

# QSIT 2015 - Questions 2

2015, HIT F 13

## 1. Density matrix of a qubit entangled with another one

The density operator formalism is used to describe a quantum system whose state is not completely known. Suppose a quantum system is in state  $|\psi_i\rangle$  with respective probability  $p_i$ . The density operator for the system is defined as

$$\rho = \sum_i p_i |\psi_i\rangle\langle\psi_i|.$$

Let us consider a system of two qubits, which is described by  $|\psi_{AB}\rangle$  and let  $\hat{O}$  be an observable of the qubit A. Then its expectation value is described by

$$\langle O \rangle = \text{tr}[\rho_A \hat{O}],$$

where  $\rho_A = \text{tr}_B[\rho_{AB}]$  is the reduced density operator of qubit A. For maximally entangled states such as the Bell states,  $\rho_A$  describes a maximally mixed state.

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- (a) Suppose that the system is in state  $|\Psi^+\rangle$ . What is the state of qubit A ignoring the state of qubit B?
  - (b) What are the expectation values for  $\sigma_x^A$ ,  $\sigma_y^A$ ,  $\sigma_z^A$  for the  $|\Psi^+\rangle$  state?

## 2. Quantum State Tomography.

To determine the state of a quantum system a specific number of measurements have to be performed on identically prepared systems. From the results of such a complete set of measurements the state can then be fully characterized.

- (a) How many measurements do you need to determine the quantum state of the system?
- (b) Write down explicitly, what measurements can be used and how you can infer the state from the results of these measurements.
- (c) How is the number of required measurements related to the normalization of the state? What does it mean, if the state is found to be not normalized?
- (d) Which measurements are required to characterize a state of two qubits?

**3. Quantum search algorithm (Grover's algorithm)**

You are given a search space with  $N = 16$  elements, and the element you are looking is encoded in the state  $|5\rangle$ .

- (a) How many qubits do you need to encode the whole search space?
- (b) Construct the needed oracle.
- (c) How many Grover iterations are needed to find the needed element?