Rydberg atoms
part 1

Tobias Thiele

Content

- Part 1: Rydberg atoms
- Part 2: 2 typical (beam) experiments

References

- T. Gallagher: Rydberg atoms

Introduction – What is „Rydberg“?

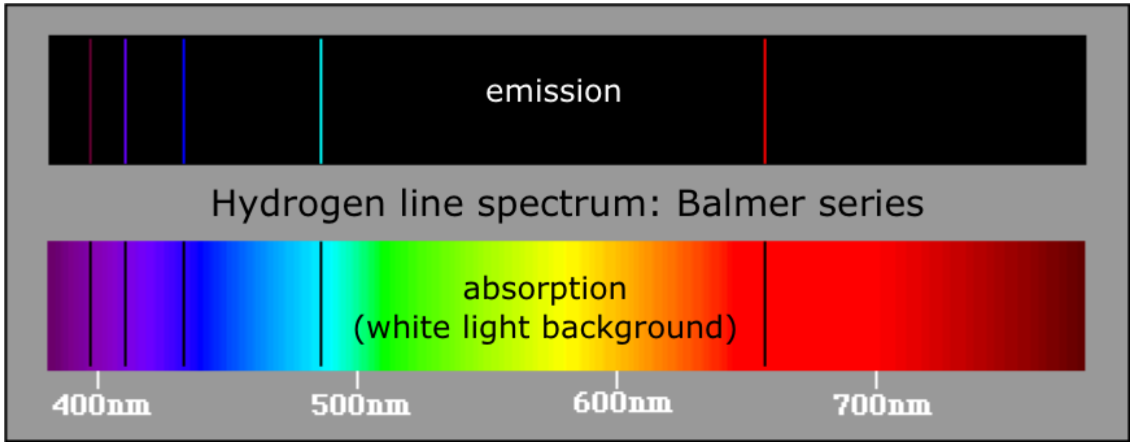
- Rydberg atoms are (any) atoms in state with high principal quantum number *n.*

- Rydberg atoms are (any) atoms with exaggerated properties

equivalent!

Introduction – How was it found?

- In 1885: Balmer series:
  - Visible absorption wavelengths of H:

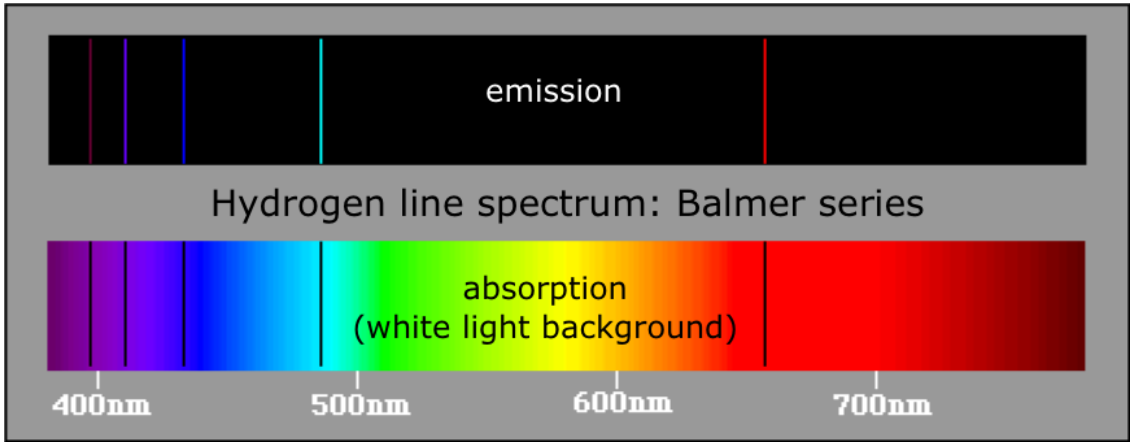


$$\lambda = \frac{bn^2}{n^2 - 4}$$

  - Other series discovered by Lyman, Brackett, Paschen, ...
  - Summarized by Johannes Rydberg: $\tilde{\nu} = \tilde{\nu}_\infty - \frac{Ry}{n^2}$

Introduction – Generalization

- In 1885: Balmer series:
  - Visible absorption wavelengths of H:



$$\lambda = \frac{bn^2}{n^2 - 4}$$

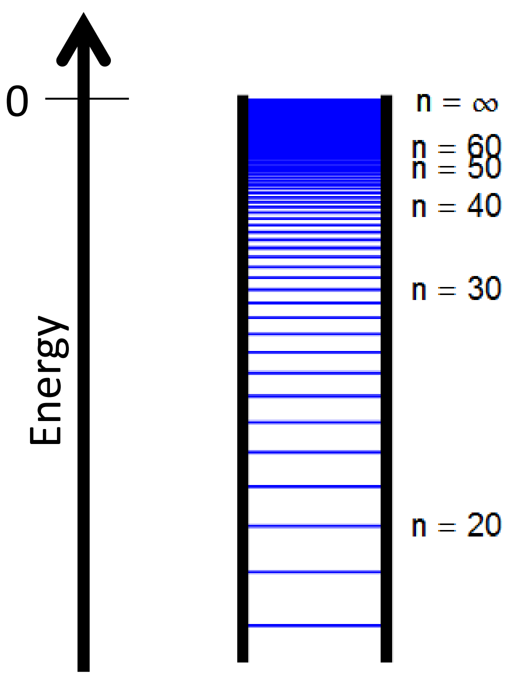
  - Other series discovered by Lyman, Brackett, Paschen, ...
  - Quantum Defect was found for other atoms: $\tilde{\nu} = \tilde{\nu}_\infty - \frac{Ry}{(n-\delta_l)^2}$

Introduction – Rydberg atom?

Hydrogen

- Energy follows Rydberg formula:

$$E = \cancelto{0}{E_\infty} - \frac{\underbrace{hRy}_{=13.6\text{ eV}}}{(n - \cancelto{0}{\delta_l})^2}$$

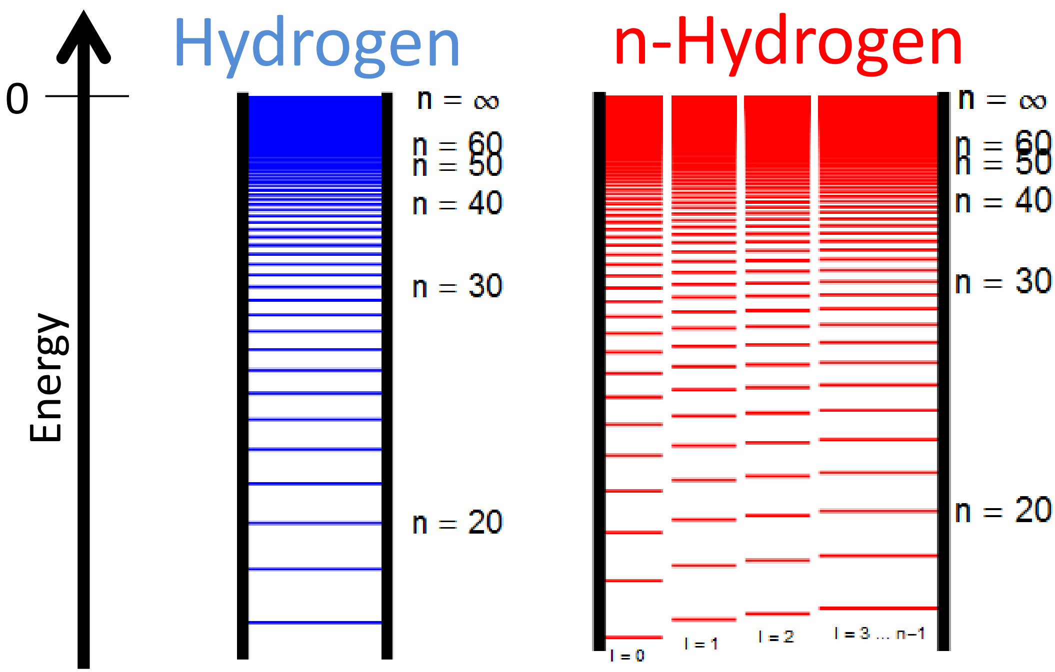


Quantum Defect?

- Energy follows Rydberg formula:

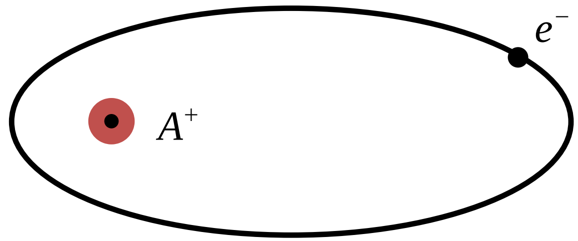
$$E = E_\infty - \frac{hRy}{(n - \delta_l)^2}$$

Quantum Defect



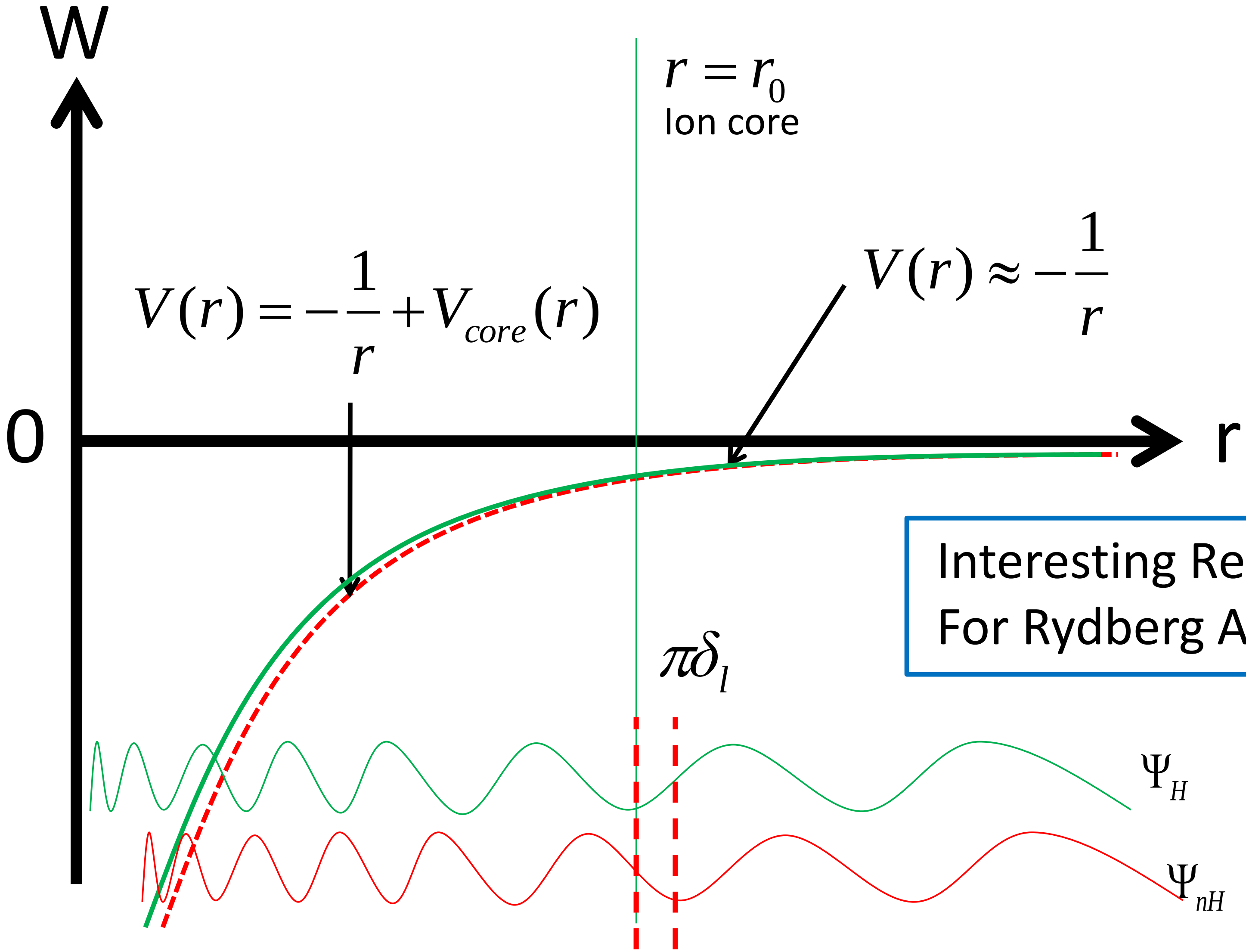
Rydberg Atom Theory

- Rydberg Atom



- Almost like Hydrogen
  - Core with one positive charge
  - One electron
- What is the difference?
  - No difference in angular momentum states

Radial parts-Interesting regions



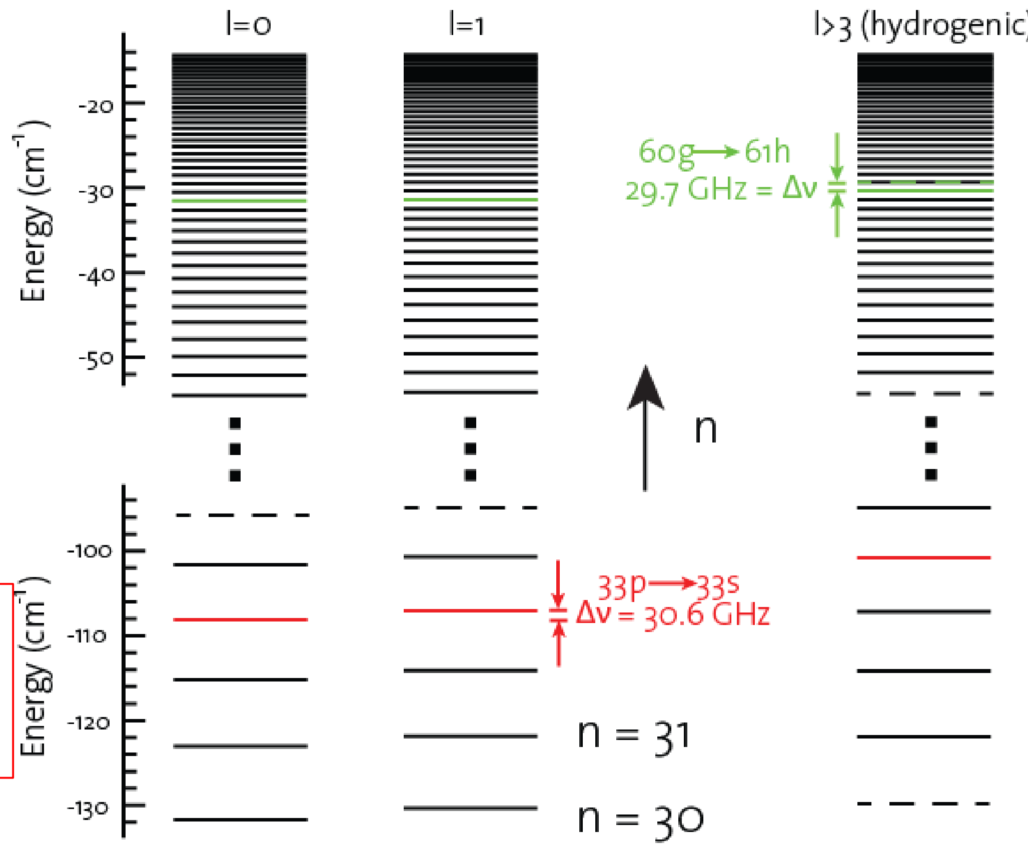
Interesting Region
For Rydberg Atoms

(Helium) Energy Structure

$$W = -\frac{1}{2(n-\delta_l)^2}$$

- δ_l usually measured
  - Only large for low l (s,p,d,f)
- He level structure
- δ_l is big for s,p

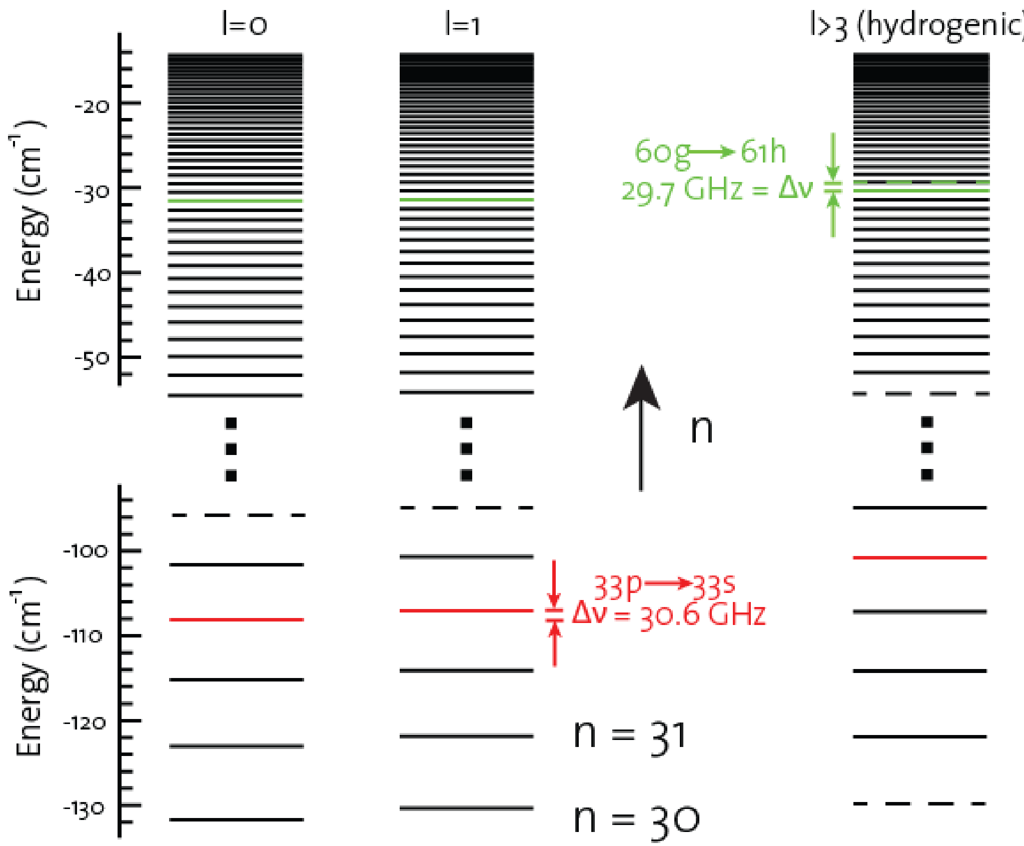
Excentric orbits penetrate into core.
Large deviation from Coulomb.
Large phase shift-> large quantum defect



(Helium) Energy Structure

$$\boxed{W = -\frac{1}{2(n-\delta_l)^2}}$$

- δ_l usually measured
  - Only large for low l (s,p,d,f)
- He level structure
- δ_l is big for s,p
- $$\boxed{\frac{dW}{dn} = \frac{1}{(n-\delta_l)^3}}$$



Electric Dipole Moment

- Electron most of the time far away from core
  - Strong electric dipole: $\vec{d} = e\vec{r}$
  - Proportional to transition matrix element $\left\langle \Psi_f \middle| \vec{d} \middle| \Psi_i \right\rangle = e\left\langle \Psi_f \middle| \vec{r} \middle| \Psi_i \right\rangle = e\left\langle \Psi_f \middle| r\cos(\theta) \middle| \Psi_i \right\rangle$
- We find electric Dipole Moment
  - $\left\langle \Psi_f \middle| \vec{d} \middle| \Psi_i \right\rangle \propto \left\langle r \right\rangle \left\langle l \pm 1 \middle| \cos(\theta) \middle| l \right\rangle \propto n^2$
- Cross Section: $\sigma \propto \left\langle r \right\rangle^2 \propto n^4$

$W = -\frac{1}{2(n-\delta_l)^2}$	$\frac{dW}{dn} = \frac{1}{(n-\delta_l)^3}$

Stark Effect $H\Psi = \left(H_0 + \vec{d}\vec{F}\right)\Psi = E\Psi$

- For non-Hydrogenic Atom (e.g. Helium)
  - „Exact“ solution by numeric diagonalization of

$$\left\langle \Psi_f \left| H \right| \Psi_i \right\rangle = \left\langle \Psi_f \left| H_0 \right| \Psi_i \right\rangle + \left\langle \Psi_f \left| \vec{d} \right| \Psi_i \right\rangle \vec{F}$$

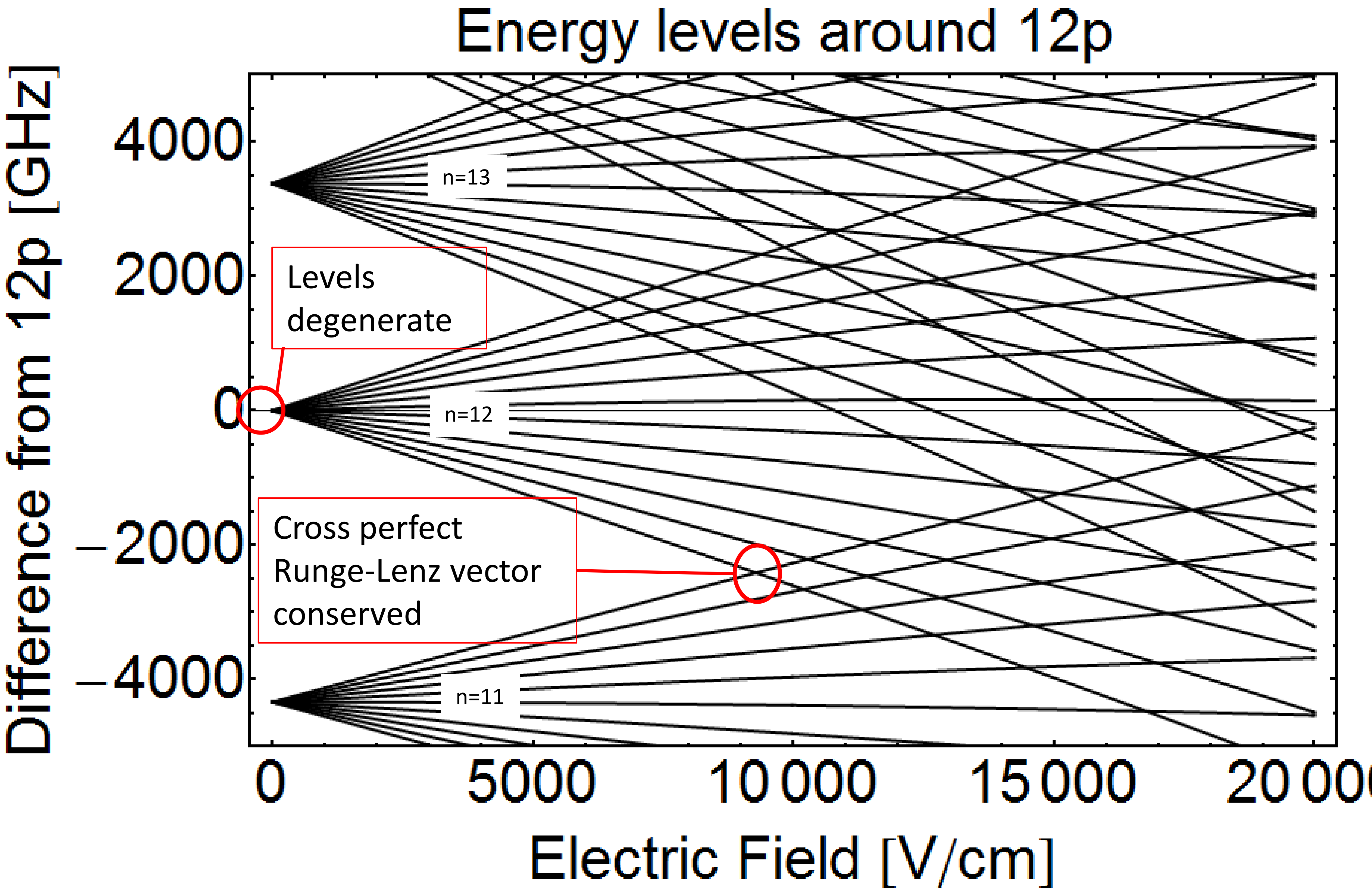
in undisturbed (standard) basis ($\hat{n}$,l,m)

$$W = -\frac{1}{2(n-\delta_l)^2}$$

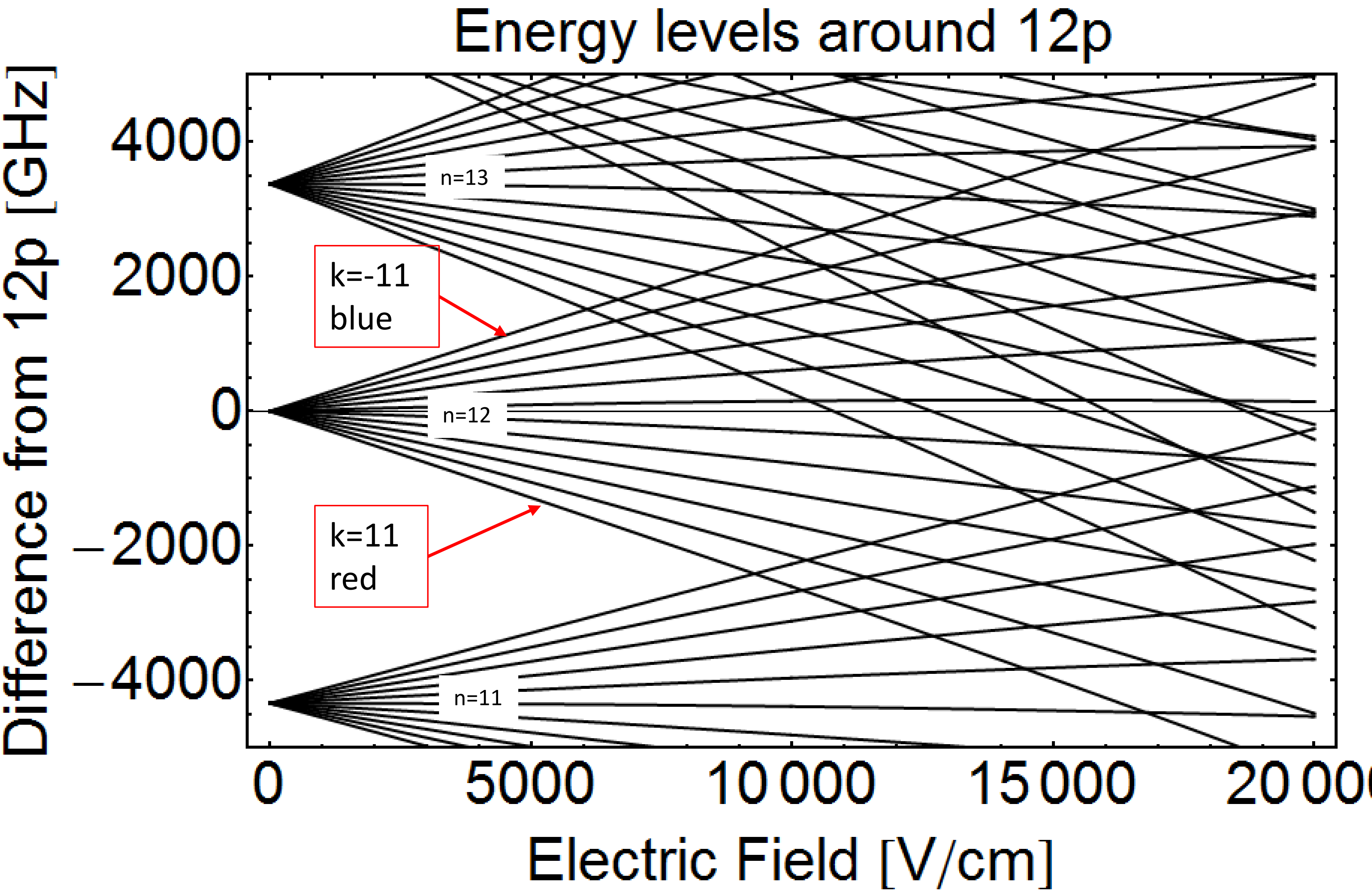
Numerov

$W = -\frac{1}{2(n-\delta_l)^2}$	$\frac{dW}{dn} = \frac{1}{(n-\delta_l)^3}$	$\left\langle \vec{d} \right\rangle \approx a_0 n^2$	$\sigma \propto n^4$

Stark Map Hydrogen



Stark Map Hydrogen



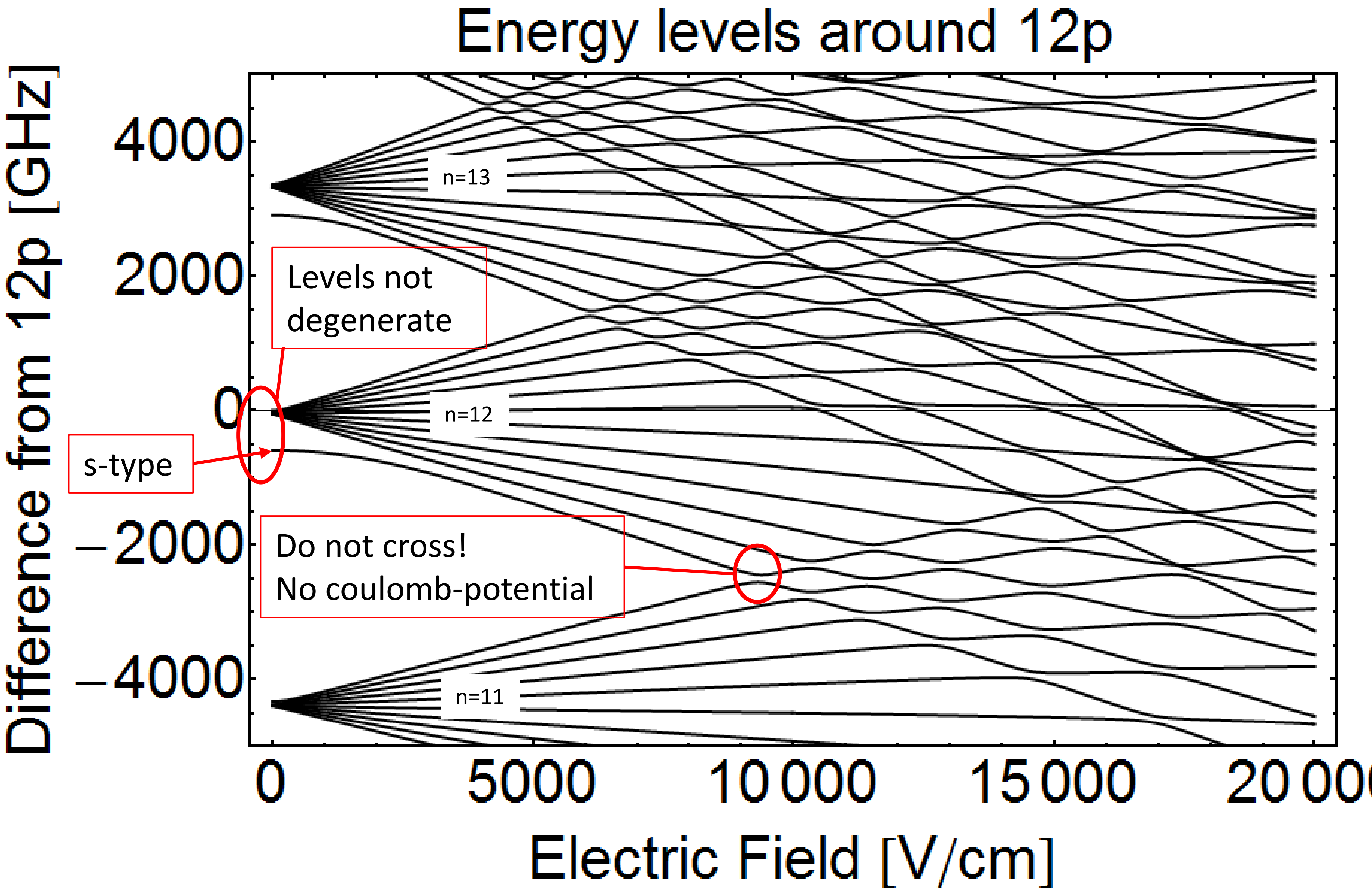
Hydrogen Atom in an electric Field

- Rydberg Atoms very sensitive to electric fields
  - Solve: $H\Psi = \left(H_0 + \vec{d}\vec{F}\right)\Psi = E\Psi$ in parabolic coordinates

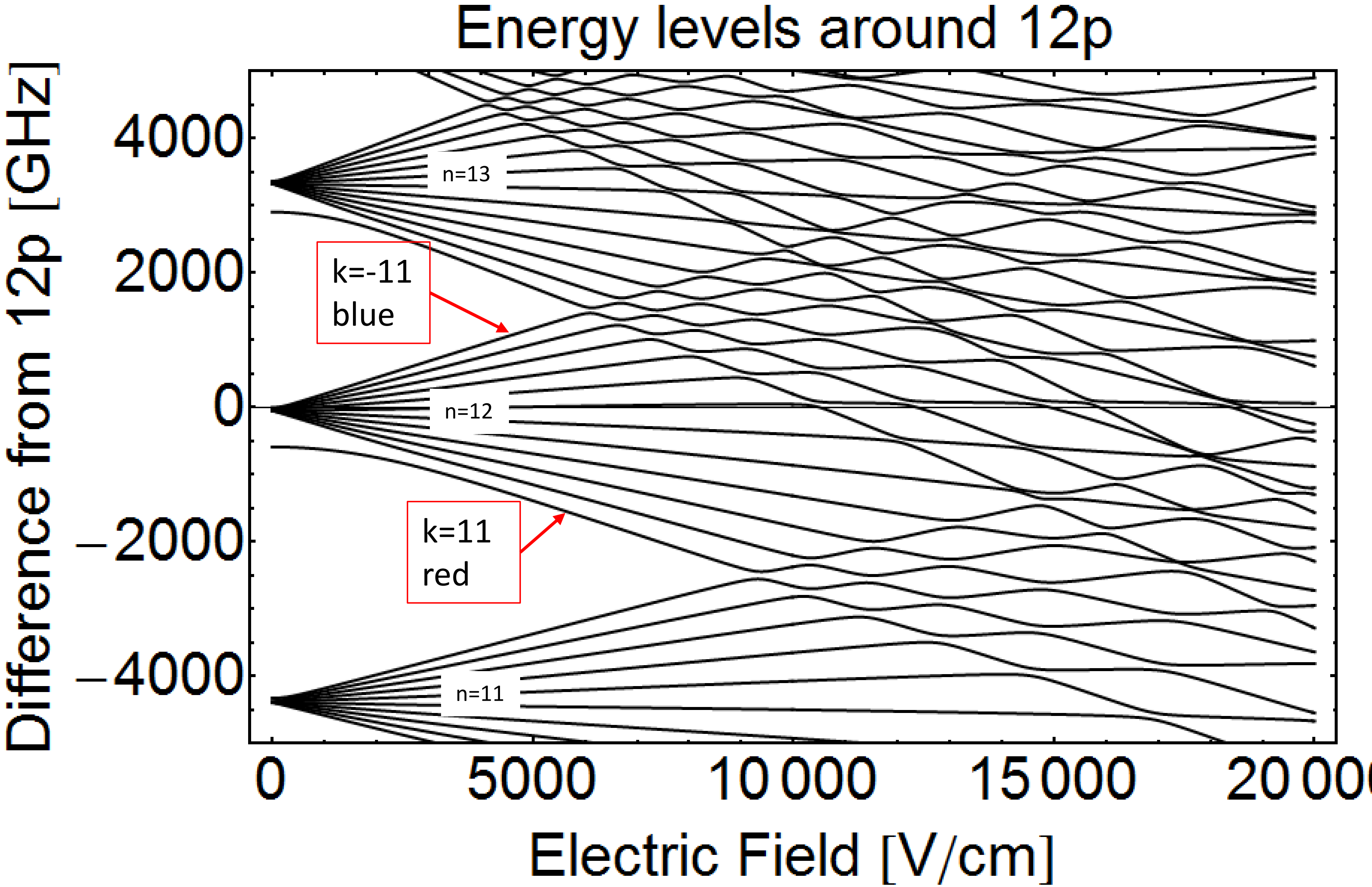
- Energy-Field dependence: Perturbation-Theory

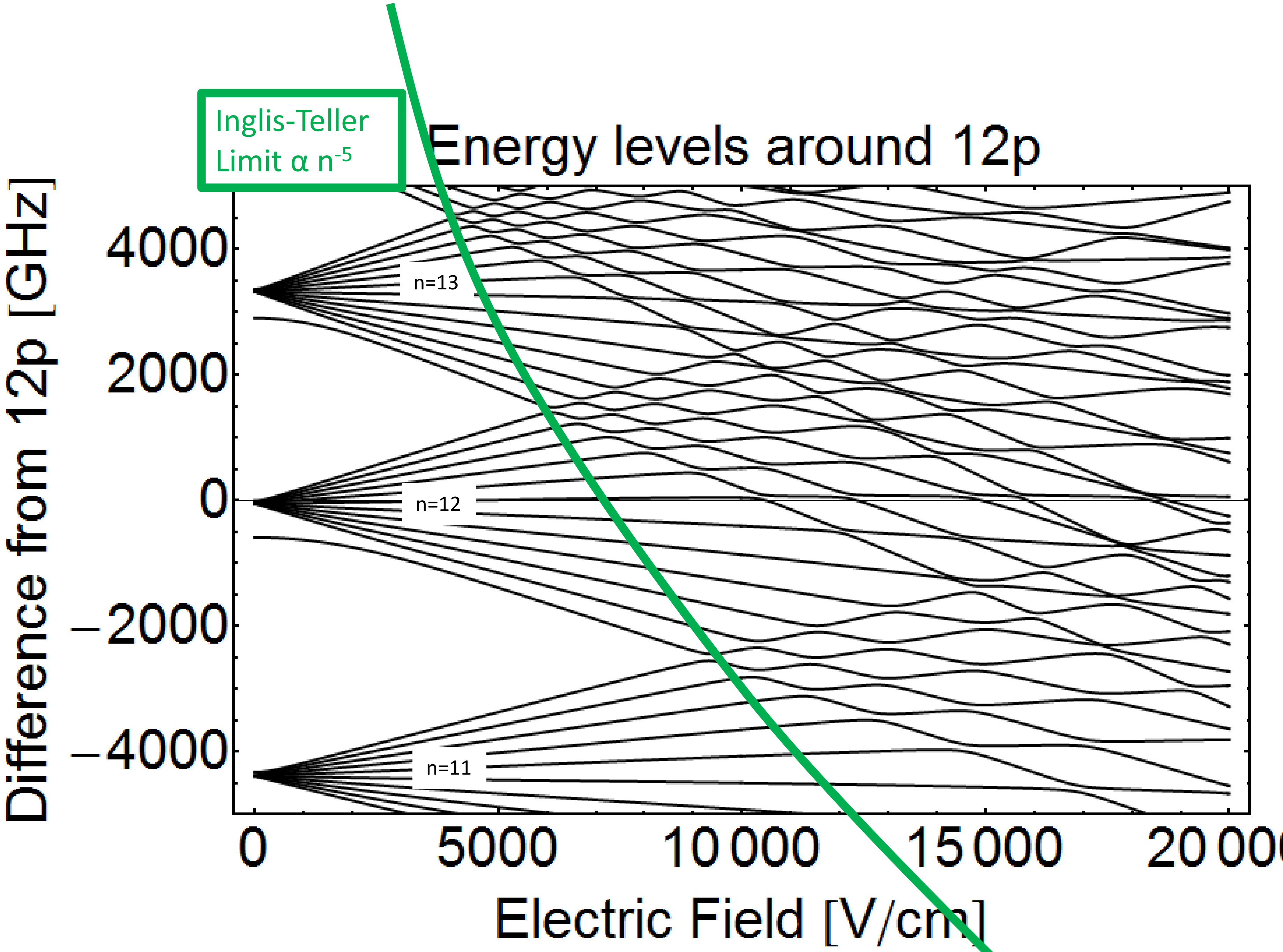
$$W = -\frac{1}{2n^2} - \frac{3}{2}F\underbrace{(n_1 - n_2)}_{k}n + \frac{F}{16}n^4\left(17n^2 - 3\underbrace{(n_1 - n_2)}_{k}^2 - 9m^2 + 19\right) + O(n^5)$$

Stark Map Helium



Stark Map Helium

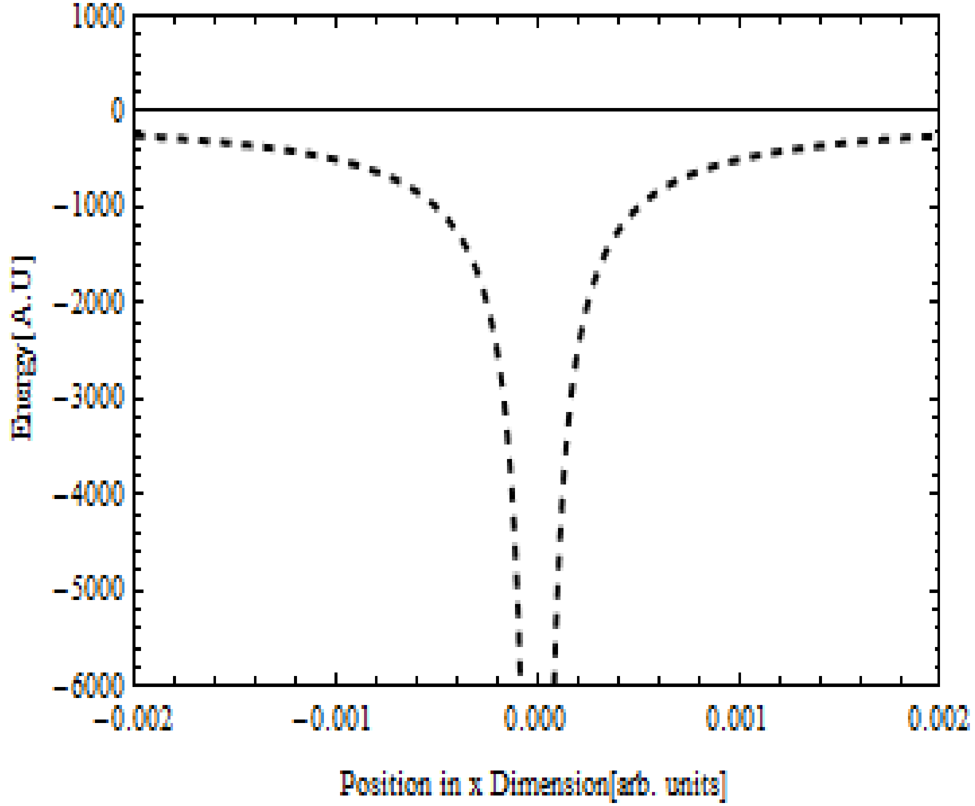




Inglis-Teller
Limit α n-5
Energy levels around 12p
Difference from 12p [GHz]
4000
2000
0
−2000
−4000
n=13
n=12
n=11
0
5000
10 000
15 000
20 000
Electric Field [V/cm]

Rydberg Atom in an electric Field

- When do Rydberg atoms ionize?
  - No field applied



$W = -\frac{1}{2(n-\delta_l)^2}$	$\frac{dW}{dn} = \frac{1}{(n-\delta_l)^3}$	$\langle \vec{d} \rangle \approx a_0 n^2$	$\sigma \propto n^4$	$F_{IT} \propto n^{-5}$

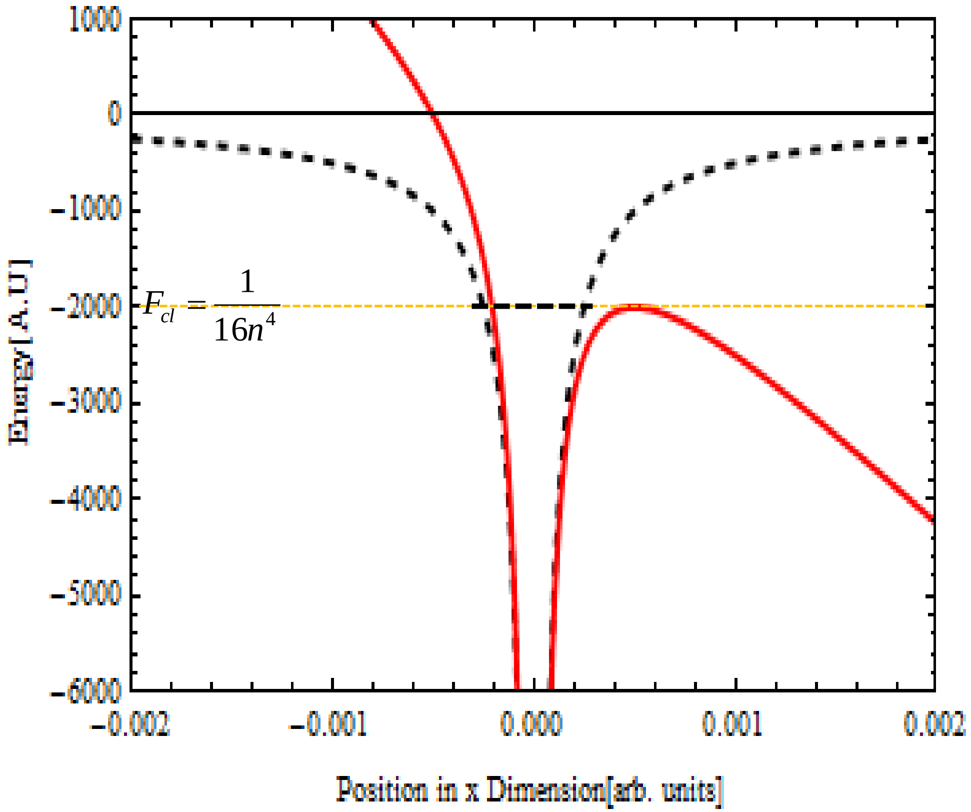
Rydberg Atom in an electric Field

- When do Rydberg atoms ionize?
  - No field applied
  - Electric Field applied
  - Classical ionization:

$$V = -\frac{1}{r} + Fz$$

$$\Rightarrow F_{cl} = \frac{W^2}{4} = \frac{1}{16n^4}$$

  - Valid only for
    - Non-H atoms if F is Increased slowly



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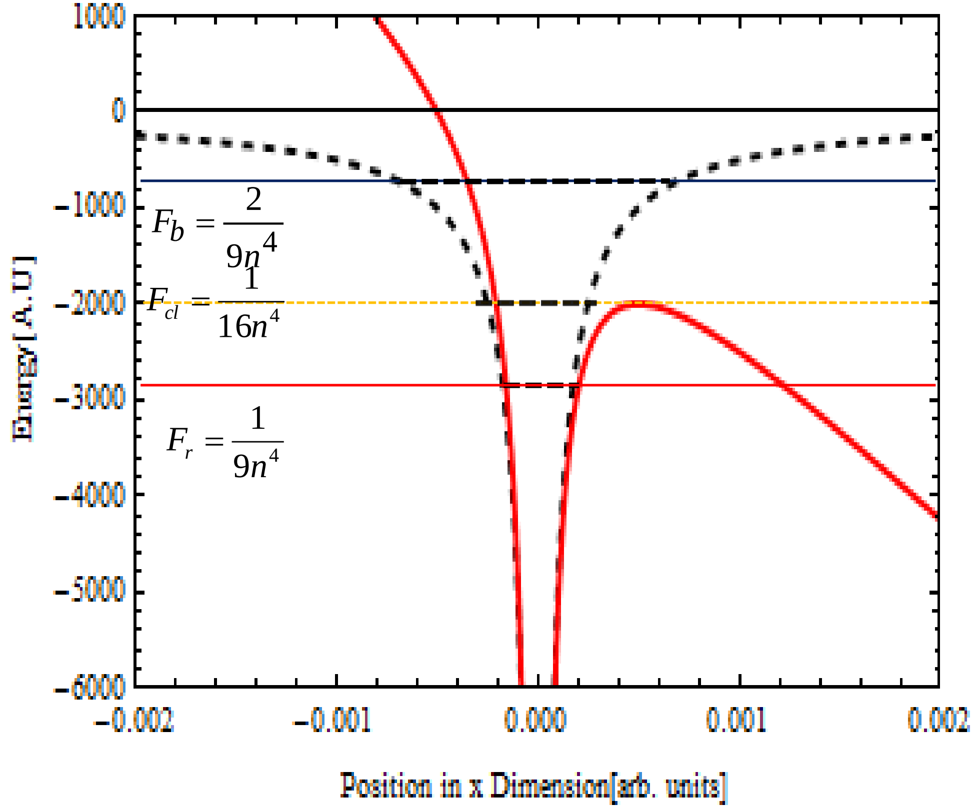
(Hydrogen) Atom in an electric Field

- When do Rydberg atoms ionize?
  - No field applied
  - Electric Field applied
  - Quasi-Classical ioniz.:

$$V(\eta) = 2\left(-\frac{Z_2}{\eta} + \frac{m^2-1}{4\eta^2} - \frac{F\eta}{4}\right)$$

$$\Rightarrow F = \frac{W^2}{4Z_2}$$



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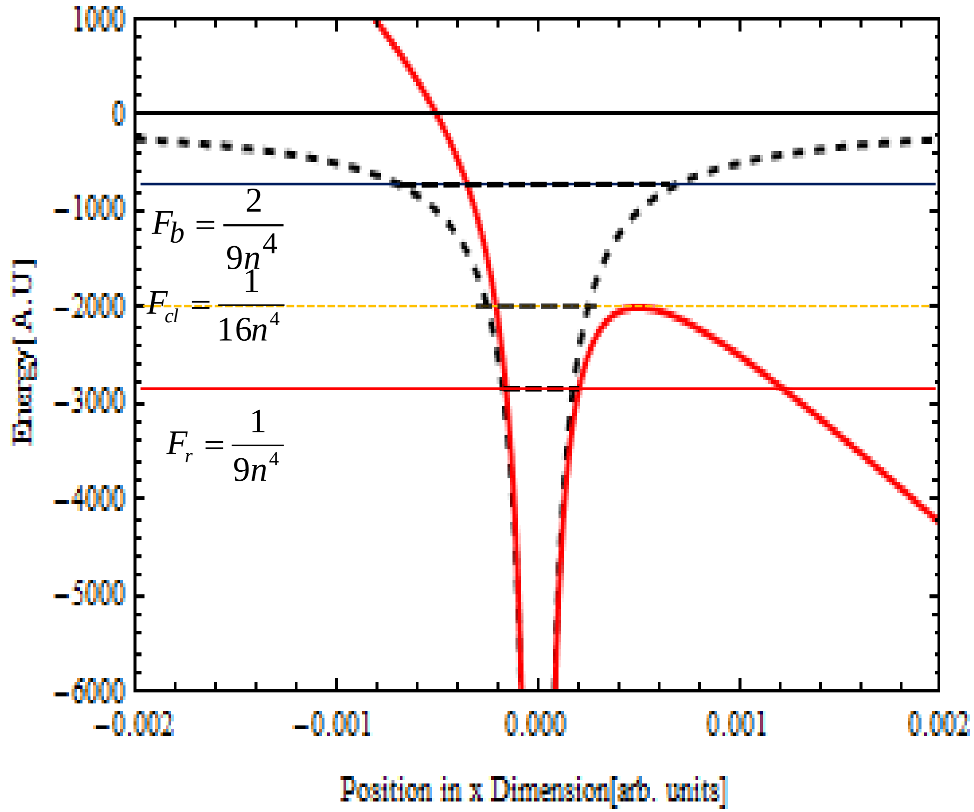
$$V(\eta) = 2\left(-\frac{Z_2}{\eta} + \frac{m^2-1}{4\eta^2} - \frac{F\eta}{4}\right)$$

$$\Rightarrow F = \frac{W^2}{4Z_2}\left|\begin{array}{l} = \dfrac{1}{9n^4} \quad \text{red} \\ \approx \dfrac{2}{9n^4} \quad \text{blue} \end{array}\right.$$



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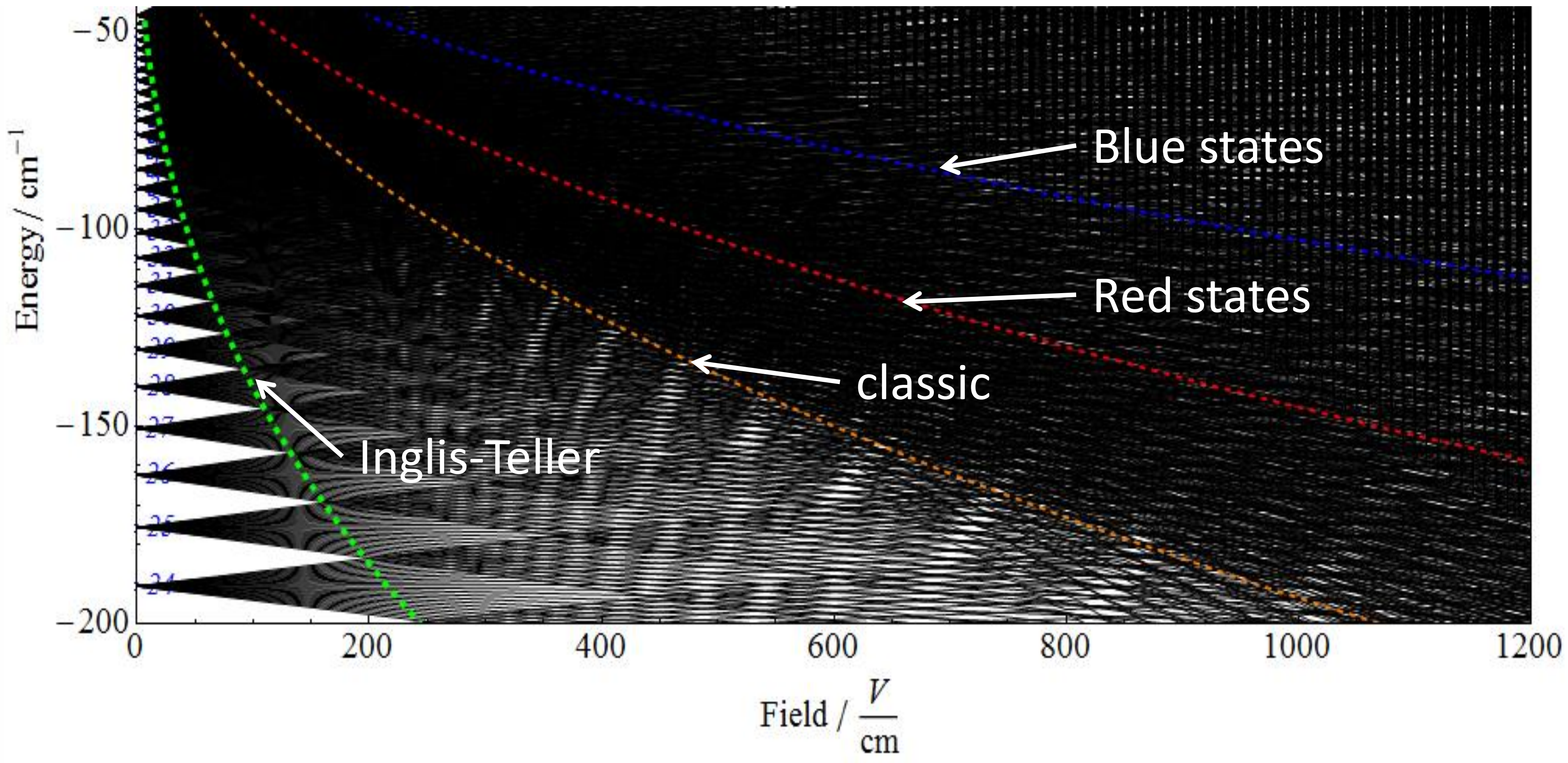
(Hydrogen) Atom in an electric Field



Lifetime

- From Fermis golden rule
  - Einstein A coefficient for two states $n, l \rightarrow n', l'$

$$A_{n',l',n,l} = \frac{4e^2 \omega^3_{n',l',n,l}}{3\hbar c^3} \frac{\max(l,l')}{2l+1} \left| \langle n'l' | r | nl \rangle \right|^2$$

  - Lifetime $\tau_{n,l} = \left(\sum_{n',l'<n,l} A_{n',l',n,l} \right)^{-1}$

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  - Lifetime $\tau_{n,l} = \left(\sum_{n',l'<n,l} A_{n',l',n,l}\right)^{-1}$

For l≈0:
Constant (dominated by decay to GS)

For l≈0: $\propto n^{-3/2}$
Overlap of WF

$W = -\frac{1}{2(n-\delta_l)^2}$	$\frac{dW}{dn} = \frac{1}{(n-\delta_l)^3}$	$\langle \vec{d} \rangle \approx a_0 n^2$	$\sigma \propto n^4$	$F_{IT} \propto n^{-5}$	$F_{cl} \propto \frac{1}{16} n^{-4}$	$\tau_{n,0} \propto n^3$

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For l ≈ n: $\propto n^2$
Overlap of WF

For l ≈ n: $\propto n^{-3}$
Overlap of WF

$W = -\frac{1}{2(n-\delta_l)^2}$	$\frac{dW}{dn} = \frac{1}{(n-\delta_l)^3}$	$\langle \vec{d} \rangle \approx a_0 n^2$	$\sigma \propto n^4$	$F_{IT} \propto n^{-5}$	$F_{cl} \propto \frac{1}{16} n^{-4}$	$\tau_{n,l} \propto n^3, n^5$

Lifetime

$$A_{n',l',n,l} = \frac{4e^2\omega^3_{n',l',n,l}}{3\hbar c^3}\frac{\max(l,l')}{2l+1}\left|\langle n'l'|r|nl\rangle\right|^2 \qquad \tau_{n,l} = \left(\sum_{n',l'<n,l} A_{n',l',n,l}\right)^{-1}$$

State	Stark State 60 p (n',l') small	Circular state 60 l=59 m=59 $(n',l') \approx (n\pm1, l\pm1)$	Statistical mixture
Scaling	n^3 (overlap of $\psi \propto n^{-3/2}$)	n^5 $\langle r\rangle \propto n^2$	$n^{4.5}$
Lifetime	7.2 µs	70 ms	≈ms

$W = -\frac{1}{2(n-\delta_l)^2}$	$\frac{dW}{dn} = \frac{1}{(n-\delta_l)^3}$	$\langle\vec{d}\rangle \approx a_0 n^2$	$\sigma \propto n^4$	$F_{IT} \propto n^{-5}$	$F_{cl} \propto \frac{1}{16} n^{-4}$	$\tau_{n,l} \propto n^3, n^5$