ETH
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Superconducting circuits: Toffoli gate and error correction

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Outline

- Introduction
- Basic theory: Toffoli Gate
- Implementation of the Toffoli Gate
- Basic theory: Error correction
- Implementation of an error correction code
- Outlook
- Discussion/Questions

Introduction

- Classical computation: one error in 10^{17} computations
- Most errors in data transmission
- Want error correction codes
- Basic idea: (classical)
  - Encode: “add redundancy“

    $0 \rightarrow 000$

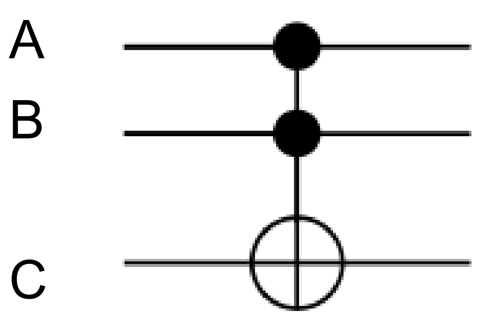
    $1 \rightarrow 111$
  - Decode: Majority voting

    e.g. 010 => 0

Introduction

- Quantum error correcting codes
- Difficulties:
  - Non-cloning theorem
  - Errors are continuous
  - Measurement destroys quantum information
- Important gate: Toffoli gate

Basic theory: Toffoli Gate



A and B control qubits

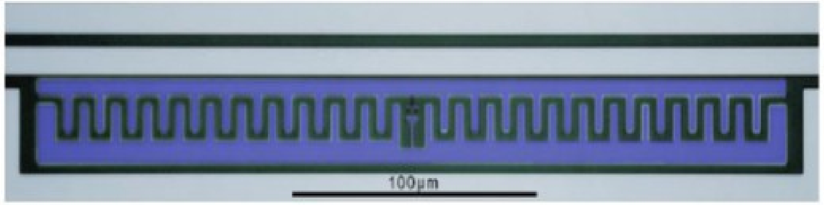
C target qubit

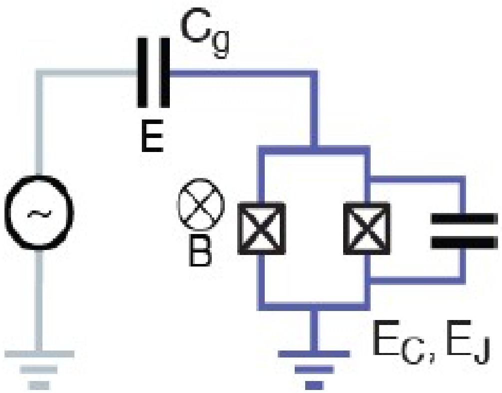
|000>	1	0	0	0	0	0	0	0
|001>	0	1	0	0	0	0	0	0
|010>	0	0	1	0	0	0	0	0
|011>	0	0	0	1	0	0	0	0
|100>	0	0	0	0	1	0	0	0
|101>	0	0	0	0	0	1	0	0
|110>	0	0	0	0	0	0	0	1
|111>	0	0	0	0	0	0	1	0

- Toffoli also known as CC-NOT
- Reversible
- Toffoli and Hadamard gate form universal set

Implementation of the Toffoli Gate

Transmon qubit:

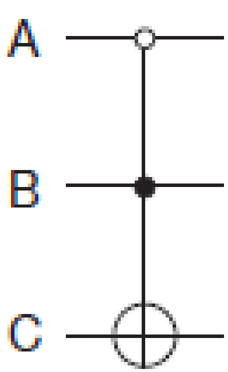




- 3 transmon qubits coupled to microwave transmission line resonator
- Different transition frequencies for the qubits
- Also use second excited state |2>. Level separation anharmonic.

Implementation of the Toffoli Gate

- ETH-Version:
  instead of |11x>
  state |01x> is flipped



Empty: Ground

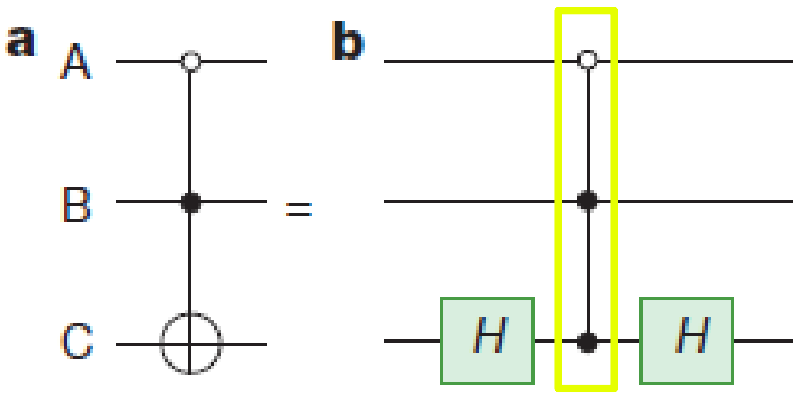
Filled: excited

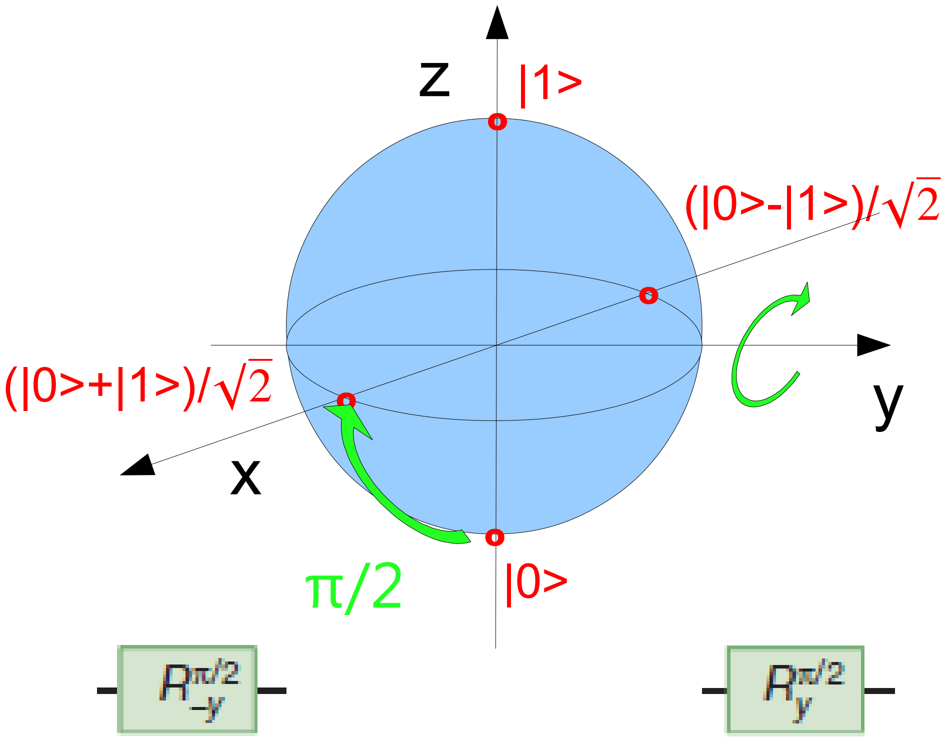
|000>	1	0	0	0	0	0	0	0
|001>	0	1	0	0	0	0	0	0
|010>	0	0	0	1	0	0	0	0
|011>	0	0	1	0	0	0	0	0
|100>	0	0	0	0	1	0	0	0
|101>	0	0	0	0	0	1	0	0
|110>	0	0	0	0	0	0	1	0
|111>	0	0	0	0	0	0	0	1

Implementation of the Toffoli Gate

- CC-Not => Hadamard
  CC-Phase
  Hadamard

Hadamard $\frac{1}{\sqrt{2}}\begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}$

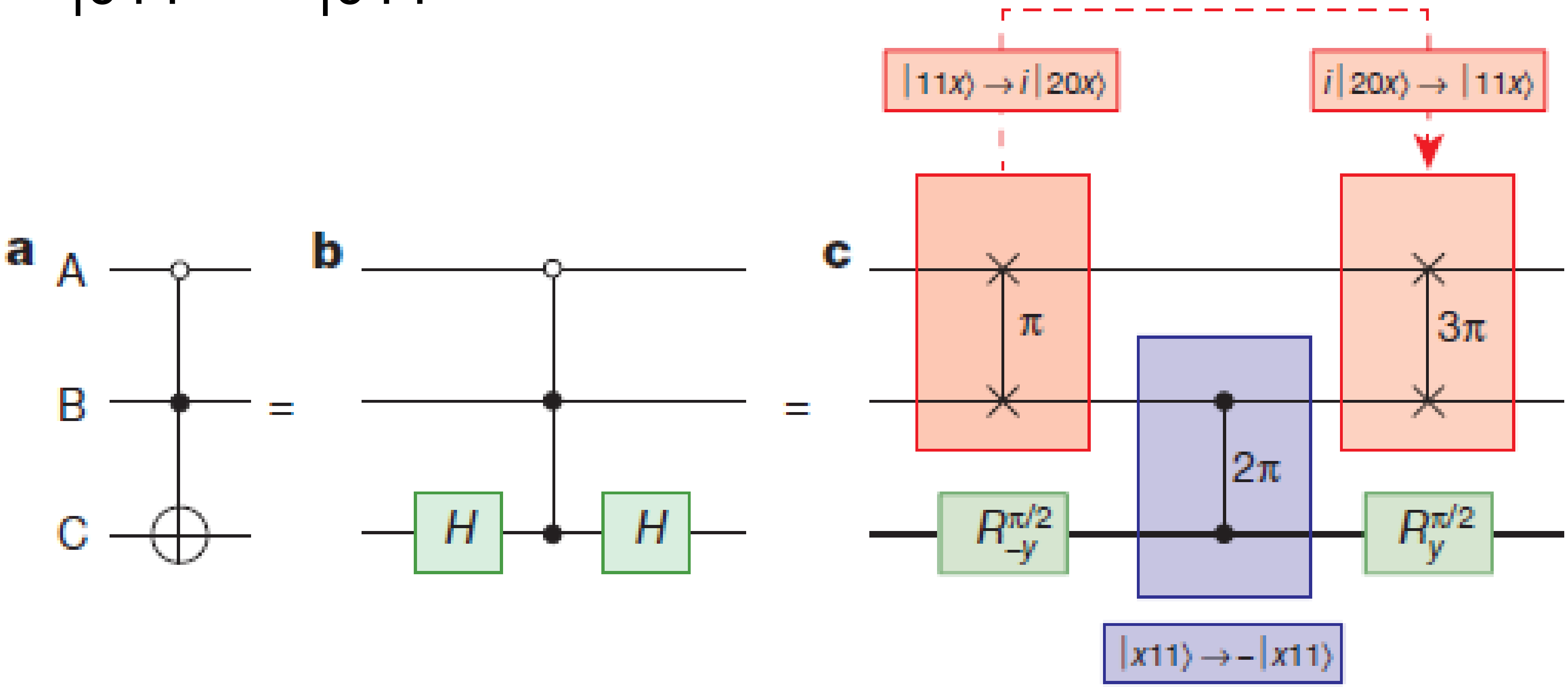




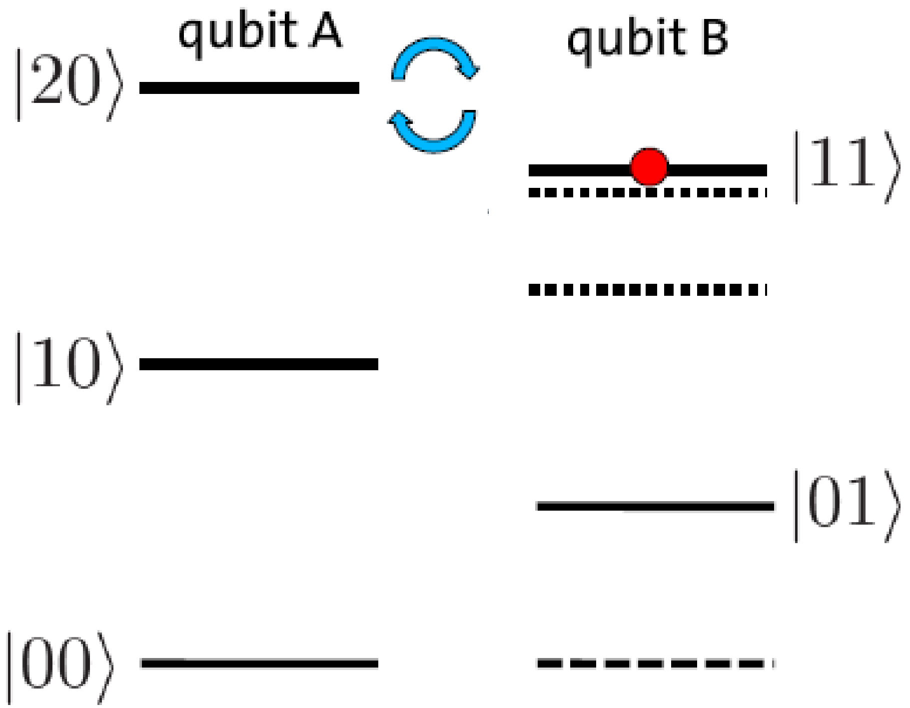
Implementation of the Toffoli Gate

- CC-Phase:

  |011> → -|011>



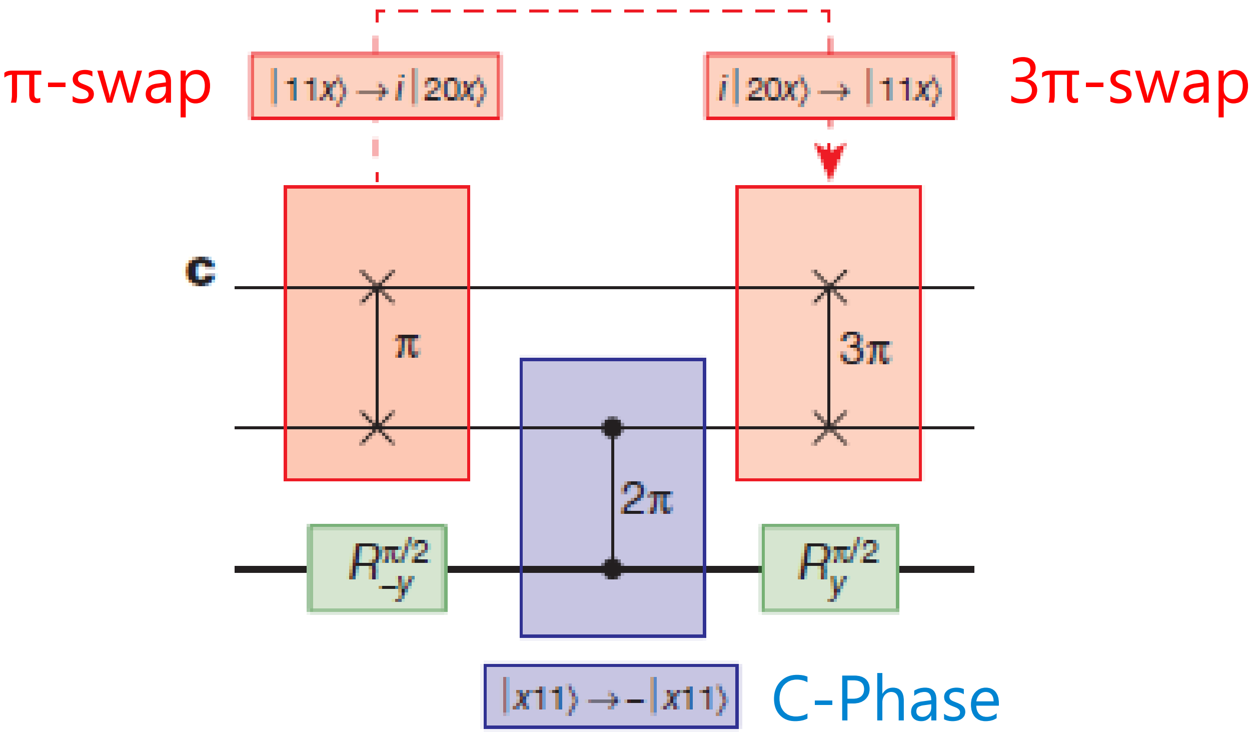
Implementation of the Toffoli Gate



$$|11\rangle \xrightarrow{\pi} i|20\rangle \xrightarrow{\pi} -|11\rangle$$

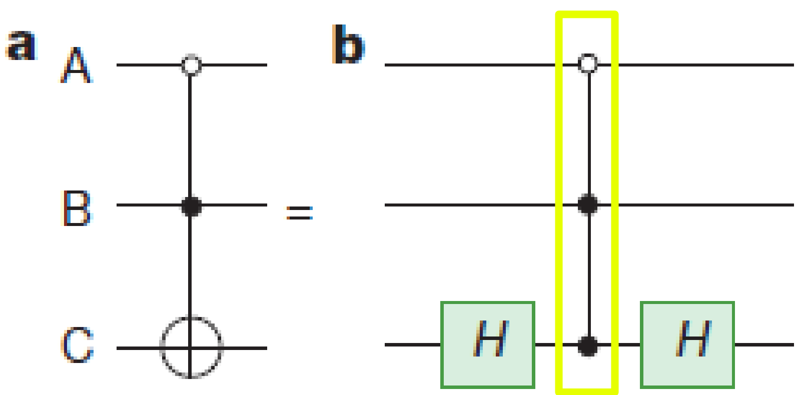
Implementation of the Toffoli Gate

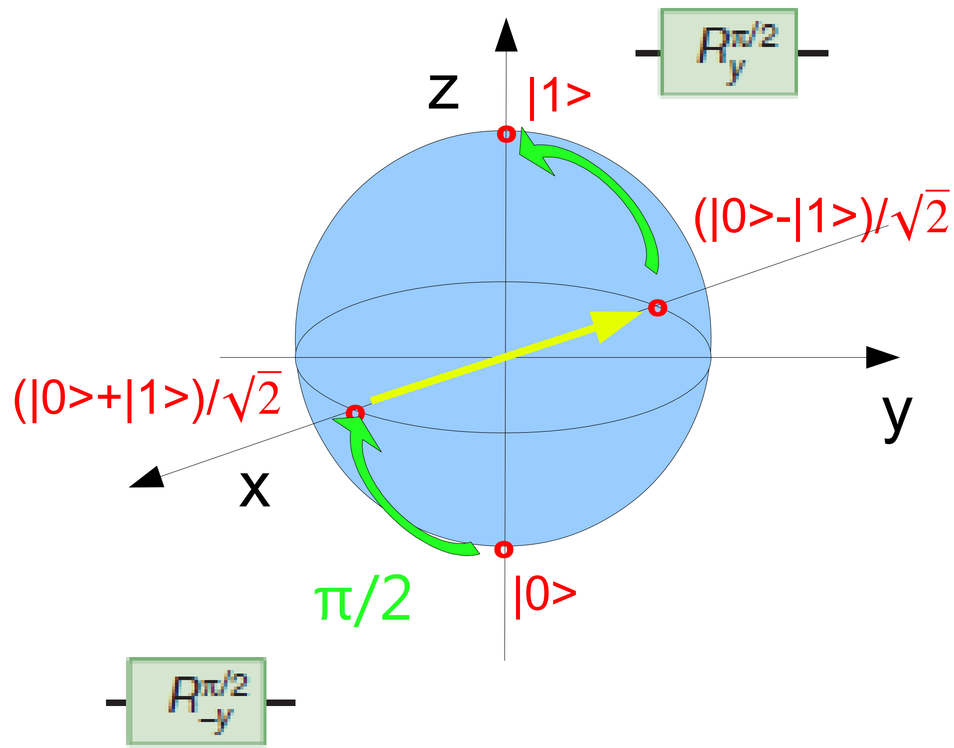
|011>	|011>	-|011>	-|011>
|11x>	i|20x>	i|20x>	|11x>
|x0y>	|x0y>	|x0y>	|x0y>
|010>	|010>	|010>	|010>



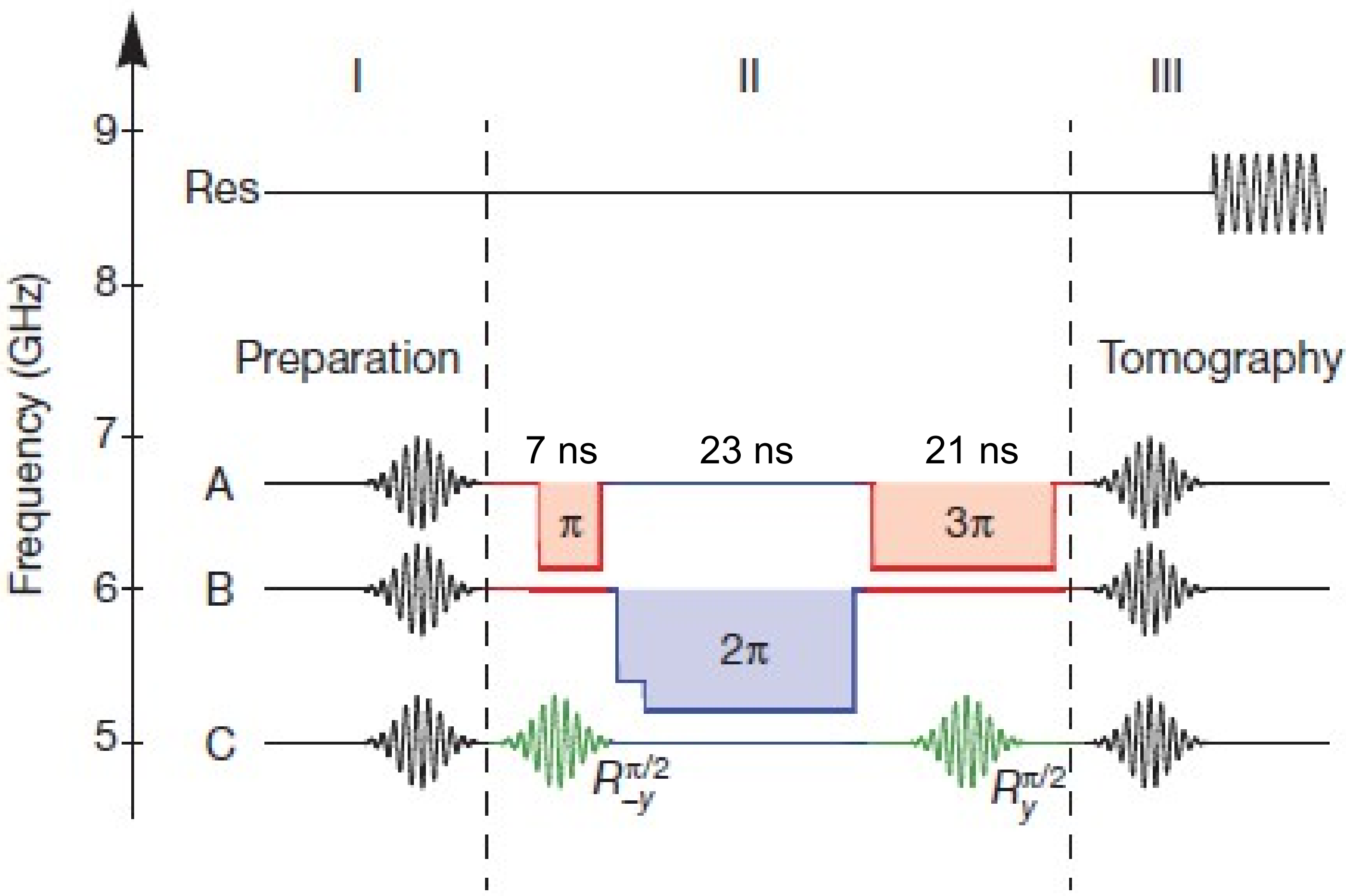
Implementation of the Toffoli Gate

- CC-Not => Hadamard
  CC-Phase
  Hadamard



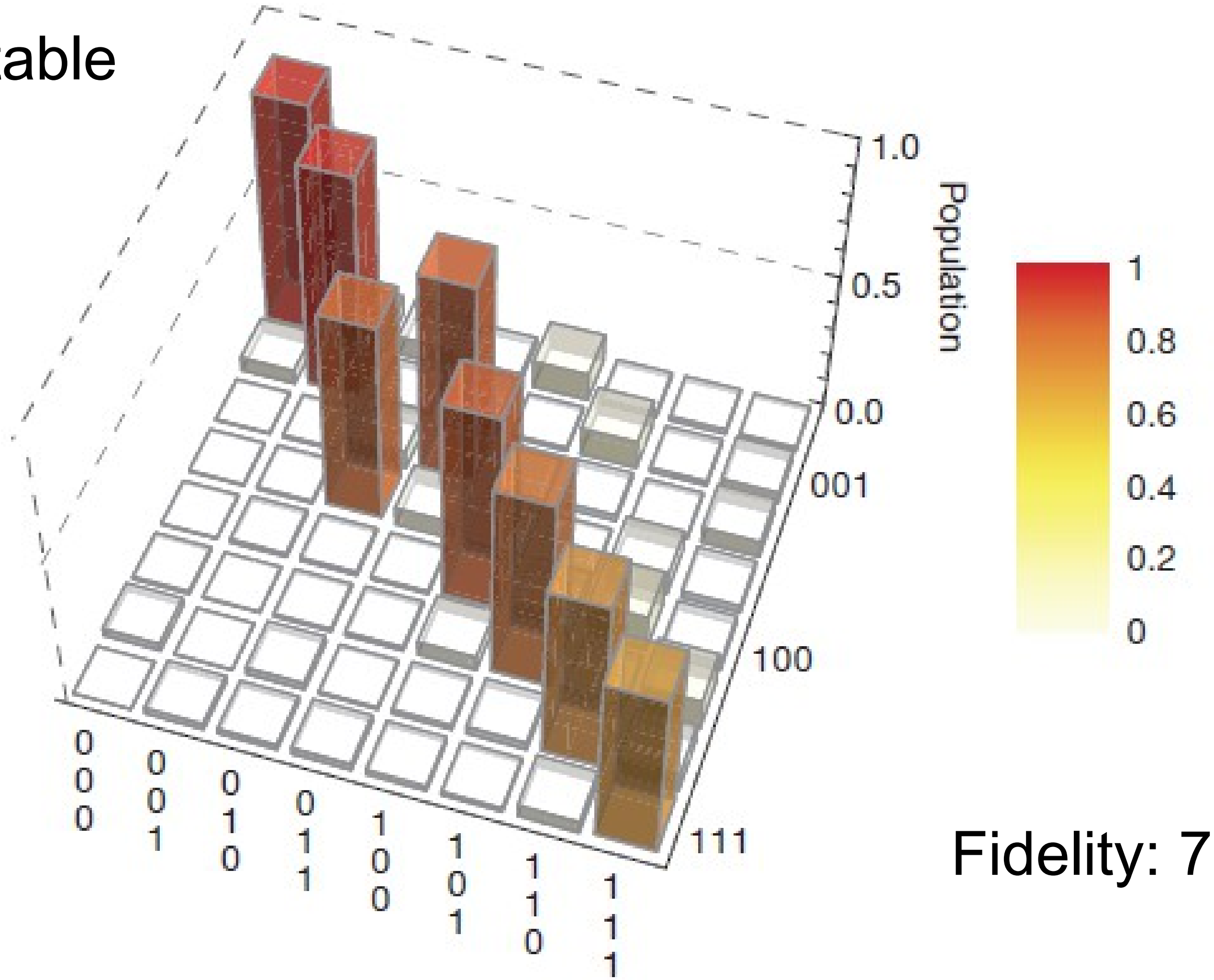


Implementation of the Toffoli Gate



Implementation of the Toffoli Gate

Truth table

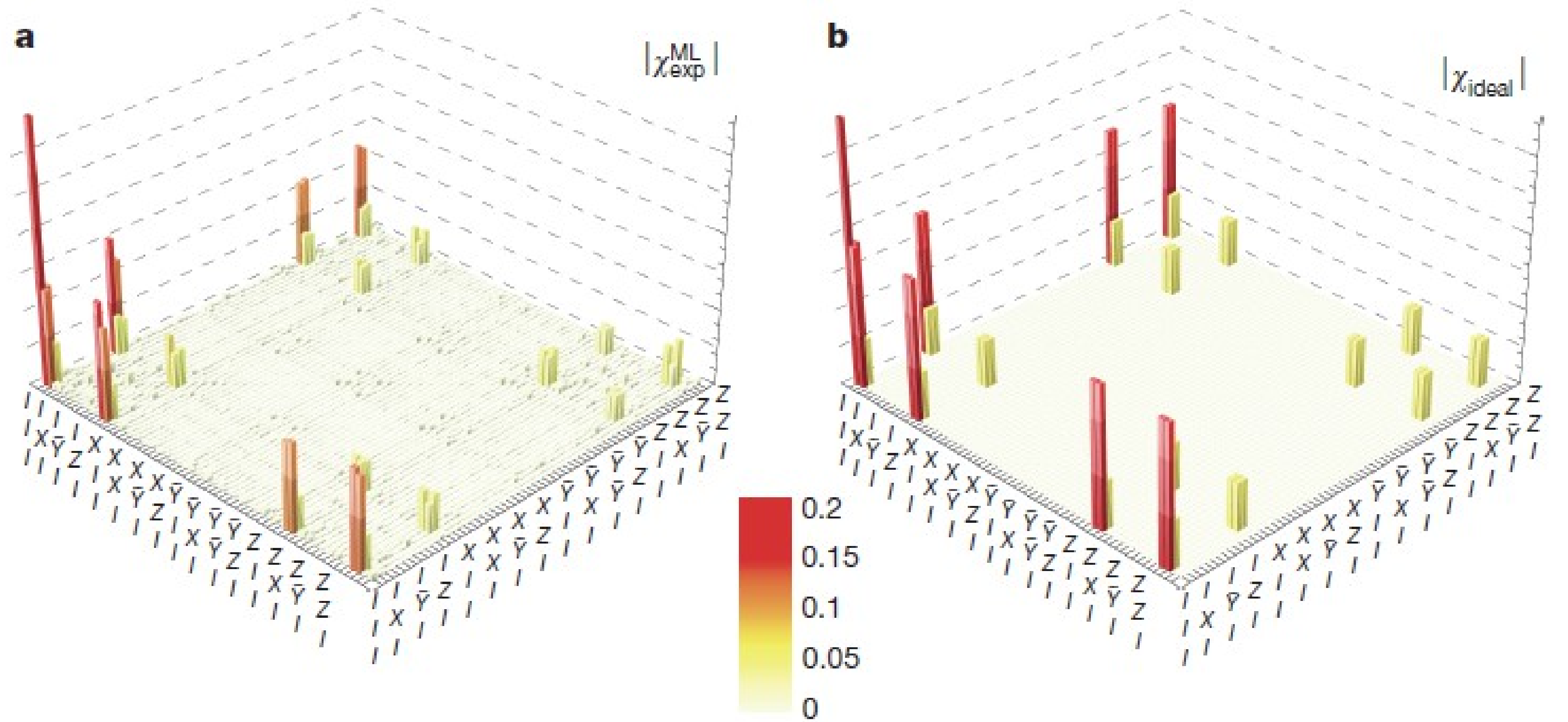


Fidelity: 76%

Implementation of the Toffoli Gate

State tomography

Fidelity: 69%



Basic Theory: Classical Error correction

- Idea: Introduce a redundancy

  $0 \rightarrow 000$

  $1 \rightarrow 111$

- If probability for a bit-flip is p, the probability for two or more bit-flips is $3p^2(1-p)+p^3=3p^2-2p^3$
- For $p<½$ the transmission reliability is increased

Basic Theory: QM Bit-Flip Error correction

- Idea:

  |0> → |000>

  |1> → |111>

  A|0> + B|1> → A|000> + B|111>

- Error detection for Bit-Flip errors:

  P_0=|000><000|+|111><111| No Error

  P_1=|100><100|+|011><011| Bit-Flip on Q1

  P_2=|010><010|+|101><101| Bit-Flip on Q2

  P_3=|001><001|+|110><110| Bit-Flip on Q3

Basic Theory: QM Bit-Flip Error correction

- Recovery: Perform the corresponding operation

  No Error → Do nothing

  Bit-Flip on Q_i → Flip Q_i
- Works if one or less Qubits are flipped,

  with probability $(1-p)^3+3p(1-p)^2=1-3p^2+2p^3$
- For $p< ½$ the reliability is increased

Basic Theory: QM Bit-Flip Error correction II

- Alternatively: Measure Z_1Z_2 and Z_2Z_3, where

  Z_1Z_2=(|00><00|+|11><11|)**Id* -(|01><01|+|10><10|)**Id*
- Z_1Z_2 gives +1 if Q_1 and Q_2 are the same and -1 else
- Use Q_2 as the target qubit, Q_1 and Q_3 as control
- Q_2 fliped if both measurements yield -1

→ For implementation, use a single qubit gate

that rotates Q_1 (Q_3) to |1> if Z_1Z_2 (Z_2Z_3) yields -1

and to |0> otherwise

Implementation of the Bit-Flip code

|Ψ>=A|0>+B|1>

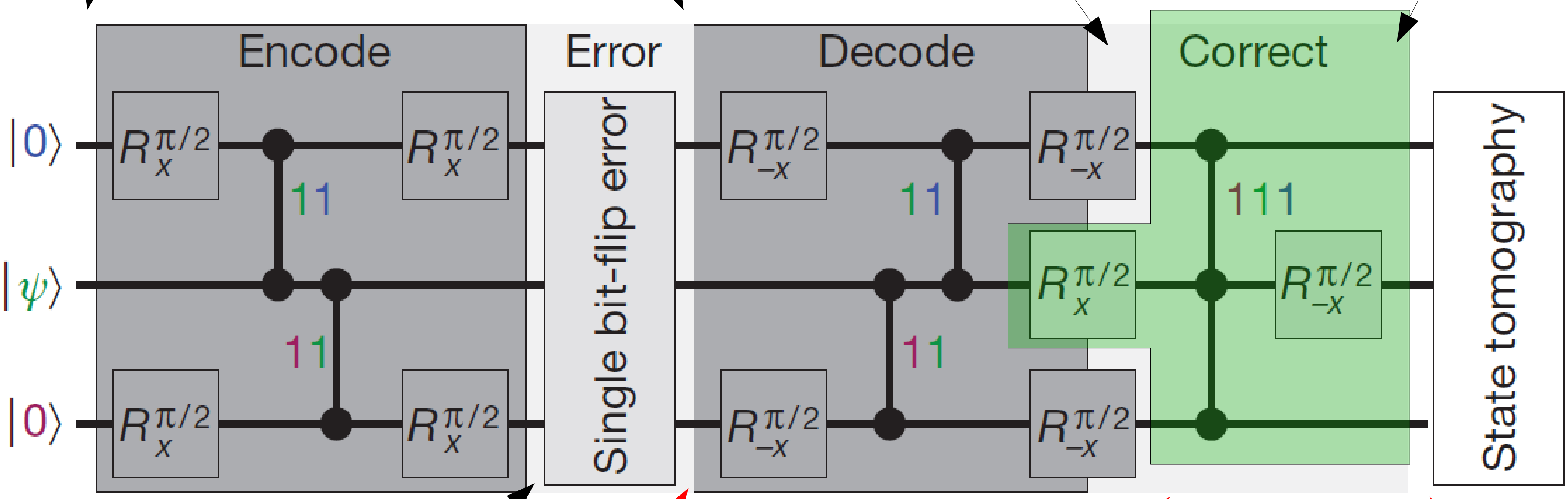
No Error:

A|000>+B|010>

A|000>+B|111>

A|000>+B|010>

Toffoli



A|000>+B|111>

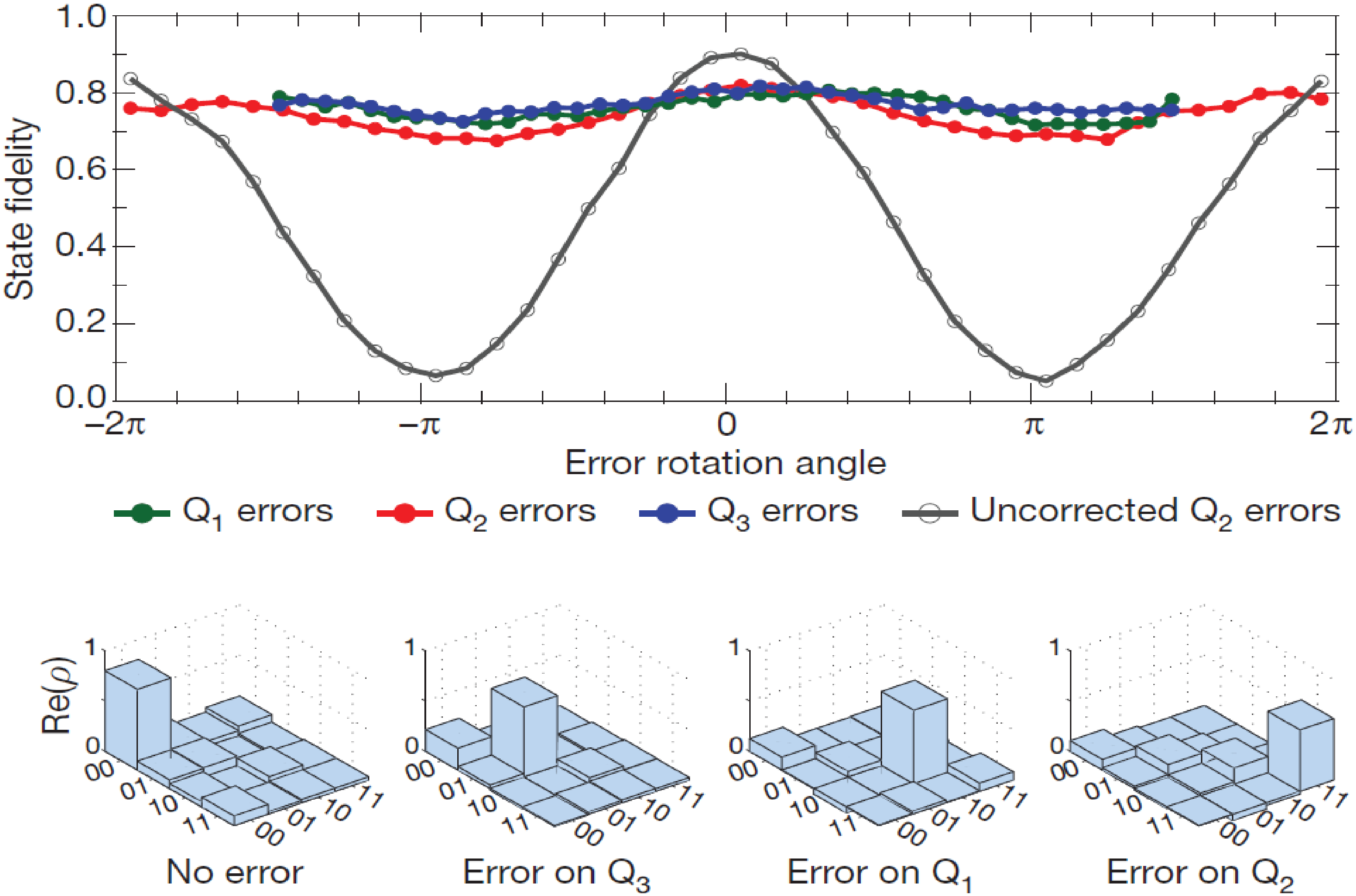
A|010>+B|101>

Bit-Flip in Q2:

A|111>+B|101>

A|101>+B|111>

Bit-Flip error correction: Results



Linear dependance in p vanished!

Basic Theory: QM Phase-Flip Error correction

- Error P flips relative phase of |0> and |1>:
  A|0> + B|1> → A|0> - B|1>
- Idea: Use the existing Bit-Flip correction scheme
- First apply a rotational gate (Hadamard)
  |±>=(|0>±|1>) → P|+>=|-> and P|->=|+>
- Map |0> → |+++> and |1> → |--->
- Use the same procedure as before, but in the |±> - Basis
- Apply the inverse rotation (Hadamard) to recover the initial state

Implementation of the Phase-Flip code

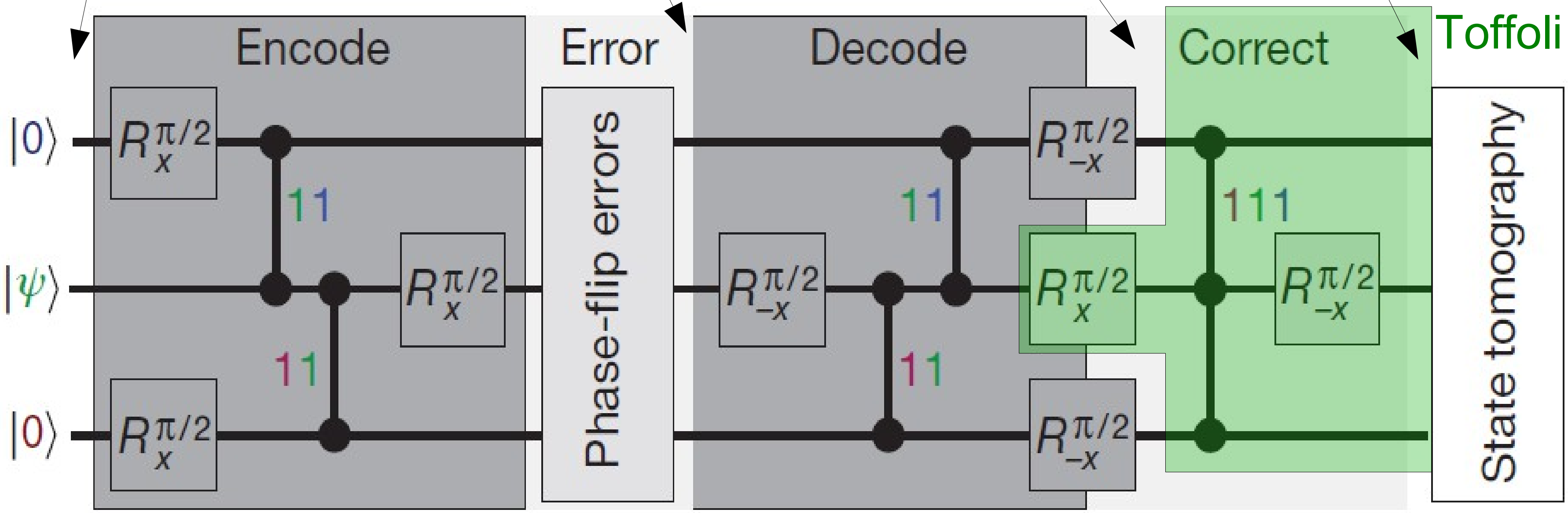
No Error:

A|000>+B|010>

A|+++>+B|--->

A|000>+B|010>

Toffoli



A|+++>+B|--->

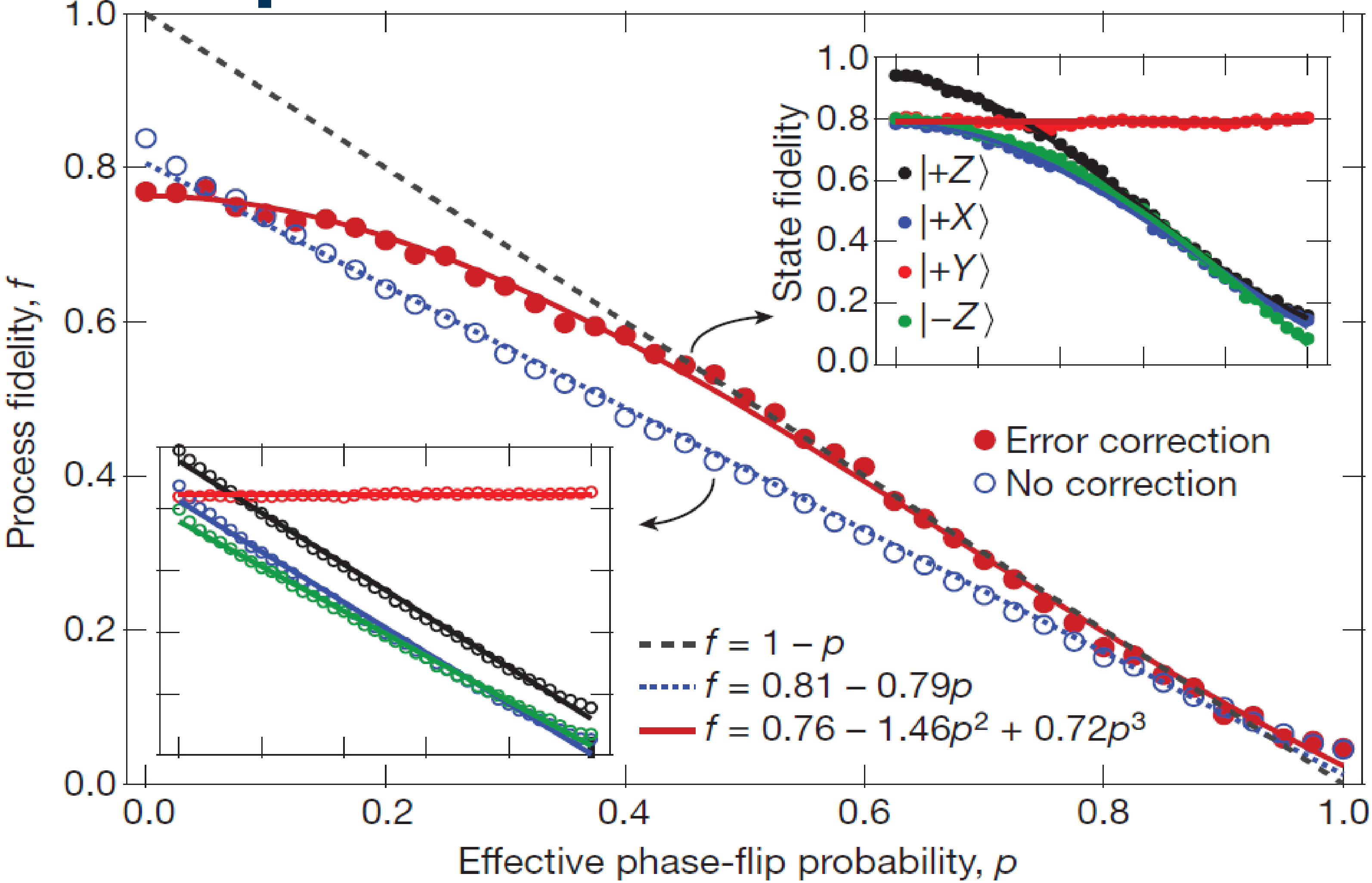
Phase-Flip in Q2:

A|+-+>+B|-+->

A|111>+B|101>

A|101>+B|111>

Phase-Flip error correction: Results



Error induced with probability $p=\sin^2(\Theta/2)$

Conclusion

- Toffoli gate implemented
- Higher excited levels reduce gate time
- Bit-Flip error correction code
- Phase-Flip error correction code

Outlook

- Error correction with Shor's 9-Qubit code
- Fault tolerant quantum computation

Discussion/Questions

References

- Papers:
  - Implementation of a Toffoli gate with superconducting circuits
    (A. Fedorov, L. Steffen, M. Baur, M.P. da Silva & A. Wallraff)
    Nature 481, 170-172 (2012)
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- Slides: QSIT lecture of A. Wallraff 2013