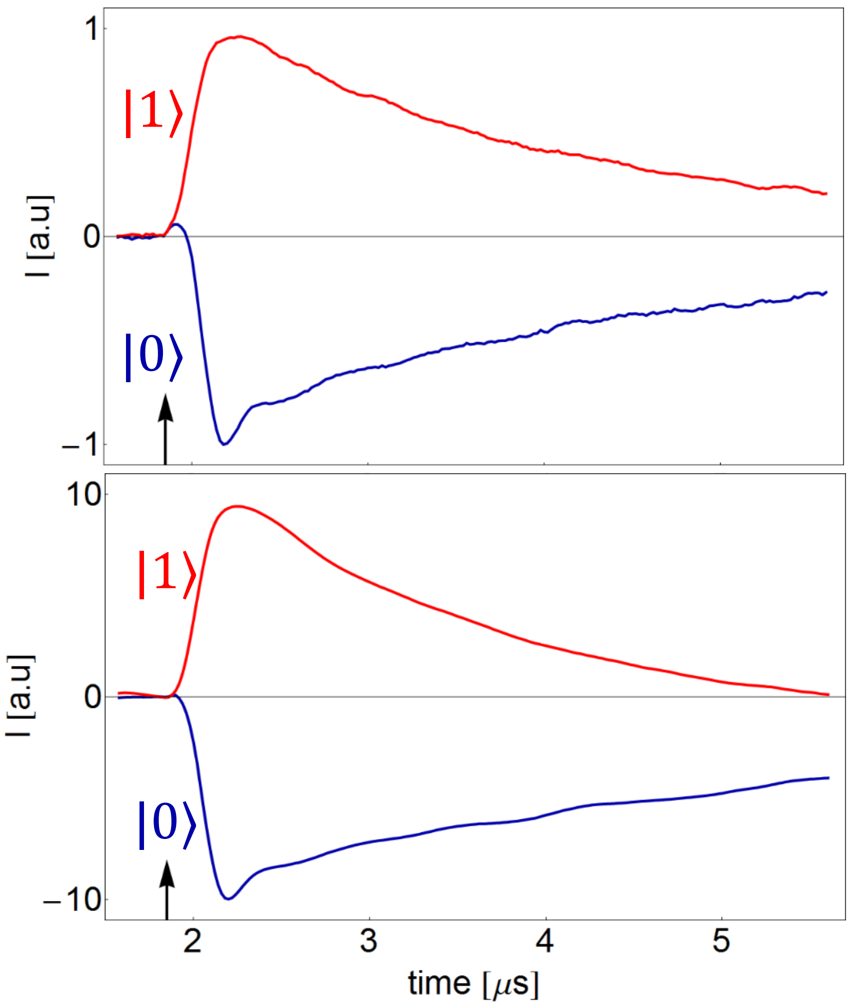
P. Kurpiers, Y. Salathe *et al*, *ETH Zurich* (2013)
R. Vijay *et al.*, *PRL* 106, 110502 (2011)

Qubit-Readout (Averaged)

single-shot measurements:

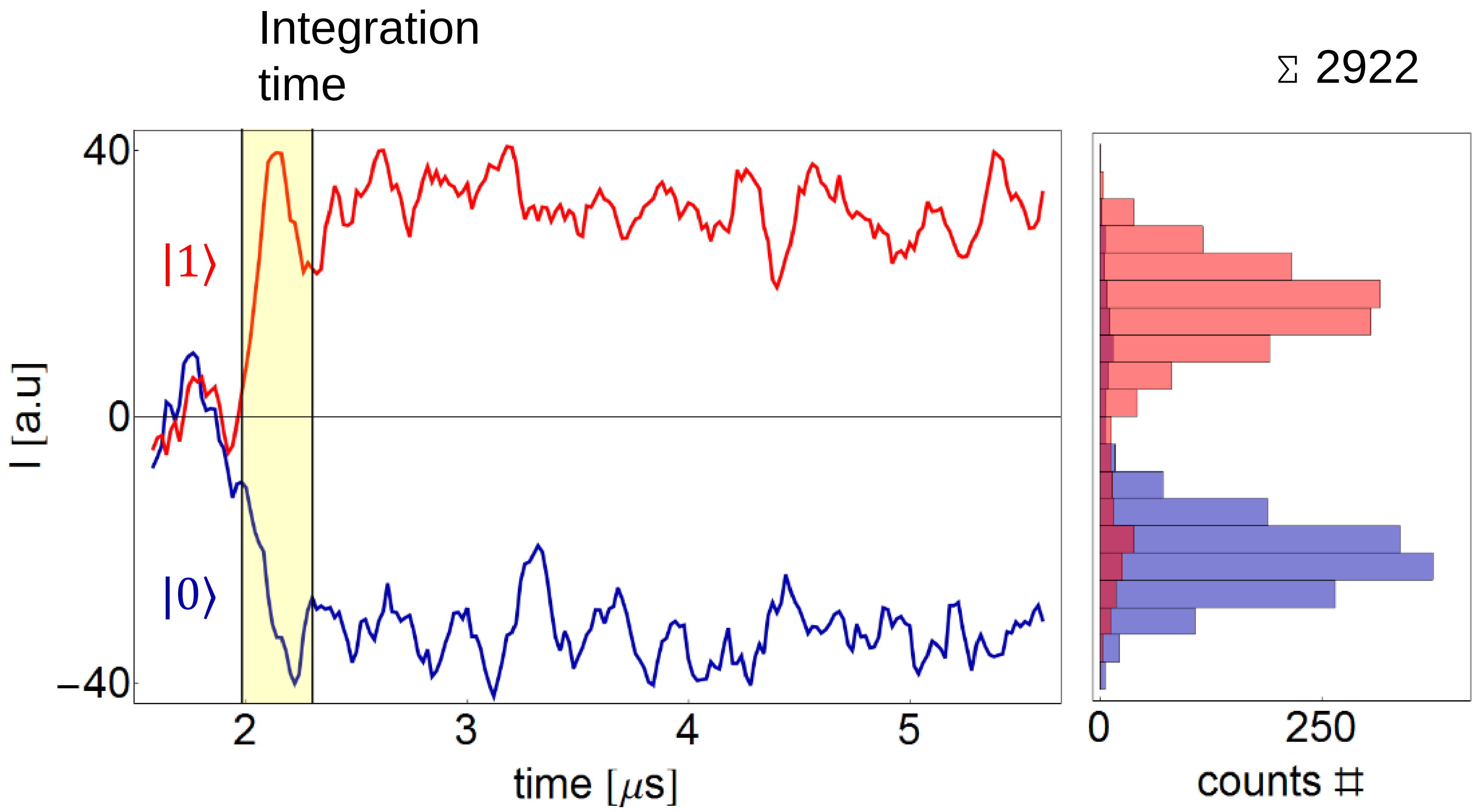


averaged measurements (8 10^4):



P. Kurpiers, Y. Salathe *et al*, *ETH Zurich* (2013)
R. Vijay *et al.*, *PRL* 106, 110502 (2011)

Statistics of Integrated Single-Shot Readout



P. Kurpiers, Y. Salathe *et al*, *ETH Zurich* (2013)

Fast, High-Fidelity, Single-Shot Readout

Readout

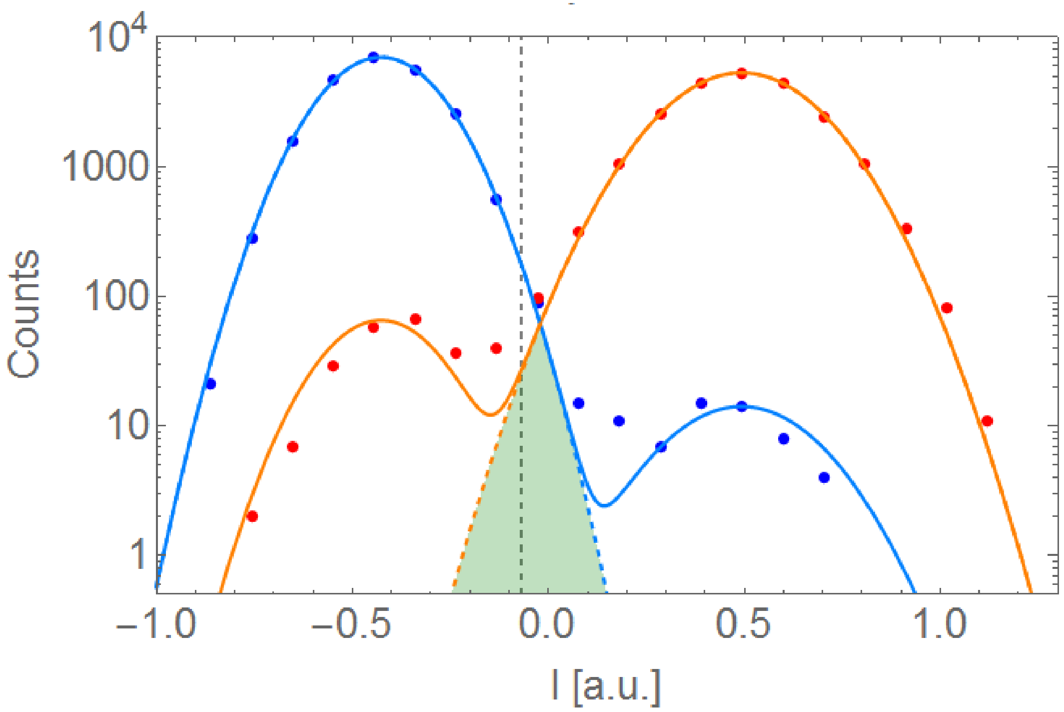
- High fidelity: 98.3 %
- Fast: 72 ns integration time

With Purcell filter …

- Read out res. BW ~ 35 MHz
- Qubit T_1 ~ 8 µs

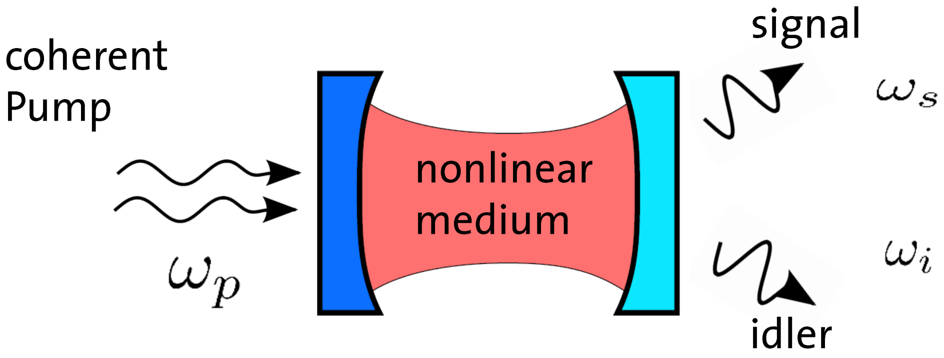
… and parametric amplifier:

- Phase sensitive
- BW ~ 20 MHz

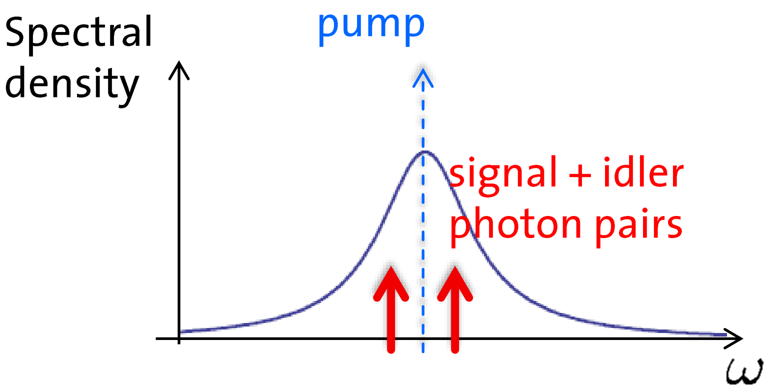


T. Walter, P. Kurpiers *et al.*, Quantum Device Lab, ETH Zurich (2016)

Parametric amplifier: Basic working principle



4-wave mixing: $\boxed{2\omega_p = \omega_s + \omega_i}$



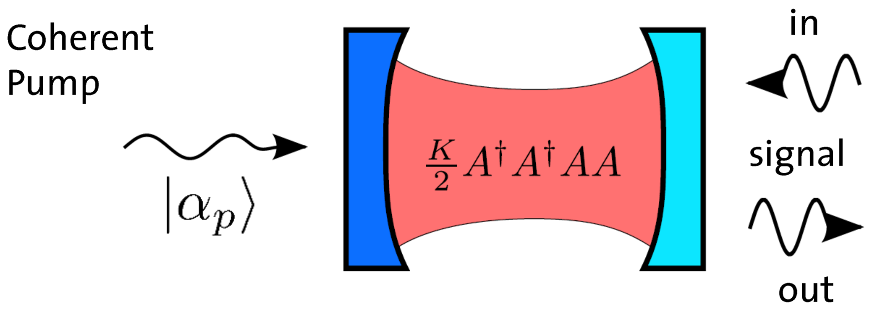
ω_s, ω_i occupy same mode → degenerate paramp

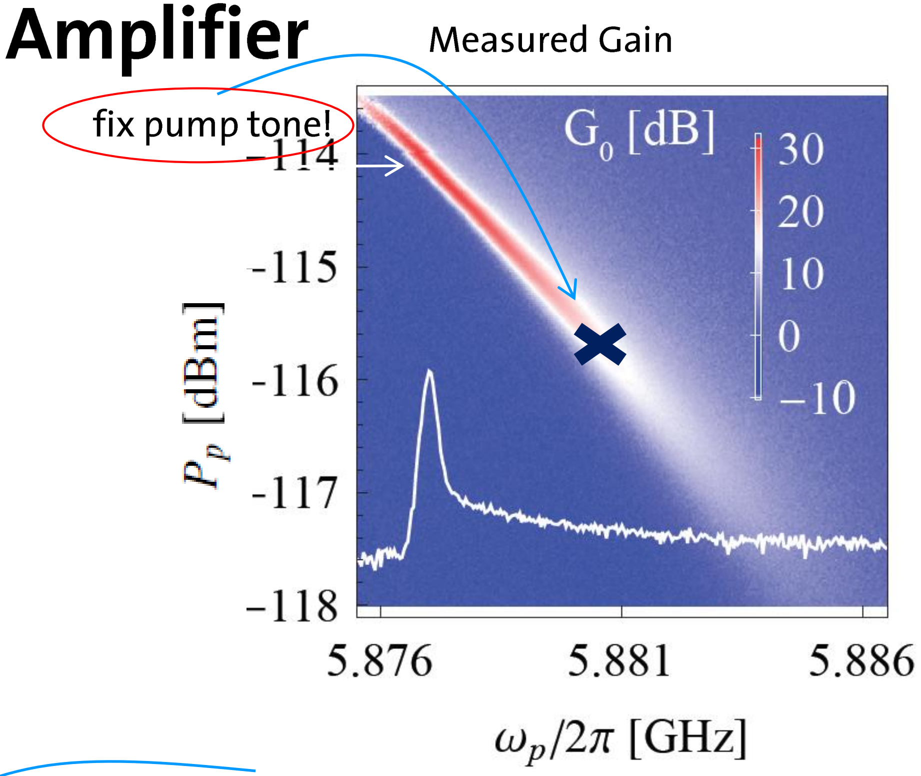
Conversion stimulated by:

1) vacuum fluctuations -> spontaneous parametric down-conversion (SPDC)
2) input (quantum) signals ->parametric amplification

Yurke *et al.*, *PRL* **60**, 764 (1988)
Castellanos-Beltran et al., *Nat. Phys.* **4**, 929 (2008)
Eichler and Wallraff, EPJ **1**, (2014)

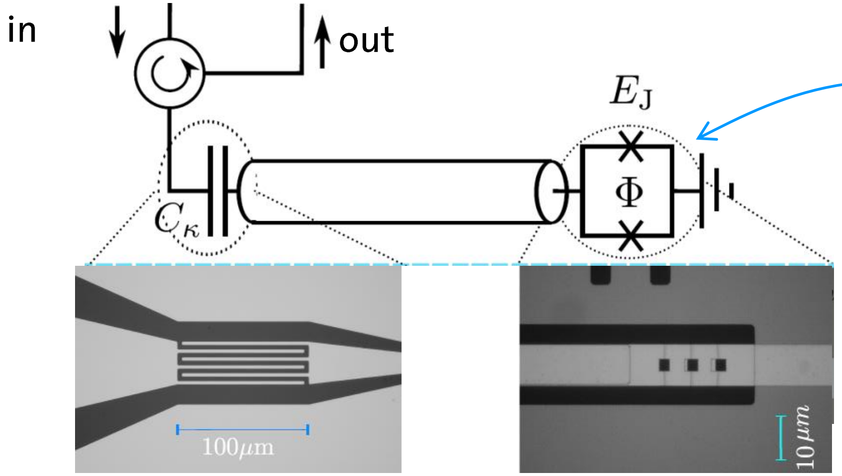
Near Quantum-Limited Parametric Amplifier




Measured Gain

fix pump tone!

Circuit QED implementation:



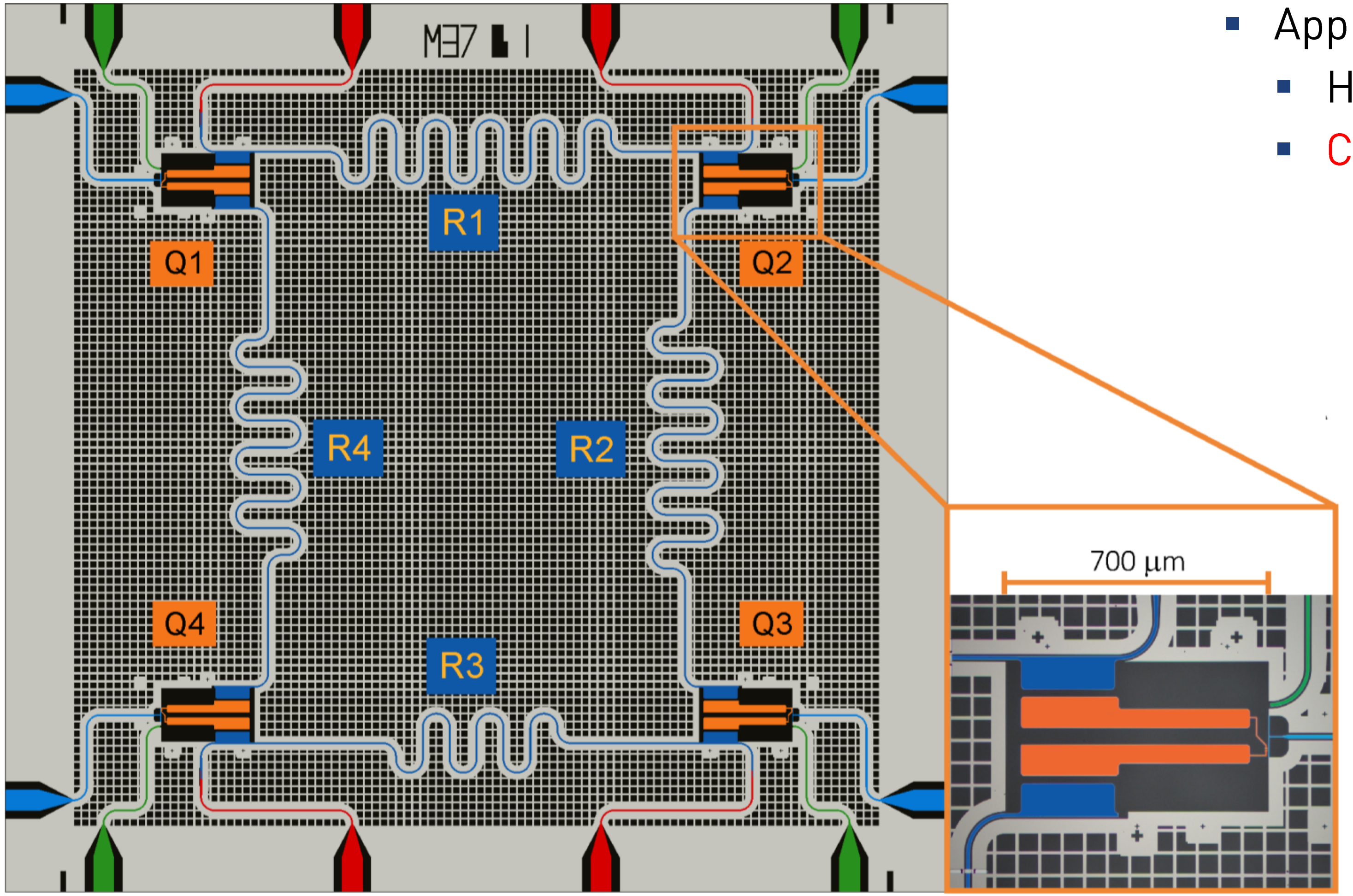
SQUID(-array) provides
required nonlinearity

Eichler *et al.*, EPJ Quantum Technology **1**, 2 (2014)
Eichler *et al.*, *Phys. Rev. Lett.* **107**, 113601 (2011)
Eichler et al., *Phys. Rev. Lett.* **113**, 110502 (2014)

Caves, *Phys. Rev. D* **26**, 1817 (1982)
Yurke and Buks, *J. Lightwave Tech.* **24**, 5054 (2006)
Castellanos-Beltran *et al.*, *Nat. Phys.* **4**, 929 (2008)

...many possible variations
of this basic principle!

Outlook:

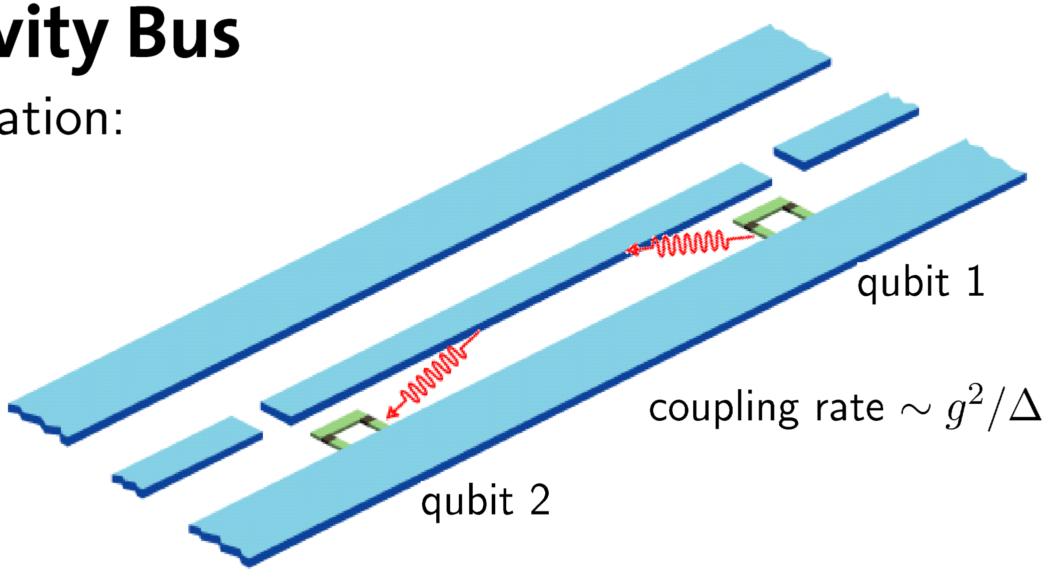


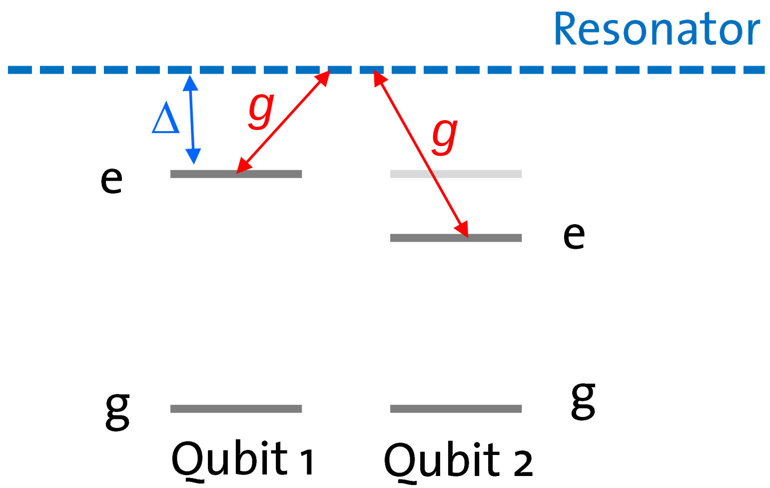
Building a quantum computer from electronic circuits

- Applications of cavity QED
  - High fidelity dispersive read-out
  - Coupling qubits and realizing 2-qubit gates

Two qubit interaction & The Cavity Bus

coupling through virtual excitation:





A. Blais et al., *PRA* **69,** 062320 (2004)
Majer et al., Nature **449** (2007)

Questions?