NV Centers in Quantum Information Technology

De-Coherence Protection & Teleportation

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The NV Center

- Point Defect in Diamond
- Interesting Physics in negatively charged state NV$^{-1}$
- Total electron spin $S=1$
- $^{14}\text{N}$ Nuclear Spin $I=1$
Di Vincenzo Criteria

1. Well-defined qubits
2. Initialization
3. $t_{\text{coherence}} > t_{\text{gate operation}}$
4. Universal set of quantum gates
5. Qubit specific read-out
6. Convert from stationary to mobile qubit
7. Faithful transmission
Relevant Ground State Energy Structure

\[ |0\rangle_e \rightarrow |1\rangle_e \quad \text{2.9 MHz} \quad |0\rangle_N \rightarrow |\uparrow\rangle_N \quad \text{5.1 MHz} \quad |1\rangle_N \rightarrow |\downarrow\rangle_N \quad \text{1.4 GHz} \]

\[ B_0 = 500 \text{ G} \]
Relevant Ground State Energy Structure

Electron Spin Modulation: MW Rabi Driving at 1.4 GHz with driving strength 40 MHz

\[ |1\rangle_e \]
\[ |0\rangle_e \]
\[ B_0 = 500 \text{ G} \]

\[ |\uparrow\rangle_N \quad |\downarrow\rangle_N \]

5.1 MHz

2.9 MHz

1.4 GHz
Relevant Ground State Energy Structure

Electron Spin Modulation:
- MW Rabi Driving at 1.4 GHz with Rabi frequency at 40 MHz
- RF Rabi Driving at 2.9 MHz with Rabi frequency at 30 kHz

- 2 qubit register
- qubit modulation via Rabi driving
- entanglement through hyperfine interaction
Spin Initialization from Excited State

1) Electron Spin using LASER pumping

\[ m_S = 0 \]
\[ m_S = \pm 1 \]

Spin Initialization from Excited State

1) Electron Spin using LASER pumping

- Energy levels diagram showing transitions at 532 nm.
  - Initial state: $m_S=0$
  - Final state: $m_S=+/−1$

2) Nuclear Spin using LASER pumping at $B = 500$ G

- States $|0, \downarrow\rangle$ and $|+1, \uparrow\rangle$ are populated.
  - The nuclear spin flips from $|+1, \uparrow\rangle$ to $|0, \downarrow\rangle$.

References:

Read-Out

PL Spectrum of optically excited NV Center:

- $m_S = 0$ is bright ($E_x$)
- $m_S = -1$ is dark ($A_1$)

Read-Out

PL Spectrum of optically excited NV Center:

- $m_S = 0$ is bright ($E_x$)
- $m_S = -1$ is dark ($A_1$)

Can also be used to read out $m_i$ by using a CNOT gate:

Decoherence

Decoherence is caused by all the undesired interactions of a quantum state with its environment which shortens its lifetime.

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- Dynamic decoupling: Periodic flipping of the qubit spin state to average out the interactions with the environment.


Dynamical Decoupling

Figure 1. Dynamical decoupling pulse sequences. The empty and solid rectangles represent 90° and 180° pulses, respectively, and \( N \) represents the number of iterations of the cycle. (a) Initial state preparation. (b) Hahn spin-echo sequence. (c) CPMG sequence. (d) CDD sequence of order \( N \), \( \text{CDD}_N = C_{n-1} \) and \( C_0 = t \).

Dynamical decoupling sequences with a single rotation axis are achieved by iteratively applying to the system a series of stroboscopic control pulses in cycles of period \( t_c \) [44]. Over that period, the time-averaged SE interaction can be described by an averaged or effective Hamiltonian [88]. The goal of DD is the elimination of the effective SE interaction. This can be seen by looking at Hahn's pioneering spin-echo experiment [31] (figure 1b). It is based on the application of a \( p \)-pulse to the spin system at a time \( t \) after the spins were left to evolve in the magnetic field. This pulse effectively changes the sign of the SE interaction—in this case, the Zeeman interaction with the magnetic field. Letting the system evolve for a refocusing period or time reversed evolution during the same duration \( t \) generates the echo. If the magnetic field is static, the dynamics is completely reversed and the initial state of the spin recovered. However, if the magnetic field fluctuates, its effect cannot be reversed completely. Thus, the echo amplitude decays as a function of the refocusing time [31, 32]. This decay contains information about the time-dependence of the environment.

To reduce the decay rate of the echo due to a time-dependent environment, Carr and Purcell introduced a variant of the Hahn spin-echo sequence, where the single \( p \)-pulse is replaced by a series of pulses separated by intervals of duration \( t \) [32]. This CP sequence reduces the changes induced by the environment if the pulse intervals are shorter than the correlation time of the environment. However, as the number of pulses increases, pulse errors tend to accumulate. Their combined effect can destroy the state of the system, rather than preserving it against the effect of the environment. This was noticed by Meiboom & Gill [33] who proposed a modification of the CP sequence for compensating pulse errors, the CPMG sequence.

Decoherence in multi-qubit gates

1) Qubits couple to each other but also to environment

Decoherence in multi-qubit gates

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2) Qubits decoupled from each other and environment

Decoherence in multi-qubit gates

1) Qubits couple to each other but also to environment

2) Qubits decoupled from each other and environment

3) Qubits only decoupled from environment

Qubit Coupling

Generally desirable

Fast coupling for fast qubit manipulation

But we pay a price

We also get faster coupling to the environment
Qubit Coupling

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Generally desirable
Fast coupling for fast qubit manipulation

But we pay a price
We also get faster coupling to the environment
"Fast" and "Slow" Qubits

Encode Physical Qubits in:

- atomic states
- superconducting circuits
- quantum dots
- NV centers
"Fast" and "Slow" Qubits

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Two Qubit Gates

Using "fast" qubit as the control bit

Question
Can we use dynamical decoupling to make a gate using the "fast" qubit as our control bit?
Two Qubit Gates

Difficult Scenario
Using "fast" qubit as the control bit
Two Qubit Gates

Difficult Scenario
Using "fast" qubit as the control bit

Question
Can we use dynamical decoupling to make a gate using the "fast" qubit as our control bit?
"Fast" and "Slow" Qubits; NV Centers

"Fast" qubit: electronic spin

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"Fast" and "Slow" Qubits; NV Centers

"Fast" qubit: electronic spin

- GHz energy splitting
"Fast" and "Slow" Qubits; NV Centers

"Fast" qubit: electronic spin

- GHz energy splitting
- $T_2 = 3.5\mu s$; Rabi $2\pi$ pulse: $20ns$
“Fast” and “Slow” Qubits; NV Centers

“Fast” qubit: electronic spin
- GHz energy splitting
- $T_2 = 3.5\,\mu s$; Rabi $2\pi$ pulse: 20\,ns

“Slow” qubit: nuclear spin

![NV center diagram]
"Fast" and "Slow" Qubits; NV Centers

"Fast" qubit: electronic spin
- GHz energy splitting
- $T_2 = 3.5 \mu s$; Rabi $2\pi$ pulse: 20ns

"Slow" qubit: nuclear spin
- MHz energy splitting
"Fast" and "Slow" Qubits; NV Centers

"Fast" qubit: electronic spin

- GHz energy splitting
- $T_2 = 3.5\mu s$; Rabi $2\pi$ pulse: $20\,ns$

"Slow" qubit: nuclear spin

- MHz energy splitting
- $T_2 = 5.3\,ms$; Rabi $2\pi$ pulse: $30\,\mu s$
Two Qubit Gates

Imagine
Two Qubit Gates

Imagine

$$\alpha |0\rangle + \beta |1\rangle$$
Two Qubit Gates

Imagine

\( \alpha |0\rangle + \beta |1\rangle \)
Two Qubit Gates

Imagine

\[ \alpha |0\rangle + \beta |1\rangle \]

\[ R_x(\pi)[\alpha |0\rangle + \beta |1\rangle] \]
Two Qubit Gates

Imagine

\[ \alpha |0\rangle + \beta |1\rangle \]
Two Qubit Gates

Not obvious whether this can work
Two Qubit Gates

Not obvious whether this can work
Building a 2-Qubit Gate

Electronic Spin

\[ m_S = 0 : |0\rangle \]
\[ m_S = -1 : |1\rangle \]

Nuclear Spin

\[ m_I = +1 : |\uparrow\rangle \]
\[ m_I = 0 : |\downarrow\rangle \]
Building a 2-Qubit Gate

Electronic Spin
\[ m_S = 0 : |0\rangle \]
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\[ m_I = +1 : |\uparrow\rangle \]
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Building a 2-Qubit Gate

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\[ m_S = 0 : |0\rangle \]
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Nuclear Spin

\[ m_I = +1 : |↑\rangle \]
\[ m_I = 0 : |↓\rangle \]

Timescales (\(\mu s\))

\[ T_{2,e} \]

\[ 3.5 \]
Building a 2-Qubit Gate

Electronic Spin

\[ m_S = 0 : |0\rangle \]
\[ m_S = -1 : |1\rangle \]

Nuclear Spin

\[ m_I = +1 : |\uparrow\rangle \]
\[ m_I = 0 : |\downarrow\rangle \]

Timescales (\(\mu s\))

\begin{align*}
T_{2,e} & : 3.5 \\
T_{\text{Rabi},n} & : 30
\end{align*}
Building a 2-Qubit Gate

Electronic Spin

\[ m_S = 0 : \vert 0 \rangle \]
\[ m_S = -1 : \vert 1 \rangle \]

Nuclear Spin

\[ m_I = +1 : \vert \uparrow \rangle \]
\[ m_I = 0 : \vert \downarrow \rangle \]

Timescales (\(\mu s\))

\[ T_{2,e} \quad T_{\text{Rabi,n}} \quad T_{2,e-\text{spin echo}} \]

\(1.4 \text{ GHz}\)
\(2.9 \text{ MHz}\)
\(5.1 \text{ MHz}\)
Building a 2-Qubit Gate

Decoupling Pulse Sequence

$\tau - X - 2\tau - Y - \tau$

Electronic Qubit in State $|0\rangle$

$\exp(-i\sigma_z \theta_0) \exp(-i\sigma_x 2\theta_1 \hbar) \exp(-i\sigma_z \theta_0)$

Electronic Qubit in State $|1\rangle$

$\exp(-i\sigma_x \theta) \exp(-i\sigma_z \theta_0) \exp(-i\sigma_x \theta_1 \hbar)$
Building a 2-Qubit Gate

Decoupling Pulse Sequence

\[ \tau - X - 2\tau - Y - \tau \]

Electronic Qubit in State \(|0\rangle\)

\[ \exp\left(\frac{-i\sigma_z\theta_0}{\hbar}\right)\exp\left(\frac{-i\sigma_x2\theta_1}{\hbar}\right)\exp\left(\frac{-i\sigma_z\theta_0}{\hbar}\right) \]
Building a 2-Qubit Gate

Decoupling Pulse Sequence
\[ \tau - X - 2\tau - Y - \tau \]

Electronic Qubit in State \(|0\rangle\)
\[ \exp\left(\frac{-i\sigma_z\theta_0}{\hbar}\right) \exp\left(\frac{-i\sigma_x 2\theta_1}{\hbar}\right) \exp\left(\frac{-i\sigma_z\theta_0}{\hbar}\right) \]

Electronic Qubit in State \(|1\rangle\)
\[ \exp\left(\frac{-i\sigma_x\theta_1}{\hbar}\right) \exp\left(\frac{-i\sigma_z 2\theta_0}{\hbar}\right) \exp\left(\frac{-i\sigma_x\theta_1}{\hbar}\right) \]
Building a 2-Qubit Gate

Special case 1

\[ \tau = (2n + 1)\pi/A \]
Building a 2-Qubit Gate

Special case 1
\[ \tau = (2n + 1)\pi / A \]

Example

[Diagram of a 2-qubit gate with a conditional rotation gate and labels for elements and time intervals]
Building a 2-Qubit Gate

Special case 1

\[ \tau = (2n + 1)\pi / A \]

Example
Building a 2-Qubit Gate

Special case 1
\[ \tau = (2n + 1)\pi/A \]

Example
Building a 2-Qubit Gate

Special case 2

\[ \tau = 2n\pi / A \]
Building a 2-Qubit Gate

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\[ \tau = 2n\pi / A \]

Example
Building a 2-Qubit Gate

Special case 2

\[ \tau = 2n\pi /A \]

Example
Building a 2-Qubit Gate

Special case 2

$$\tau = 2n \pi / A$$

Example
Building a 2-Qubit Gate

Combine special cases 1 and 2
obtain a conditional rotation gate

[Diagram of a quantum gate with labels 'el.', 'nucl.', and 'R_x(\theta)']
Experimental Results

\[ CNOT \text{ Gate (} \theta = \pi \text{)} \]

Process fidelity:
\[ F_p = \text{Tr}(\chi_{\text{ideal}} \chi) = 83\% \]

For a State \[ \left| \psi \right\rangle = \alpha \left| 0 \right\rangle + \beta \left| 1 \right\rangle \]

\[ \rho = \left| \psi \right\rangle \langle \psi \right| \]

For an Operator \[ A = \alpha I + \beta \sigma_x + \gamma \sigma_y + \delta \sigma_z \]

\[ \varepsilon(\rho) = A \rho A^\dagger = \sum_{i,j} \chi_{ij} E_i \rho E_j^\dagger \]
Experimental Results

CNOT Gate \( (\theta = \pi) \)

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CNOT Gate \((\theta = \pi)\)

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Testing Gate Robustness

Inject noise into the diamond
Reduce $T_{2,SE}$ from 251$\mu s$ to 50$\mu s$
Testing Gate Robustness

Inject noise into the diamond
Reduce $T_{2,SE}$ from $251\mu s$ to $50\mu s$

Reduce RF drive power to nuclear spin
Gate time increases to $120\mu s$
Testing Gate Robustness

Inject noise into the diamond
Reduce $T_{2,SE}$ from 251 $\mu$s to 50 $\mu$s
Reduce RF drive power to nuclear spin
Gate time increases to 120 $\mu$s

Single qubit decoupling
apply $(\tau - \pi - \tau)^N$
Testing Gate Robustness

Inject noise into the diamond
Reduce $T_{2,SE}$ from 251$\mu$s to 50$\mu$s
Reduce RF drive power to nuclear spin
Gate time increases to 120$\mu$s

Single qubit decoupling
apply $(\tau - \pi - \tau)^N$

$T_{2,N=16} = 234\mu$s
Testing Gate Robustness

Apply CNOT

Input state
\((|0\rangle + i|1\rangle) \otimes |\uparrow\rangle\)

Desired output state
\(|\psi\rangle = (|0 \uparrow\rangle + |1 \downarrow\rangle)/\sqrt{2}\)
Testing Gate Robustness

Apply CNOT

Input state
$$(|0⟩ + i|1⟩) \otimes |↑⟩$$

Desired output state
$$|ψ⟩ = (|0⟩ \uparrow + |1⟩ \downarrow)/\sqrt{2}$$
Testing Gate Robustness

Apply CNOT

Input state

\((|0⟩ + i|1⟩) \otimes |↑⟩\)

Desired output state

\(|\psi⟩ = (|0⟩ |↑⟩ + |1⟩ |↓⟩)/\sqrt{2}\)

State Fidelity

\(N = 16 : F = \sqrt{\langle ψ | ρ | ψ \rangle}\) reaches 96%
Running Grover’s Algorithm

Recall: Search Algorithm

- Find entry in list of $N$ elements
- Number of oracle calls scales as $\sqrt{N}$
Running Grover’s Algorithm

Recall: Search Algorithm

- Find entry in list of \( N \) elements
- Number of oracle calls scales as \( \sqrt{N} \)
Running Grover’s Algorithm

\[ \alpha = \pi/2 \text{- pulse on electron} \quad \pi \text{- pulse on electron} \]

CPhase (1↓)

\[ a = \text{nuclear } \pi/2 \text{ rotation around axis } a \]

CPhase (0↑)

Total time, 322 μs
Running Grover’s Algorithm

Final State Fidelity > 90%
Summary

- Can construct 2-qubit gate protected from decoherence
- Especially useful when control bit is “fast”
- Achieved process fidelities above 80%, and state fidelities above 90% using an NV center
- Ultimate goal: $10^{-4}$
Summary

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- Ultimate goal: $< 10^{-4}$
Quantum Teleportation

NV - Centers
Framework

• Unconditional teleportation
  • Any state can be transmitted

• Remoteness
  • Sender and receiver are reasonably separated (3m)
Entanglement

- Remote entanglement between NV electrons
  - Local entanglement: Spin rotation / Spin-selective excitation
    **Electron-Photon**
  - Local entanglement: Quantum interference photon detection
    **Photon-Photon**
Teleporter Setup

Configuration

• Alice NV-Center:
  Transmission Qubit (1) \textit{Nuclear spin}
  Messenger Qubit (2) \textit{Electron spin}

• Bob NV-Center:
  Receiver Qubit (3) \textit{Electron spin}

• Qubits 2 & 3 entangled in \(|\Psi^-\rangle_{23}\)
Teleporter Setup

Initialization

• Transmission Qubit initialized in $|1\rangle_1$

• Projective measurement of Messenger

• Prior to entanglement

• Source State $|\psi\rangle_1 = \alpha|0\rangle_1 + \beta|1\rangle_1$

• After entanglement to avoid Dephasing
Teleporter Setup

Final State

- Final State in Bell basis:

\[
|\psi\rangle_1 \otimes |\Psi^-\rangle_{23} = \frac{1}{2} \left[ |\Phi^+\rangle_{12} (\alpha|1\rangle_3 - \beta|0\rangle_3) + |\Phi^-\rangle_{12} (\alpha|1\rangle_3 + \beta|0\rangle_3) + |\Psi^+\rangle_{12} (-\alpha|0\rangle_3 + \beta|1\rangle_3) + |\Psi^-\rangle_{12} (-\alpha|0\rangle_3 - \beta|1\rangle_3) \right]
\]
Teleportation

- Interaction between Qubits 1 and 2
- CNOT followed by $\pi/2$ Y-rotation of Transmitter
- Projective measurements
- Conditional Pauli-rotations
Teleportation

Interaction

• Nuclear rotations controlled by Electron excitation level:

  • Controlled $\pi/2$ Y-rotation (on 1 controlled by 2)
  $\pi$ Y-rotation (unconditional on 2)
  Controlled $\pi/2$ Y-rotation (on 1 controlled by 2)

  Effectively: $\pi/2$ Y-rotation (unconditional on 1)
Teleportation Interaction

- Overall state after interaction:

\[
R_{y1}(\pi/2)U_{CNOT}(|\psi\rangle_1 \otimes |\Psi^-\rangle_{23}) = \\
\frac{1}{2} \left[ |11\rangle_{12}(\alpha|1\rangle_3 - \beta|0\rangle_3) + |01\rangle_{12}(\alpha|1\rangle_3 + \beta|0\rangle_3) + |10\rangle_{12}(\alpha|0\rangle_3 - \beta|1\rangle_3) + |00\rangle_{12}(\alpha|0\rangle_3 + \beta|1\rangle_3) \right]
\]
Teleportation

Interaction

• Overall state after interaction:

\[
R_{y1}(\frac{\pi}{2}) U_{CNOT}(|\psi\rangle_1 \otimes |\Psi^-\rangle_{23}) = \frac{1}{\sqrt{2}} \left[ |11\rangle_{12} (\sigma_{xz} |\psi\rangle_3 )
+ |01\rangle_{12} (\sigma_x |\psi\rangle_3 )
+ |10\rangle_{12} (\sigma_z |\psi\rangle_3 )
+ |00\rangle_{12} (I |\psi\rangle_3 ) \right]
\]
Teleportation

Measurement

- Direct measurement on messenger
- Projective measurement on transmitter
- CNOT on $|0\rangle_2$ electron (on reinitialized messenger, controlled by transmitter)
  Direct measurement on messenger
Teleportation

Pauli rotations

• Depending on measurement:

\[ |00\rangle_{12} \mapsto \mathbb{1} \]
\[ |10\rangle_{12} \mapsto \sigma_z \]
\[ |01\rangle_{12} \mapsto \sigma_x \]
\[ |11\rangle_{12} \mapsto \sigma_{xz} \]
Results

- Tomography for Y on Bob’s side to confirm alignment of reference frames

- 6 unbiased states transmitted. **Fidelity 0.77**
Outlook

• Remote Entanglement
  Mutliple Qubits per node:
  
  • NV Centers are a good candidate for Quantum networks
  
  • Entanglement fidelity high enough to close detection loophole of Bell Inequality