

QSIT 2013 - Questions 4

12. April 2013, HIT F 13

1. Two-level approximation for a Cooper-pair box

The Hamiltonian for a Cooper-pair box is given by

$$H_{CPB} = \sum_n \left[E_C (\hat{n} - n_g)^2 |n\rangle\langle n| - \frac{E_J}{2} (|n\rangle\langle n+1| + |n+1\rangle\langle n|) \right].$$

- Plot first four eigenvalues of this Hamiltonian as a function of the gate charge n_g .
- Write down the Hamiltonian for the two-dimensional qubit subspace in terms of the Pauli matrices σ_x and σ_z by restricting the quantum states to $n = 0, 1$.
- What is the transition frequency between ground and excited state?

2. Measurement of relaxation and dephasing

Typically all experiments starts from the qubit being in its ground state $|0\rangle$. The state of the qubit can be manipulated with the help of the microwave pulses that can induce rotations by an angle ϕ about any of the axis x, y according to $R_{x,y}(\phi) = \exp[-i(\phi/2)\sigma_{x,y}]$ (note that the manipulation are performed in the rotating frame). In order to extract the relaxation time T_1 and T_2 we can use the following schemes:

- Make a π rotation (π -pulse) about x-axis, wait a time interval of Δt and measure the population of the excited state $|1\rangle$. Repeat experiment for different Δt .
- Make a $\pi/2$ rotation ($\pi/2$ -pulse) about x-axis, wait a time interval of Δt , make another $\pi/2$ -pulse and measure the population of the excited state $|1\rangle$. Repeat experiment for different Δt .

Using Bloch equations answer the following questions:

- (a) What is the evolution of the Bloch vector for these experiments?
- (b) How do we extract the timescales T_1 , T_2 ?
- (c) What happens if the microwave pulses are detuned from the frequency of the qubit by $\delta\omega$ for the second experiment. (Ramsey experiment)
- (d) What happens if the one insert π pulse exactly in the middle between $\pi/2$ pulses for the second experiment (Echo experiment).