QSIT FS 2015 Questions 1 - Solutions

1. State space in Quantum Mechanics
   a. A neutron is a spin 1/2 particle. In a constant magnetic field it will have two non-degenerate states. Hence, The Hilbert space describing such a neutron in a magnetic field is two dimensional
   b. In a gradient field, the splitting of the neutron's eigenstate energies will depend on the position. Thus, the relevant Hilbert space dimention is uncountably infinite.
   c. A small mirror attached to a spring can be described as a harmonic oscillator. A harmonic oscillator, has infinitely many equidistant states. The corresponding Hilbert space has countable infinite dimension
   d. Here we add a laser radiation. The total Hilbert space is a product of two Hilbert spaces. Namely, that of small mirror attached to a spring and of laser. Laser can also be described a harmonic oscillator.
   e. Hydrogen has infinitely many states. However, if we consider only the ground state then the Hilbert space has dimensionality of 2, because of two spin states.
   f. A laser beam of wavelength 121 nm induces transition of a Hydrogen atom between n=1, J=1/2 (two states) and n=2, J=3/2 (four states). Thus the relevant Hilbert space is six dimensional
   g. A Hydrogen atom at room temperature can be well approximated to be in ground state. So for N atoms we will have 2^N dimensional Hilbert .

2. Bloch sphere:

$$|\psi\rangle = \alpha|0\rangle + \beta|1\rangle$$

Azimutal angle: $\theta = 2\arctan\left|\frac{\beta}{\alpha}\right|$

Polar angle: $\varphi = \arg\left(\frac{\beta}{\alpha}\right)$

a) $\theta \approx 109.4^\circ$ $\quad \varphi = 0$

b) $\theta \simeq 109.4^\circ$ $\quad \varphi = \frac{3}{2}\pi$

c) $\theta \simeq 70.6^\circ$ $\quad \varphi = \frac{3}{2}\pi$

d) $\theta \simeq 109.4^\circ$ $\quad \varphi = \frac{3}{2}\pi$ (global phase is factored out)