Recap

exercise 5/6

Full qubit Hamiltonian for j^{th} qubit:

(1) $H_{q_j} = \frac{E_{el,j}}{2}\sigma_{x,j} - \frac{E_{J,j}}{2}\sigma_{z,j}$ $\rightarrow E_{el} = 4E_{c_j}(1-2n_{g,j})$, $E_{c_j} = \frac{e^2}{2C_{\Sigma_j}}$, $n_{g,j} = \frac{C_{g_j} V_{g_j}}{2e}$

$\rightarrow E_{J,j} = E_J \cdot \cos(\pi \frac{\phi_J}{\phi_0})$

(2) Qubits j coupled to 1 resonator (mode) (no vacuum field)

$$H = \omega_r a^\dagger a + \sum_{j=1}^{N} \frac{\omega_{a,j}}{2}\sigma_{z,j} - \sum_{j=1}^{N} g_j \left(\mathcal{N}_j - \cos(\theta_j)\sigma_{z,j} + \sin(\theta_j)\sigma_{x,j}\right)(a^\dagger + a)$$

with $\omega_{a,j}^2 = E_{J,j}^2 + [4E_{c,j}(1-2n_{g,j})]^2$,

$g_j = e\frac{C_{g,j}}{C_{\Sigma,j}} \frac{V^0_{rms}}{\hbar}$

$\mathcal{N}_j = (1-2n_{g,j})$

$\theta_j = \arctan\left(\frac{E_J}{E_{c,j}(1-2n_{g,j})}\right) = \frac{\pi}{2}$ for $n_{g,j} = 0.5$ degeneracy point.

Classical drive of cavity $\underset{\text{coherent state}}{D(\alpha)} = \exp(\alpha a^\dagger - \alpha^* a)$, 1 qubit coupled to resonator

see last exercise

$\Rightarrow \tilde{\mathcal{H}} = D^\dagger(\alpha) H D(\alpha) - i D^\dagger(\alpha)\dot{D}(\alpha)$ (via transformation of $i\hbar\partial_t\psi = H\psi$)

$$\approx \underset{\text{mode}}{\omega_r a^\dagger a} + \underset{\text{qubit}}{\frac{\omega_a}{2}\sigma_z} - \underset{\text{qubit-mode}}{g(a^\dagger\sigma_- + \sigma_+ a)} - \underset{\text{qubit-drive (via cavity)}}{g(\alpha^*\sigma_- + \alpha\sigma_+)}$$

$\dot{\alpha} = -i\omega_r\alpha + i\varepsilon(t)e^{-i\omega_d t}$

$\Rightarrow$
- $\varepsilon(t) = \text{const}$
- rotating frame ω_d
- $\omega_r - \omega_d \gg 0$

$$\tilde{\mathcal{H}} = \Delta_r a^\dagger a + \frac{\Delta_a}{2}\sigma_z - g(a^\dagger\sigma_- + \sigma_+ a) + \frac{\Omega_R}{2}\sigma_x$$

drive qubit with coherent drive! (off resonant)

$\Delta_r = \omega_r - \omega_d$, $\Delta_a = \omega_a - \omega_d$, $\Omega_R = 2\frac{\varepsilon g}{\Delta_r} \approx 2g\sqrt{\bar{n}}$ ← mean photon number in cavity.

$\bar{n} = \left(\frac{\varepsilon}{\Delta_r}\right)^2$ for $\Delta_r \gg \kappa$

Bit-flip (on resonance but qubit detuned from resonator) developed parameter $\frac{g}{\Delta} \ll 1$

$U = \exp\left(\frac{g}{\Delta}(a^\dagger\sigma_- - a\sigma_+)\right) = \exp(\lambda X) \Rightarrow \underset{H_x}{e^{-\lambda X} H e^{\lambda X}} \underset{\text{Hausdorff}}{=} H + \lambda[H, X] + \frac{\lambda^2}{2}[[H,X],X] + \dots$, $\lambda = \frac{g}{\Delta}$

$$H_x = \Delta_r a^\dagger a + \frac{1}{2}\Big(\overbrace{\Delta_a + 2\chi\left[a^\dagger a + \tfrac{1}{2}\right]}^{\tilde{\Delta}_a}\Big)\sigma_z + \frac{\Omega_R}{2}\sigma_x \approx \Delta_r a^\dagger a + \frac{\tilde{\Delta}_a}{2}\sigma_z + \frac{\Omega_R}{2}\sigma_x$$

Dispersive limit!

$\chi = \frac{g^2}{\Delta}$, $\Delta = \omega_r - \omega_a$

$\langle a^\dagger a\rangle = 0$

$(a^\dagger + a)\sigma_z = 0$

$\tilde{\Delta}_a = 0 \Leftrightarrow$ bit flip operation

$\Rightarrow$ changing $\tilde{\Delta}_a$, Ω_R + phase

$\Rightarrow$ all single qubit rotations possible!

off-resonant case

$$U = \exp(\beta^* a_+ - \beta a_-) \Rightarrow H_2 = U H_1 U^+ \simeq \Delta_r a^\dagger a + \frac{1}{2}\left(\tilde{\Delta}_a + \frac{1}{2}\frac{\Omega_R^2}{\Delta_a}\right)\sigma_z$$

$\frac{\Omega_R}{2\Delta_a}$ drive detuned from qubit

keep constant

Stark shift $\simeq 2g\sqrt{n}\,\frac{\Omega_R}{2\Delta_a}$

make larger $\Rightarrow$ fast rotations $\sim$ ns

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2 qubit gates

dispersive limit

$$U = \exp\left(\sum_{i=1}^{2}\frac{g_i}{\Delta_i}(a^\dagger \sigma_{-,i} - a\sigma_{+,i})\right) \Rightarrow H_{2q} = \omega_r a^\dagger a + \sum_{j=1,2}\frac{\tilde{\omega}_{a,j}}{2}\sigma_{z,j} + \underbrace{\frac{g_1 g_2}{2}\left(\frac{\Delta_1+\Delta_2}{\Delta_1\Delta_2}\right)(\sigma_{+1}\sigma_{-2} + \sigma_{-1}\sigma_{+2})}_{J\cdot J\text{-coupling}}$$

$\langle a^\dagger a\rangle \approx 0$

$\tilde{\omega}_{a,j} = \omega_{a,j} + \chi_j(a^\dagger a + \frac{1}{2})$

$\rightarrow$ qubit 1 couples via g_1 to cavity and cavity via g_2 to qubit 2 $\Rightarrow J = \sqrt{g_1 \cdot g_2}$

$\rightarrow$ actual coupling: $\frac{J^2}{g}$

dispersive $\rightarrow \frac{J^2}{\Delta} = \frac{g_1 g_2}{(\Delta_1^{-1}+\Delta_2^{-1})^{-1}}$

$\Rightarrow$ Bring both qubits into resonance and wait for $\frac{\pi\Delta_1\Delta_2}{2g_1g_2(\Delta_1+\Delta_2)}$ $\Rightarrow \sqrt{i\,SWAP}$ $\leftarrow$ "slow" since $\Delta_i \gg g_i$ ≈ 100ns

$\Rightarrow$ Universal set w. single qubit gates