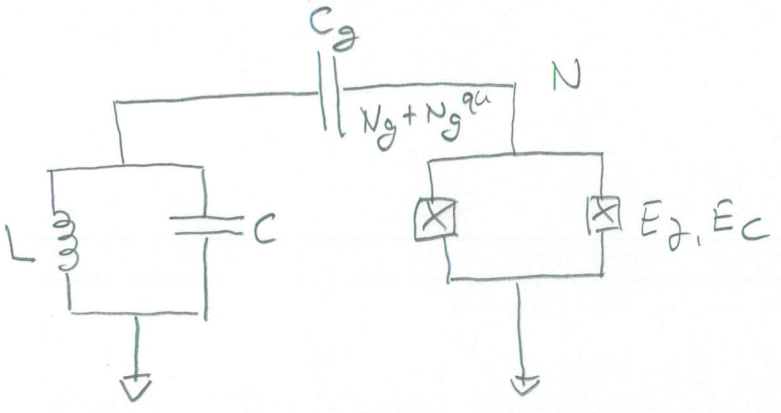


Jaynes - Cummings Hamiltonian in Circuit QED



$$\hat{H} = \underbrace{\frac{\hat{Q}^2}{2C} + \frac{\hat{\Phi}^2}{2L}}_{\text{H.O.}} + \frac{E_c}{2} \underbrace{(1 - 2(N_g + N_g^{qu}))}_{=0 \text{ at } N_g = \frac{1}{2}} \hat{\sigma}_z - \frac{E_J}{2} \hat{\sigma}_x$$

N_g^{qu} : quantum fluctuations of charge on capacitor C_g
 $N_g = \frac{1}{2}$: consider charge degeneracy

quantum fluctuations of harmonic oscillator

$$\hat{H}_{HO} = \frac{1}{2} C \hat{V}^2 + \frac{1}{2} L \hat{I}^2$$

- consider only voltage fluctuations with operator \hat{V}
- variance of voltage fluctuations

$$\hat{V} = \sqrt{\frac{\hbar \omega_r}{2C}} (\hat{a}^\dagger + \hat{a})$$

$$\Delta V^2 = \langle 0 | \hat{V}^2 | 0 \rangle - \underbrace{\langle 0 | \hat{V} | 0 \rangle^2}_{\text{mean voltage} = 0 \text{ for } |n\rangle = |0\rangle}$$

$$= \frac{\hbar \omega_r}{2C} \underbrace{\langle 0 | (\hat{a}^\dagger + \hat{a})^2 | 0 \rangle}_{=0} = \frac{\hbar \omega_r}{2C}$$

- with quantum fluctuations of charge

$$N_g^{qu} = \frac{C_g}{2e} \hat{V}^{qu} = \frac{C_g}{2e} \sqrt{\frac{\hbar \omega_r}{2C}} (\hat{a}^\dagger + \hat{a})$$

