QSIT 2013 - Questions 5

19. April 2013, HIT F 13

1. **Microwave drive of a CPB**
   Consider a Cooper-pair box subject to an external microwave drive coupled through the gate capacitor.

   (a) Starting from the full Hamiltonian of the CPB derive the following Hamiltonian in the two level approximation:

   $$H = -\frac{\hbar\omega_0}{2}\sigma_z + A\cos(\omega t)\sigma_x$$

   (b) What will be the Hamiltonian in a frame rotating around z-axis at the same frequency as the drive.

   (c) Which terms can be neglected and why?

2. **Resonator-qubit interaction in dispersive regime**
   The coupling of a qubit to a resonator in the rotating wave approximation is written as

   $$H = \hbar\omega_r\left(a^\dagger a + \frac{1}{2}\right) - \frac{\hbar\omega_q}{2}\sigma_z + \hbar g(a^\dagger\sigma^- + a\sigma^+).$$

   Here, ω_r and ω_q are the frequencies of the resonator and the qubit, respectively, and g is their coupling strength.

   In the dispersive regime when the qubit and the resonator frequencies are far detuned ($|\Delta| = |\omega_q - \omega_r| \gg g$), diagonalization of this Hamiltonian to the lowest order in g leads to

   $$H \approx \left(\hbar\omega_r + \frac{\hbar g^2}{\Delta}\sigma_z\right)\left(a^\dagger a + \frac{1}{2}\right) - \frac{\hbar\omega_q}{2}\sigma_z$$

   .

(a) How can we measure the qubit state using a resonator which is dispersively coupled to the qubit?

(b) How does the qubit energy depend on the resonator state?

(c) Can we measure the state of the resonator using the qubit?